Review of Economic Dynamics 18 (2015) 843–862

Contents lists available at ScienceDirect

Review of Economic Dynamics

www.elsevier.com/locate/red

Politico-economic equivalence ☆

Martín Gonzalez-Eiras [a,b], Dirk Niepelt [c,d,*]

[a] University of Copenhagen, Denmark [1]
[b] CONICET, Argentina
[c] Study Center Gerzensee, Switzerland [2]
[d] University of Bern, Switzerland

ARTICLE INFO

Article history:
Received 5 September 2014
Received in revised form 4 August 2015
Available online 20 August 2015

JEL classification:
E62
H55
H63

Keywords:
Equivalence
Politico-economic equilibrium
Tax policy
Government debt
Social security reform

ABSTRACT

Traditional "economic equivalence" results, like the Ricardian equivalence proposition, define equivalence classes over exogenous policies. We derive "politico-economic equivalence" conditions that apply in environments where policy is endogenous and chosen sequentially. A policy regime and a state are equivalent to another such pair if both pairs give rise to the same allocation in politico-economic equilibrium. The equivalence conditions help to identify factors that render institutional change non-neutral and to construct politico-economic equilibria in new policy regimes. We exemplify their use in the context of several applications, relating to social security reform, tax-smoothing policies and measures to correct externalities.

1. Introduction

Important neutrality results in public economics, macroeconomics and other fields establish classes of "economically equivalent" policies that give rise to the same equilibrium allocation (conditional on initial states). For example, in a simple model of household choice, policies relying on different combinations of consumption, capital-income and labor-income taxes form equivalence classes, and in the standard overlapping-generations model, pay-as-you-go social security policies are economically equivalent to certain policies relying on taxes and explicit government debt.

While proving very useful in a variety of contexts, these neutrality results only apply when policy is exogenous. In politico-economic models or theories with a Ramsey planner, policy constitutes an equilibrium outcome and the primitives of the analysis include policy regimes rather than policies. These regimes constrain the choice sets of political decision makers and are reflected in admissibility restrictions that limit the available policy instruments.[3]

☆ We thank the editor (Marco Bassetto), two anonymous referees, John Hassler, Roland Hodler, Enrique Kawamura, Per Krusell, Andrés Neumeyer, Torsten Persson, Laura Sunder-Plassmann, Jaume Ventura as well as seminar and conference participants at EPRU (University of Copenhagen), ESEM, ESSIM, IIES (Stockholm University), LAMES, SED, Universidad de San Andrés and Universidad Torcuato di Tella for helpful comments.

* Corresponding author.
E-mail addresses: mge@alum.mit.edu (M. Gonzalez-Eiras), dirk.niepelt@szgerzensee.ch (D. Niepelt).
[1] Øster Farimagsgade 5, DK-1553 Copenhagen, Denmark.
[2] P.O. Box 21, CH-3115 Gerzensee, Switzerland.

[3] Policy regimes are distinct from political institutions. The latter determine the aggregation of preferences in the political process.

http://dx.doi.org/10.1016/j.red.2015.08.001
1094-2025/

This raises the question how equivalence of policy regimes can be defined, and what the conditions for such equivalence are. Answers to this question are of direct relevance for important policy debates. Consider for example the proposal to "privatize" social security and debt finance the transition. From a narrow economic point of view, moving from a pay-as-you-go financed social security regime to a regime with taxes and explicit government debt could be deemed irrelevant because specific pay-as-you-go and debt policies are economically equivalent. From a politico-economic point of view, however, such a regime change might be expected to result in a different equilibrium allocation. In fact, the disagreement about the merits of social security reform suggests that many policy makers hold exactly that view.

In this paper, we propose answers to the question posed above. We define a policy regime and state to be "politico-economically equivalent" to another such pair if both pairs give rise to politico-economic equilibria and the same equilibrium allocation. We derive conditions under which politico-economic equivalence follows. And we apply the conditions to environments with taxes, public debt or corrective policy measures. To the best of our knowledge, the notion of politico-economic equivalence is novel in the literature. It differs from the standard economic equivalence notion—with Ricardian equivalence as a special case—because it conceptualizes equivalence classes of *policy regimes* rather than *individual policies*.[4]

Our results are derived within a general dynamic framework comprising a household sector, firms and a government. We do not impose restrictions on the commitment power of political decision makers. As a consequence, the results hold in environments with either sequential policy choice or policy choice once and for all, as for example with Ramsey policies. Nor do we impose restrictions on political objective functions except that these functions must not directly depend on regime specific policy instruments. Our results therefore hold under a variety of assumptions about the political process or the structure of government, political parties and interest groups.

In a first step, we define economic equivalence of exogenous policies (conditional on states) and we extend well-known economic neutrality propositions (e.g., Barro, 1974; Sargent, 1987; Rangel, 1997; Coleman, 2000; Ghiglino and Shell, 2000; Bassetto and Kocherlakota, 2004) to derive a general economic equivalence result. In the second step, we define politico-economic equivalence of policy regimes (conditional on states) and derive sufficient conditions for politico-economic equivalence. In parallel to the economic equivalence result which emphasizes the implications of exogenous policy for the choice sets of households and firms, the politico-economic equivalence result emphasizes the consequences of policy regimes for the choice sets of political decision makers. These choice sets are constrained fourfold: By the state; the admissibility restrictions on policy instruments under the control of political decision makers; the continuation policy functions of subsequent political decision makers; and the requirement that policies implement a competitive equilibrium. Accordingly, our conditions for politico-economic equivalence relate to state spaces and policy spaces.

Intuitively, these conditions guarantee that the choice set of political decision makers in the "new" policy regime is sufficiently large but not too large relative to the choice set in the "initial" regime: The equilibrium allocation in the initial regime must also be implementable in the new regime, but political decision makers in the new regime must not be able to implement allocations on or off the equilibrium path that cannot be implemented in the initial regime. If these requirements are satisfied then revealed preference implies politico-economic equivalence.

To bring this intuitive argument to bear it is necessary to render the state spaces comparable across policy regimes while allowing for regime specific policy instruments and commitment structures. To that end, we define a relation between states across regimes: A state in the initial regime is *strongly associated* with a state in the new regime if for every admissible and feasible sequence of policy instruments in the initial regime there exists an admissible sequence of policy instruments in the new regime such that the two pairs of states and sequences are economically equivalent.

The first equivalence condition requires that the initial states under both regimes are associated. The choice set of political decision makers in the new regime then is sufficiently large and the first intuitive criterion is met. The second equivalence condition stipulates that for every possible state in the new regime there exists a unique strongly associated state in the initial regime. The set of equilibrium allocations that can be implemented in the new regime (conditional on a state) then is weakly smaller than the set that can be implemented in the initial regime (conditional on the strongly associated state). Accordingly, the second intuitive criterion is met as well and politico-economic equivalence follows.

When state spaces are strongly associated (as they are in some examples considered in the applications section) politico-economic equivalence follows independently of the equilibrium policy functions in the initial regime. This has two important consequences. First, *robust* politico-economic equivalence for arbitrary political aggregation mechanisms is guaranteed. And second, equivalence can be established even if no information about the equilibrium in the initial regime is available and without having to first characterize that equilibrium. Both these features prove very useful in applied work.

For situations where strong association is too strong a condition to impose, we provide an alternative equivalence result that relaxes the second condition but requires information about the policy functions in the initial regime instead. The weaker condition builds on the notion of association rather than strong association of state spaces: A state in the initial regime is *associated* with a state in the new regime if the latter and a sequence of admissible policy instruments in the new regime are economically equivalent to the former and the *equilibrium* sequence of policy instruments in the initial regime.

The second equivalence condition now requires that for every state in the new regime there exists a unique associated state in the initial regime. If this requirement is met an equivalent continuation policy function can be defined; it maps

[4] It also differs from Correia et al.'s (2008) notion of equivalent *economic environments* with different frictions (with identical optimal allocations) and from Chari et al.'s (2007) notion of equivalent *frictions* (giving rise to identical wedges).

a state under the new regime into the sequence of policy instruments that is economically equivalent to the equilibrium continuation policy sequence in the initial regime. An additional condition formalizes the requirement that the choice set of political decision makers in the new regime is not too large: Any allocation that can be implemented in the new regime, subject to future policy choices being determined according to the equivalent continuation policy function, must also be implementable in the initial regime. Politico-economic equivalence then follows.

The politico-economic equivalence results serve several purposes. On the one hand, they constitute a useful tool for researchers interested in characterizing politico-economic equilibria. When high dimensional state and policy spaces render such a task difficult, the equivalence results can help by relating the unknown equilibrium of interest to another equilibrium in a "simpler model" that is easier to characterize. We illustrate this in an example where the equilibrium in the "simpler model" is also unknown to start with but can easily be derived from an existing model in the literature. On the other hand, the results help to identify factors that render institutional change non-neutral. We demonstrate this in the context of several applications, relating to social security reform, tax-smoothing policies and measures to correct externalities.

Regarding social security reform, we ask whether economic equivalence of certain pay-as-you-go social security policies and debt policies extends to the political sphere. We contrast existing politico-economic models of social security (Cooley and Soares, 1999; Tabellini, 2000; Boldrin and Rustichini, 2000; Forni, 2005; Gonzalez-Eiras and Niepelt, 2008) with novel models in which political decision makers may issue debt and choose the repayment rate on maturing debt.[5] We show that certain politico-economic theories of social security that have been proposed in the literature may be re-interpreted as novel politico-economic theories of government debt, and our analysis therefore contributes to a small but growing literature on debt in politico-economic equilibrium (e.g., Battaglini and Coate, 2008; Díaz-Giménez et al., 2008; Yared, 2010; Song et al., 2012; Niepelt, 2014). Other theories cannot be re-interpreted in that light. We identify (sufficiently rich) heterogeneity among households and differential tightness of admissibility restrictions across policy regimes as factors undermining politico-economic equivalence, and we argue that those factors can help rationalize why interest groups might favor or oppose the privatization of social security.

Regarding tax smoothing policies, we start from Bassetto and Kocherlakota's (2004) observation that the timing of distorting tax collections may be allocation neutral if taxes can be levied on contemporaneous and lagged incomes.[6] We find that this economic equivalence result extends to the political sphere if policy is chosen once and for all. With sequential policy choice, in contrast, a policy regime allowing for the taxation of current and lagged incomes generally is not politico-economically equivalent to a regime allowing for the taxation of contemporaneous income only. Intuitively, when policy is chosen sequentially the choice set of political decision makers in the former regime is larger than in the latter because the elasticity of the tax base with respect to ex-post taxation equals zero.

Our final application concerning measures to correct externalities compares policy regimes with Pigovian taxes and cap-and-trade restrictions. We find that the two policy regimes are politico-economically equivalent, provided that permits in the cap-and-trade regime can be allocated in a way that replicates the distributive implications of the Pigovian tax and transfer scheme.

The remainder of the paper is structured as follows. Section 2 offers a simple example that introduces central concepts of the analysis and conveys the basic intuition. Section 3 lays out the model and discusses economic equivalence. Section 4 contains the paper's main result on politico-economic equivalence. Section 5 discusses applications and Section 6 concludes. Appendix A contains formal definitions and proofs.

2. A simple example

To motivate the analysis in the following sections, we start with a simple example of a two period economy with an OLG structure. Time is indexed by $t = 0, 1$. The economy is inhabited by workers and retirees. Workers in period $t = 0$ are retirees in period $t = 1$; workers born in the second period and retirees die at the end of the period. The ratio of workers to retirees equals $\nu > 0$. Workers supply labor inelastically, pay taxes, consume and save. Retirees consume the return on their savings as well as resources they might receive from government. Firms use labor supplied by workers and physical capital owned by retirees to produce the output good.

We compare a social security regime and a debt regime. In the social security regime, political decision makers may levy a non-negative social security tax τ_t on labor income and distribute the proceeds among retirees. In the debt regime, political decision makers may levy a tax τ'_t on labor income, issue debt b'_{t+1} and use the proceeds to repay outstanding debt b'_t (held by retirees) at the non-negative repayment rate z'_t. (Throughout the paper, we differentiate regimes by denoting variables in the second regime by a "prime.") Since the repayment rate on debt, and thus its equilibrium price, is endogenous we can without loss of generality fix the face value of debt per retiree at an exogenous level, $\bar{b}'_t > 0$.

Economic equivalence relations link pairs of states and policy sequences that give rise to the same competitive equilibrium allocation. In the example, the capital stock k is the single endogenous state variable. The state k_0 and a social security policy sequence (τ_0, τ_1) are economically equivalent to the state k'_0 and a debt policy sequence $(\tau'_0, z'_0, \tau'_1, z'_1)$ if $k'_0 = k_0$

[5] Cukierman and Meltzer (1989) analyze a model with commitment to social security taxes and debt. We discuss their findings in the applications section.

[6] Bassetto and Kocherlakota (2004) extend Barro's (1974) equivalence result to environments with distorting taxes.

(identical production possibilities); $\bar{b}'_0 z'_0 = \tau_0 w_0 \nu$ and $\bar{b}'_1 z'_1 = \tau_1 w_1 \nu$ (identical payments from the government to retirees at initial equilibrium prices); $\tau'_0 = \tau_0 - \frac{\tau_1 w_1 \nu}{R_1 w_0}$ (identical life time tax burdens for workers born in the first period at initial equilibrium prices); and $\tau'_1 = \tau_1$ (identical life time tax burdens for workers born in the second period). Here, R_t and w_t denote gross interest rates and wages in the social security equilibrium, respectively.

Political decision makers cannot commit and policy therefore is chosen sequentially, subject to *admissibility restrictions* that define the instruments political decision makers may use (labor income taxes in either regime and the debt repayment rate in the debt regime) as well as numerical restrictions on those instruments ($\tau_t \geq 0$ and $z'_t \geq 0$ since lump sum taxes on retirees are ruled out), and subject to the constraint that their policy choices be *feasible* that is, implement a competitive equilibrium.[7] Political decision makers in the first period also must take the continuation policy function of their successors into account. A politico-economic equilibrium in the social security regime conditional on initial capital stock k_0 is given by policy functions $\tau_0(\cdot), \tau_1(\cdot)$ and a competitive equilibrium with capital stock k_1 such that k_0, $\tau_0^\star = \tau_0(k_0)$ and $\tau_1^\star = \tau_1(k_1)$ implement this competitive equilibrium and the policy choices are optimal. A politico-economic equilibrium in the debt regime is defined similarly, with the policy functions given by $\tau'_0(\cdot), z'_0(\cdot), \tau'_1(\cdot), z'_0(\cdot)$.

Suppose that conditional on k_0 the social security regime gives rise to a politico-economic equilibrium with sequentially chosen equilibrium policy sequence $(\tau_0^\star, \tau_1^\star)$. We want to assess *politico-economic equivalence* of the social security and debt regimes. More specifically, we want to assess whether (conditional on an initial capital stock k_0) the debt regime gives rise to a politico-economic equilibrium with equilibrium policy sequence $(\tau_0'^\star, z_0'^\star, \tau_1'^\star, z_1'^\star)$ such that the pairs $(k_0, (\tau_0^\star, \tau_1^\star))$ and $(k_0, (\tau_0'^\star, z_0'^\star, \tau_1'^\star, z_1'^\star))$ are economically equivalent.

A brute force approach to checking this relies on solving for the politico-economic equilibrium in the debt regime. An alternative approach compares the choice sets of political decision makers. Consider the situation in period $t = 1$. Since any equilibrium social security policy choice $\tau_1^\star$ is non-negative the economically equivalent debt policy choice as implied by the economic equivalence relations, namely $(\tau'_1, z'_1) = (\tau_1^\star, \frac{\tau_1^\star w_1(k_1)\nu}{\bar{b}'_1})$, necessarily is admissible.[8] Moreover, for any admissible and feasible debt policy choice (τ'_1, z'_1) there exists an admissible social security policy choice, $\tau_1 = \tau'_1$, such that (k_1, τ_1) and $(k_1, (\tau'_1, z'_1))$ are economically equivalent. Conditional on a given capital stock in the second period the admissibility restrictions in the debt regime thus are sufficiently loose for the equilibrium allocation in the social security regime to be implementable in the debt regime as well; and they are sufficiently tight that any allocation that can be implemented in the debt regime can also be implemented in the social security regime. If political objectives only relate to the allocation, the debt policy implementing the allocation implemented by the equilibrium social security policy must be an equilibrium outcome in the debt regime as well. Politico-economic equivalence in the second period follows.

Proceeding to the first period, the brute force characterization of the debt equilibrium would have to deal with the fact that the equilibrium price of debt reflects expectations about the equilibrium repayment choice $z_1'^\star$, workers are indifferent between accumulating capital or buying debt, and government spending is financed out of taxes and funds raised from newly issued debt. Focusing on the choice sets of political decision makers again simplifies the task although continuation policy functions now have to be taken into account as well. From the perspective of political decision makers in the debt regime, these continuation policy functions are given by *equivalent continuation policy functions* $\tilde{\tau}'_1(\cdot)$ and $\tilde{z}'_1(\cdot)$ that implement the same competitive equilibrium allocation as the equilibrium policy choice in the social security regime.[9] This follows directly from the finding above that the equilibrium debt policy choice in the second period is economically equivalent to the equilibrium policy choice in the social security regime.

Since the continuation policy functions in the debt regime, $\tilde{\tau}'_1(\cdot), \tilde{z}'_1(\cdot)$, are equivalent to the continuation policy function in the social security regime, $\tau_1(\cdot)$, the allocation implemented by k_0, $\tau_1(\cdot)$ and the social security tax rate $\tau_0^\star$ can also be implemented by k_0, $\tilde{\tau}'_1(\cdot), \tilde{z}'_1(\cdot)$ and a debt policy choice $(\tau_0'^\star, z_0'^\star)$. This follows from the economic equivalence relations when the continuation policy functions are evaluated at $k_1^\star$. In fact, this debt policy choice $(\tau_0'^\star, z_0'^\star)$ is admissible.[10] The economic equivalence relations also imply that any allocation that can be implemented by k_0, $\tilde{\tau}'_1(\cdot), \tilde{z}'_1(\cdot)$ and some admissible debt policy choice (τ'_0, z'_0) can equally be implemented by k_0, the continuation policy function $\tau_1(\cdot)$ and an admissible social security policy choice τ_0.[11] Conditional on the same initial capital stock, the equilibrium allocation in the social security regime therefore can also be implemented in the debt regime, and any allocation that can be implemented in the debt regime can also be implemented in the social security regime. Politico-economic equivalence therefore follows.

This simple example illustrates the general logic behind the politico-economic equivalence conditions derived in Section 4. In line with the above reasoning, these conditions require that the choice sets of political decision makers in the new regime (the debt regime in the example) are sufficiently large, but not too large. To compare these choice sets, we rely on economic equivalence relations. A complication we have to address in Section 4 relates to state spaces. In the example considered here, both regimes feature the same endogenous state—the capital stock—but this need not be the case in

[7] The general analysis also applies to settings where political decision makers can commit, for example because a Ramsey planner chooses policy.

[8] We denote by $w_t(k_t)$ and $R_t(k_t)$ the equilibrium wage and gross interest rate, respectively, as a function of the capital stock.

[9] The equivalent continuation policy functions are given by $\tilde{\tau}'_1(\cdot) = \tau_1(\cdot)$ and $\tilde{z}'_1(\cdot) = \tau_1(\cdot) w_1(\cdot)\nu/\bar{b}'_1$. Continuation policy functions determine policy choices over the complete continuation history. In the example here, this history only comprises the second period.

[10] It is given by $(\tau_0'^\star, z_0'^\star) = (\tau_0^\star - \frac{\tau_1(k_1^\star) w_1(k_1^\star)\nu}{R_1(k_1^\star) w_0(k_0)}, \tau_0^\star w_0(k_0)\nu/\bar{b}'_0)$. Note that $\tau_0^\star \geq 0$ implies $z_0'^\star \geq 0$.

[11] The policy choice is given by $\tau_0 = \frac{z'_0(k_0)\bar{b}'_0}{w_0(k_0)\nu}$. Note that $z'_0 \geq 0$ implies $\tau_0 \geq 0$.

general. As a prerequisite for the definition of equivalent continuation policy functions we therefore have to impose certain requirements on state spaces and the policy space in the new regime.

3. Economic equivalence

We consider a deterministic, discrete-time economy with time indexed by $t = 0, 1, \ldots$.[12] The economy is inhabited by a government, households, and firms. The latter are owned by households, close down at the end of a period and reopen at the beginning of the next. Both households and firms may be heterogeneous. Let $\mathcal{I}$ denote the set of households and firms, and let $\mathcal{N}$ denote the set of household and firm types. The set $\mathcal{I}$ can be partitioned into subsets of households or firms of identical type; each such subset contains a continuum of homogeneous households or firms.

Central objects in our analysis are states, policy sequences and competitive equilibrium allocations. Let $\mu_t \in \mathcal{M}_t$ denote the *state* in period t.[13] It may encompass both economic and political restrictions. Examples of the former type include physical or human capital stocks; household choices in previous periods if current or future taxes are functions of those; or financial asset holdings. Examples of political state variables include policy instruments chosen in the past by political decision makers that could commit. For instance, under (partial) commitment, the contemporaneous income tax schedule or the repayment rate on maturing government debt may be part of the state. We denote by μ_t^i the state of a household or firm $i \in \mathcal{I}$.

Let $p^{t-1} \in \mathcal{P}^{t-1}$ denote the *policy sequence* from period t onward (that is, after period $t-1$). Absent commitment on the part of political decision makers, p^{t-1} could for example include contemporaneous and future income tax schedules or the repayment rate on government debt maturing in period t.

A policy sequence p^{t-1} is *feasible* conditional on μ_t if the best responses by households and firms to the pair (μ_t, p^{t-1}) and to each others' best responses satisfy the resource constraints in the economy. In this case, the pair (μ_t, p^{t-1}) is said to implement a *competitive equilibrium allocation* as of period t, with the timing being first, that a feasible (conditional on the state) policy sequence from period t onward is fixed and second, that households and firms choose their best responses. A formal definition that follows Stokey (1991) is given in Appendix A.1. We denote a competitive equilibrium allocation as of period t (or a set thereof) that is implemented by a pair of state and feasible policy sequence, by $\text{CE}(\mu_t, p^{t-1})$. Competitive equilibrium prices, denoted by q_t, directly follow from the marginal rates of substitution and transformation that are implied by the equilibrium allocation. A *competitive equilibrium* as of period t consists of a competitive equilibrium allocation as of period t as well as the corresponding equilibrium prices, q_t.

The competitive equilibrium allocation as of period t, $\text{CE}(\mu_t, p^{t-1})$, implies μ_{t+1}. This next-period state and the policy sequence p^t (that is, the policy sequence p^{t-1} without its period-t component) implement a competitive equilibrium allocation as of period $t+1$, $\text{CE}(\mu_{t+1}, p^t)$. The latter constitutes the continuation competitive equilibrium allocation of $\text{CE}(\mu_t, p^{t-1})$.

The objective of a firm in period t is to maximize profits. The utility function of a household $i \in \mathcal{I}$ of type $n \in \mathcal{N}$ as of period t is described by the function Ω_t^n (see Appendix A.1 for a formal definition). We do not need to impose any substantive restrictions on Ω_t^n except that we have to rule out the possibility that preferences directly depend on regime-specific policy instruments. But to simplify the discussion we proceed as if preferences only depended on the allocation. We also assume that the household's objective is time-consistent.[14]

Equivalence classes in economic models relate policy sequences (and, at least implicitly, states) that implement the same equilibrium allocation. For example, when taxes are not distorting and other conditions are satisfied, the Ricardian equivalence proposition defines classes of tax policies with constant present discounted tax revenue that implement the same equilibrium allocations. We refer to the equivalence of policy sequences (and states) as *economic equivalence*, as defined below. The definition allows for the possibility that a state and policy sequence implement multiple equilibrium allocations.

Definition 1. A state and policy sequence, (μ_t, p^{t-1}), are *economically equivalent* to another state and policy sequence, (μ'_t, p'^{t-1}), if

i. p^{t-1} is feasible conditional on μ_t;
ii. p'^{t-1} is feasible conditional on μ'_t;
iii. $\text{CE}(\mu_t, p^{t-1}) = \text{CE}(\mu'_t, p'^{t-1})$.

The direct approach to establishing economic equivalence of (μ_t, p^{t-1}) and (μ'_t, p'^{t-1}) relies on characterizing the competitive equilibrium allocations for each pair (if they exist) and comparing them. An indirect approach relies on a comparison of households' and firms' choice sets, as formalized in Proposition 1 below. The choice set of a household encompasses all

[12] The extension to the stochastic case is immediate if the number of states in each period is finite. Our analysis applies both to finite and infinite horizons. We distinguish between the two cases where necessary.

[13] The set $\mathcal{M}_t$ and other sets introduced in this section are defined later. For now, they can be treated as exogenous.

[14] Alternatively, we could assume that households can commit in their role as economic agents even if they may not be able to commit as political actors.

restrictions imposed by the dynamic and intertemporal budget constraints as well as other constraints, for instance the consumption set or quotas instituted by policy. The choice set of a firm is defined by its production function and, potentially, restrictions imposed by policy on the level of inputs and/or outputs.

Proposition 1. *Consider a state and policy sequence, (μ_t, p^{t-1}), that implement a competitive equilibrium allocation as of period t (or a set thereof),* $\mathrm{CE}(\mu_t, p^{t-1})$*, with corresponding prices q_t. Consider a new state and policy sequence, (μ'_t, p'^{t-1}), that satisfy the following conditions:*

i. *state variables that determine production possibilities are identical across μ_t and μ'_t;*
ii. *restrictions on inputs and/or outputs of firms are identical across p^{t-1} and p'^{t-1};*
iii. *households' choice sets are identical for each type $n \in \mathcal{N}$;*
iv. *at the equilibrium allocation and prices (or at each allocation and prices in the set of equilibrium allocations and prices), (μ'_t, p'^{t-1}) satisfy the government budget constraints.*

Then, (μ'_t, p'^{t-1}) are economically equivalent to (μ_t, p^{t-1}).

A formal statement of the proposition and its proof are contained in Appendix A.2. The intuition for the proof is straightforward: With choice sets unchanged and preferences independent of policy, equilibrium choices are unaltered and these choices continue to be feasible. The pair (μ'_t, p'^{t-1}) therefore implements the same competitive equilibrium allocation, $\mathrm{CE}(\mu_t, p^{t-1})$.

Proposition 1 provides sufficient conditions for economic equivalence.[15] We emphasize the result because the strategy of comparing choice sets rather than directly characterizing equilibria mirrors the strategy we adopt below to establish politico-economic equivalence.

4. Politico-economic equivalence

As discussed in the introduction, policy regimes rather than policy sequences constitute primitives of the analysis when policy is endogenous. In models with an endogenous policy choice, it therefore seems reasonable to consider equivalence classes of policy regimes (and states). In this section, we define equivalence of policy regimes and derive sufficient conditions for it.

Our analysis applies to settings where policy is chosen once and for all (as, for example, when a benevolent government chooses a Ramsey policy) and to environments with sequential policy choice. To keep the notation simple, we present the case where policy choices are made in every period. The case with full commitment or intermediate cases require slight adjustments in the definition of politico-economic equilibrium and the politico-economic equivalence conditions. Specifically, the equivalence conditions then only need to be satisfied in periods when policy choices actually are made.

In period t, political decision makers are confronted with state μ_t and opt for a *policy choice* p_t. The private sector then learns about this policy choice and anticipates the *continuation policy choice* p^t. Jointly, the policy choice and continuation policy choice form the policy sequence $p^{t-1} = (p_t, p^t)$. If this policy sequence is feasible conditional on μ_t, then a competitive equilibrium with allocation $\mathrm{CE}(\mu_t, p^{t-1})$ results.

Let $\mathcal{P}_t$ denote the set of *admissible* policy choices p_t. The restrictions embedded in $\mathcal{P}_t$ specify the policy instruments under the control of political decision makers in period t as well as restrictions on the numerical values of those instruments. A *policy regime* is defined by $\mathcal{P} \equiv \prod_{t\geq 0} \mathcal{P}_t$ and the set of admissible continuation policy choices in period t is denoted by $\mathcal{P}^t \equiv \prod_{s\geq t+1} \mathcal{P}_s$.

Recall that an admissible continuation policy choice $p^t \in \mathcal{P}^t$ is feasible conditional on μ_{t+1} if the pair implements a competitive equilibrium allocation. Let $\mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$ denote the set of admissible and feasible continuation policy choices conditional on μ_{t+1}. An admissible policy choice $p_t \in \mathcal{P}_t$ is feasible conditional on μ_t if there exists an admissible continuation policy $p^t \in \mathcal{P}^t$ such that $p^{t-1} = (p_t, p^t)$ is feasible conditional on μ_t. Let $\mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$ denote the set of admissible and feasible policy choices conditional on μ_t. Every admissible and feasible continuation policy choice at time 0, $p^{-1} = (p_0, p^0) \in \mathcal{P}^{-1}(\mu_0)$, and the allocation it implements correspond with a sequence of the state, $\{\mu_t\}_{t\geq 0}$. Let $\mathcal{M}_t(\mu_0, \mathcal{P})$ denote the set of values that the state may take in period t across all such admissible and feasible continuation policy choices if the state in the initial period is μ_0. Similarly, let $\mathcal{M}_t(\mathcal{M}_0, \mathcal{P})$ denote the set of values that the state may take in period t across all admissible and feasible continuation policy choices if the state in the initial period lies in the set $\mathcal{M}_0$. For ease of notation, we will often suppress the arguments of $\mathcal{M}_t$ if this does not create confusion.

Sequential decision making implies that policy choices in period t are functions of the state with the *policy function* $p_t(\cdot)$ mapping $\mathcal{M}_t$ into $\bigcup_{\mu_t \in \mathcal{M}_t} \mathcal{P}_t(\mu_t) \subseteq \mathcal{P}_t$. Similarly, a *continuation policy function* $p^t(\cdot)$ is a mapping from $\mathcal{M}_{t+1}$ into $\bigcup_{\mu_{t+1} \in \mathcal{M}_{t+1}} \mathcal{P}^t(\mu_{t+1}) \subseteq \mathcal{P}^t$.

[15] It summarizes and extends (economic) equivalence results in the literature, e.g., Barro (1974), Sargent (1987, Ch. 8), Bassetto and Kocherlakota (2004), Rangel (1997), Niepelt (2005).

The time-consistency requirement encapsulated in the continuation policy function implies a restriction on the policy function. To see this, consider an admissible and feasible policy choice, $p_t \in \mathcal{P}_t(\mu_t)$. While by definition, there exists an admissible continuation policy $p^t \in \mathcal{P}^t$ such that (p_t, p^t) is feasible conditional on μ_t, this continuation policy may not be consistent with the continuation policy function, that is, (p_t, p^t) may not be time-consistent. The choice set of political decision makers conditional on state μ_t therefore may be strictly smaller than $\mathcal{P}_t(\mu_t)$. We denote this choice set by $\mathcal{P}_t(\mu_t; p^t(\cdot)) \subseteq \mathcal{P}_t(\mu_t)$: the set of admissible policy choices $p_t \in \mathcal{P}_t$ such that there exists an admissible continuation policy $p^t \in \mathcal{P}^t$ that is consistent with the continuation policy function $p^t(\cdot)$ and such that $p^{t-1} = (p_t, p^t)$ is feasible conditional on μ_t. Appendix A.3 contains a formal definition.

The law of motion $g_s(\cdot)$ describes the evolution of the state in equilibrium as a function of the policy choice $p_s \in \mathcal{P}_s(\mu_s; p^s(\cdot))$:

$$\mu_{s+1} = g_s(\mu_s, p_s; p^s(\cdot)), s \geq t. \tag{1}$$

The law of motion is parametrized by the continuation policy function because this function maps the next-period state into policy which in turn affects current policy and private sector choices, which in turn affect next period's state. That is, the fixed point implicit in the law of motion is shaped by the continuation policy function; see Appendix A.3 for a formal definition.

As discussed in the previous section, household preferences are functions of the competitive equilibrium allocation. Since an allocation is a function of the state and current and future policy, $\text{CE}(\mu_t, p^{t-1})$, household preferences induce indirect preferences over policy. In the political process, these indirect preferences are aggregated into an objective function, $\Omega_t(\text{CE}(\mu_t, (p_t, p^t(\mu_{t+1}))); \mu_t)$ where μ_{t+1} satisfies (1). Through its second argument, we allow the objective function also to directly depend on the state—exclusive of political state variables.[16] For example, demographic shocks might affect the aggregation of preferences in the political process. Our specification of the objective function is consistent with standard approaches to modeling the political process. In particular, it captures any theory where political decision makers maximize a weighted sum of utility functions as long as the weights can be represented as functions of the state. This is the case, for example, under the assumption of probabilistic voting or a decisive median voter. Under mild restrictions it is also the case under the assumption of legislative bargaining.[17]

We are now ready to define politico-economic equilibrium.

Definition 2. A *politico-economic equilibrium* as of period t conditional on μ_t and $\mathcal{P}$, denoted by $\text{PEE}(\mu_t, \mathcal{P})$, consists of a sequence of policy functions, $\{p_s(\cdot)\}_{s\geq t}$; a sequence of continuation policy functions, $\{p^s(\cdot)\}_{s\geq t-1}$; a sequence of laws of motion, $\{g_s(\cdot)\}_{s\geq t}$; policy choices, $p^{\star t-1}$; and a competitive equilibrium allocation $\text{CE}(\mu_t, p^{\star t-1})$ such that[18]

i. policy functions are optimal subject to continuation policy functions:

$$p_s(\mu_s) \in \arg\max_{p_s \in \mathcal{P}_s(\mu_s; p^s(\cdot))} \Omega_s(\text{CE}(\mu_s, (p_s, p^s(g_s[\mu_s, p_s; p^s(\cdot)])))) \text{ for all } \mu_s \in \mathcal{M}_s(\mu_t, \mathcal{P}), s \geq t;$$

ii. continuation policy functions are consistent with policy functions:

$$p^{s-1}(\mu_s) = (p_s(\mu_s), p^s(g_s[\mu_s, p_s(\mu_s); p^s(\cdot)])) \text{ for all } \mu_s \in \mathcal{M}_s(\mu_t, \mathcal{P}), s \geq t;$$

iii. laws of motion are consistent with the policy and continuation policy functions according to (1);

iv. equilibrium policy choices are generated by the continuation policy function,

$$p^{\star t-1} = p^{t-1}(\mu_t),$$

and $(\mu_t, p^{\star t-1})$ implements the competitive equilibrium allocation $\text{CE}(\mu_t, p^{\star t-1})$.

The definition of politico-economic equilibrium allows policy functions to depend on time. In environments with an infinite horizon and a recursive, time-autonomous structure the policy and continuation policy functions may be time-autonomous functions of the state as well, $\psi(\cdot)$ and $\Psi(\cdot)$ say. The consistency requirement in part ii. of the above definition then reads

$$\Psi(\mu_s) = (\psi(\mu_s), \psi[g(\mu_s, \psi(\mu_s); \Psi(\cdot))], \psi\{g(g(\mu_s, \psi(\mu_s); \Psi(\cdot)), \psi[g(\mu_s, \psi(\mu_s); \Psi(\cdot))]; \Psi(\cdot))\}, \ldots)$$

where the law of motion is modified to $\mu_{s+1} = g(\mu_s, p_s; \Psi(\cdot))$. Clearly, the function $\psi(\cdot)$ is sufficient for $\Psi(\cdot)$ in that case; the law of motion therefore can be re-expressed as $\mu_{s+1} = g(\mu_s, p_s; \psi(\cdot))$; and conditions i. and ii. of the definition of politico-economic equilibrium can be combined to the fixed point requirement[19]

[16] The objective function $\Omega_t(\cdot)$ is the one assumed by Krusell et al. (1997) subject to the restriction that it does not depend on policy instruments.

[17] With legislative bargaining, the political program maximizes the utility of the agenda setter subject to the constraint of securing a minimum winning coalition. With transferable utility, this is equivalent to maximizing the utility of a subset of legislators (see, for example, Battaglini and Coate, 2007).

[18] To simplify the notation, we drop the second argument of $\Omega_s(\cdot)$.

[19] To avoid confusion, we keep time subscripts.

$$\psi(\mu_s) \in \arg \max_{p_s \in \mathcal{P}_s(\mu_s; p^s(\cdot))} \Omega_s(\text{CE}(\mu_s, (p_s, p^s)))$$
$$\text{s.t. } p^s = (\psi[g(\mu_s, p_s; \psi(\cdot))], \psi\{g(g(\mu_s, p_s; \psi(\cdot)), \psi[g(\mu_s, p_s; \psi(\cdot))]; \psi(\cdot))\}, \ldots)$$

for all $\mu_s \in \mathcal{M}_s(\mu_t, \mathcal{P})$ and $s \geq t$.

Returning to the motivating question, consider an "initial" policy regime $\mathcal{P}$ with associated politico-economic equilibrium PEE$(\mu_0, \mathcal{P})$, and a "new" policy regime $\mathcal{P}'$. We are interested in conditions that guarantee politico-economic equivalence, defined as follows:

Definition 3. A state and policy regime, $(\mu_t, \mathcal{P})$, are *politico-economically equivalent* to another state and policy regime, $(\mu'_t, \mathcal{P}')$, if

i. $(\mu_t, \mathcal{P})$ implements a politico-economic equilibrium PEE$(\mu_t, \mathcal{P})$ with policy choices $p^{\star t-1}$;
ii. $(\mu'_t, \mathcal{P}')$ implements a politico-economic equilibrium PEE$(\mu'_t, \mathcal{P}')$ with policy choices $p'^{\star t-1}$;
iii. for each sequence of policy choices $p^{\star t-1}$ in i. there exists a sequence of policy choices $p'^{\star t-1}$ in ii. such that $(\mu_t, p^{\star t-1})$ is economically equivalent to $(\mu'_t, p'^{\star t-1})$, and vice versa.

Note that politico-economic equivalence is defined with respect to pairs of a state and policy regime whereas economic equivalence is defined with respect to pairs of a state and policy sequence. This reflects the fact that the primitives of competitive equilibrium on the one hand and politico-economic equilibrium on the other differ. Note also that Definition 3 allows for multiplicity of politico-economic equilibria. Such multiplicity may arise for a unique equilibrium policy choice and equilibrium continuation policy function if the pair implements multiple sequences of the state with different associated equilibrium allocations; or it may arise if the equilibrium policy function and equilibrium continuation policy function themselves are not unique. Importantly, multiplicity of equilibrium allocations conditional on an exogenous policy sequence (as allowed for in Definition 1) need not imply multiplicity of politico-economic equilibrium. When different allocations imply different sequences of the state a given policy function typically maps those different sequences into different policy choices, undermining the possibility of multiple politico-economic equilibria.

A sufficient condition for politico-economic equivalence is that the choice set of political decision makers in the new regime satisfies two requirements. On the one hand, it must be sufficiently large in the sense that political decision makers in the new regime can implement those competitive equilibrium allocations that political decision makers in the initial regime find optimal to implement. On the other hand, the choice set in the new regime must not be too large: Political decision makers in the new regime must not be able to implement competitive equilibrium allocations on or off the equilibrium path that cannot be implemented in the initial regime. If both requirements are satisfied, then revealed preference and the regime independent political objective function imply that political decision makers in the new regime choose policies that implement the same competitive equilibrium allocation as in the initial regime.

We start by deriving an equivalence result that relies on a strong version of these intuitive conditions. It requires that any admissible and feasible policy sequence under the initial regime—not only equilibrium policy sequences—can effectively be replicated under the new regime, and vice versa. The equality of choice sets then follows directly, and it follows for arbitrary objective functions since no assumptions about the equilibrium policy functions are imposed. Subsequently, we relax this strong condition and derive a more general equivalence result that applies more broadly but requires information about the equilibrium policy functions in the initial regime. Since the choice sets of political decision makers (represented by $\mathcal{P}_s(\mu_s; p^s(\cdot))$) are constrained by the state; admissibility restrictions; continuation policy functions; and the competitive equilibrium requirement, the equivalence conditions relate to state spaces and policy spaces as well.

To state the condition for the first equivalence result we need to define a relation between states under the different regimes:

Definition 4. For a state μ'_t under the policy regime $\mathcal{P}'$, a state μ_t under the policy regime $\mathcal{P}$ is a *strongly associated state* if for every $p^{t-1} \in \mathcal{P}^{t-1}(\mu_t)$ there exists a $p'^{t-1} \in \mathcal{P}'^{t-1}$ such that (μ_t, p^{t-1}) is economically equivalent to (μ'_t, p'^{t-1}).

Definition 4 serves as the basis for the following condition. It requires that strong association generates a one-to-one mapping between the two state spaces:

Condition 1. The following holds true for all t:

i. For every $\mu_t \in \mathcal{M}_t(\mathcal{M}_0, \mathcal{P})$ there exists a unique, strongly associated $\mu'_t \in \mathcal{M}'_t(\mathcal{M}'_0, \mathcal{P}')$;
ii. for every $\mu'_t \in \mathcal{M}'_t(\mathcal{M}'_0, \mathcal{P}')$ there exists a unique, strongly associated $\mu_t \in \mathcal{M}_t(\mathcal{M}_0, \mathcal{P})$;
iii. μ_t is strongly associated with μ'_t if and only if μ'_t is strongly associated with μ_t.

Moreover, the initial state in the initial policy regime, μ_0, is strongly associated with the initial state in the new policy regime, μ'_0.

Condition 1 implies that one can construct policy and continuation policy functions in the new regime that effectively replicate the equilibrium outcomes in the initial regime. Moreover, these functions reflect the equilibrium behavior in the new regime since the choice sets of political decision makers effectively are identical across regimes if Condition 1 is met. Politico-economic equivalence therefore follows.

Proposition 2. *Consider a state and policy regime, $(\mu_0, \mathcal{P})$, that implement a politico-economic equilibrium* PEE$(\mu_0, \mathcal{P})$. *Consider a new state and policy regime, $(\mu'_0, \mathcal{P}')$, and suppose that Condition 1 is satisfied. Then, $(\mu_0, \mathcal{P})$ is politico-economically equivalent to $(\mu'_0, \mathcal{P}')$.*

Proof. See Appendix A.4. □

As mentioned earlier, Proposition 2 also applies in the special case where policy choices are made once and for all, as for example when a benevolent government chooses a Ramsey policy. In this special case, $\mathcal{P}_0 = \mathcal{P}$ and political decision makers choose a policy sequence $p^{\star -1}$ that together with the initial state, implements a competitive equilibrium allocation maximizing their objective function. Since choices are made once and for all, Condition 1 only requires that for every feasible p^{-1}, there exists an admissible p'^{-1} such that (μ_0, p^{-1}) is economically equivalent to (μ'_0, p'^{-1}), and vice versa.

Proposition 2 can be extended to deal with environments with trigger strategies. This requires two modifications. First, appropriate state variables need to be added. Second, for a given trigger strategy in the initial regime, a corresponding trigger strategy in the new regime must be specified in such a way that the policies prescribed by the strategies under the two regimes (together with the states) are economically equivalent, see Gonzalez-Eiras and Niepelt (2012). Proposition 2 can also be extended to models where political restrictions take the form of incentive compatibility constraints in a planner's problem.[20]

Condition 1 does not impose any restrictions on the equilibrium policy functions in the initial regime nor does it require any information about them. This has two important consequences. First, Proposition 2 can be used to establish politico-economic equivalence even if the politico-economic equilibria in the regimes under consideration have not been characterized. When it is easier to verify Condition 1 than to characterize equilibrium, Proposition 2 thus substantially simplifies a researcher's task.

Second, if Condition 1 is met, Proposition 2 can be used to establish politico-economic equivalence for arbitrary political objective functions (satisfying the maintained assumptions) that is, *robust politico-economic equivalence*. The robustness result is a direct consequence of the fact that Condition 1 requires *any* implementable equilibrium allocation in the initial regime also to be implementable in the new regime, not only a particular equilibrium allocation.

This very feature points to a possibility to relax Condition 1. When information about the equilibrium policy functions in the initial regime is available then Condition 1 may be weakened by requiring that only *equilibrium* allocations in the initial regime can also be implemented in the new regime (and similarly vice versa). In the following, we formalize this intuition. We start by defining a weaker relation than strong association between states:

Definition 5. For a state μ'_t under the policy regime $\mathcal{P}'$, a state μ_t under the policy regime $\mathcal{P}$ is an *associated state* if there exists a $p'^{t-1} \in \mathcal{P}'^{t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to (μ'_t, p'^{t-1}).

Condition 2. The following holds true for all t: For every $\mu'_t \in \mathcal{M}'_t(\mathcal{M}'_0, \mathcal{P}')$, there exists a unique associated $\mu_t \in \mathcal{M}_t(\mathcal{M}_0, \mathcal{P})$.[21]

If Condition 2 is satisfied then we can define an *equivalent continuation policy function* $\tilde{p}'^{t-1}(\cdot)$ that maps the state μ'_t into a continuation policy choice $\tilde{p}'^{t-1}(\mu'_t) \in \mathcal{P}'^{t-1}$ such that $(\mu_t, p^{t-1}(\mu_t))$ is economically equivalent to $(\mu'_t, \tilde{p}'^{t-1}(\mu'_t))$ where μ_t is associated with μ'_t. Similarly, we can define an *equivalent policy function* $\tilde{p}'_t(\cdot)$ that maps the state μ'_t into a policy choice $\tilde{p}'_t(\mu'_t) \in \mathcal{P}'_t$ that corresponds to the time-t component of $\tilde{p}'^{t-1}(\mu'_t)$. Both functions have domain $\mathcal{M}'_t(\mathcal{M}'_0, \mathcal{P}')$.[22]

Condition 3 requires that the initial states under both regimes are associated. This implies that under full commitment, the equilibrium allocation in the initial regime can also be implemented in the new regime and in that sense, the choice set of political decision makers in the new regime is sufficiently large.

Condition 3. The initial state in the initial policy regime, μ_0, is associated with the initial state, μ'_0.

Finally, Condition 4 formalizes the requirement that the choice set in the new regime not be too large. It stipulates that, conditional on associated states and subject to future policy choices in the new regime being determined according

[20] For a recent example of such a model, see Farhi et al. (2012).

[21] The uniqueness requirement in Condition 2 can be relaxed. Multiple states $\mu_t, \hat{\mu}_t$ may be associated with μ'_t as long as CE$(\mu_t, p^{t-1}(\mu_t)) =$ CE$(\hat{\mu}_t, p^{t-1}(\hat{\mu}_t))$.

[22] If policy instruments in the new policy regime are redundant then the equivalent continuation policy function and the equivalent policy function generally are correspondences rather than functions. To keep notation simple, we disregard this possibility in what follows.

to the equivalent continuation policy function, any allocation that can be implemented in the new regime must also be implementable in the initial regime. To simplify notation, we write $\mu_{t+1}(p_t)$ instead of $g_t(\mu_t, p_t; p^t(\cdot))$, leaving the current state and the continuation policy function implicit. Similarly, we write $\mu'_{t+1}(p'_t)$ for $g_t(\mu'_t, p'_t; \tilde{p}'^t(\cdot))$.

Condition 4. The following holds true for all $\mu'_t \in \mathcal{M}'_t(\mathcal{M}'_0, \mathcal{P}')$ and all t: Let $\mu_t \in \mathcal{M}_t(\mathcal{M}_0, \mathcal{P})$ be associated with μ'_t. If there exists a $p'_t \in \mathcal{P}'_t$ such that μ'_t, p'_t and $\tilde{p}'^t(\cdot)$ implement $\mathrm{CE}(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1}(p'_t))))$, then there exists a $p_t \in \mathcal{P}_t$ such that $(\mu_t, (p_t, p^t(\mu_{t+1}(p_t))))$ is economically equivalent to $(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1}(p'_t))))$.

We can now state the modified politico-economic equivalence result:

Proposition 3. *Consider a state and policy regime, $(\mu_0, \mathcal{P})$, that implement a politico-economic equilibrium* $\mathrm{PEE}(\mu_0, \mathcal{P})$. *Consider a new state and policy regime, $(\mu'_0, \mathcal{P}')$, and suppose that Conditions 2–4 are satisfied. Then, $(\mu_0, \mathcal{P})$ are politico-economically equivalent to $(\mu'_0, \mathcal{P}')$.*

Proof. See Appendix A.5. □

While Condition 3 clearly is a necessary condition, Conditions 2 and 4 are sufficient for politico-economic equivalence but not necessary. Failure of Condition 2 implies that equivalent continuation policy functions cannot be defined for all $\mu'_t \in \mathcal{M}'_t$. While our strategy to prove equivalence cannot be pursued in this case, equivalence nevertheless may hold. Failure of Condition 4 implies that some allocations may only be implementable in the new regime such that the choice set of political decision makers in the new regime is not a subset of the choice set in the initial regime. Equivalence may still hold since the *equilibrium* allocation in the new regime may be implementable in the initial regime as well. Since Condition 1 implies Conditions 2 and 4, it is clear that Condition 1 is violated as soon as Condition 2 or 4 fail to hold.

In parallel to Proposition 2, Proposition 3 applies in the special case where policy choices are made once and for all, as for example when a benevolent government chooses a Ramsey policy. Condition 2 can be dropped in this case since the relevant part of it is implied by Condition 3. Moreover, if choices are made once and for all, Condition 4 only requires that for every feasible p'^{-1}, there exists an admissible p^{-1} such that (μ_0, p^{-1}) is economically equivalent to (μ'_0, p'^{-1}). Also in parallel to Proposition 2, Proposition 3 can be extended to deal with environments with trigger strategies or incentive compatibility constraints in a planner's problem.

We will now turn to applications and demonstrate how the theoretical apparatus can be put to work. Depending on circumstances, we will use either Proposition 2 or Proposition 3 to establish politico-economic equivalence.

5. Applications

We consider three applications, relating to social security reform, tax-smoothing policies and measures to correct externalities. Unless otherwise noted, we let w_t, l_t and l^i_t denote the wage, labor supply of the representative worker, and labor supply of worker i in period t, respectively; $r_{t,s}$ the inverse of the gross interest rate between periods t and $s, s \geq t$; k_t the capital stock per worker; and ν_t the ratio of workers to retirees.

The applications illustrate three different strategies of using the equivalence results. The first and second strategy rely on straightforward implementations of Proposition 2 or 3. If the conditions of Proposition 2 are satisfied which is easy to check, then robust politico-economic equivalence follows for arbitrary political aggregation mechanisms. If the conditions of Proposition 3 are satisfied which can only be established on a case by case basis for each policy regime, then politico-economic equivalence follows for the specific political aggregation mechanism underlying the initial politico-economic equilibrium.

The third strategy relies on a modified use of Proposition 3. In some applications the admissibility restrictions in an initial regime $\mathcal{P}$ are "tighter" than in a given new regime $\mathcal{P}'$ such that Conditions 2 and 3 are satisfied but Condition 4 is not, rendering a straightforward implementation of Proposition 3 impossible. But in such a situation, it might still be possible to exploit Proposition 3 to characterize the politico-economic equilibrium in the new regime. The approach follows three steps:

First, the admissibility restrictions in the initial policy regime are relaxed to $\hat{\mathcal{P}} \supset \mathcal{P}$ say. Second, the politico-economic equilibrium subject to the relaxed admissibility restrictions, $\mathrm{PEE}(\mu_0, \hat{\mathcal{P}})$, is characterized. This is trivial if the equilibrium policy functions in $\mathrm{PEE}(\mu_0, \hat{\mathcal{P}})$ can be deduced from those in $\mathrm{PEE}(\mu_0, \mathcal{P})$ in a straightforward manner. Finally, the equivalence conditions are verified that is, in particular, one checks whether the admissibility restrictions were relaxed appropriately. With the conditions met, Proposition 3 can be used to directly characterize the equilibrium in the new policy regime, $\mathrm{PEE}(\mu'_0, \mathcal{P}')$. Since the second step often is straightforward, this approach constitutes an attractive alternative to characterizing $\mathrm{PEE}(\mu'_0, \mathcal{P}')$ from scratch. We demonstrate this approach in Subsection 5.1.2 by means of a simple example.

5.1. Social security reform

As noted in the introduction and the introductory example in Section 2, pay-as-you-go financed social security policies are economically equivalent to certain debt-and-tax policies. At the same time, disagreement among political decision makers about the merits of social security reform suggests that "privatizing" and "pre-funding" social security may result in a change of allocation in politico-economic equilibrium. Our first application examines this apparent contradiction in more detail.

The analysis identifies a basic economic environment—the workhorse overlapping generations model with minimal household heterogeneity and non-distorting taxes—that robustly generates politico-economic equivalence, for arbitrary political aggregation mechanisms. In this basic environment, social security reform always is allocation neutral and therefore politically uncontentious. This is no longer the case if more realistic extensions of the basic environment are considered.

In the first extension with elastic labor supply and distorting taxes, the natural admissibility restrictions are asymmetrically tight across policy regimes. As a consequence, Condition 3 or 4 may be violated, depending on the political aggregation mechanism in place, and thus Condition 1 fails to hold. Politico-economic equivalence therefore may or may not hold. However, a politico-economic equilibrium in the debt regime may still be characterized using the third strategy sketched above (rather than constructing the equilibrium from scratch).

In the second extension with richer household heterogeneity in combination with lack of commitment, the debt ownership structure constitutes a non-trivial state variable although the repayment rate on the debt is chosen ex post. Different debt ownership structures imply different sets of wealth distributions and allocations that the government can implement. In the social security regime, in contrast, the government's policy instruments do not provide such flexibility. As a consequence, Conditions 2 and 4 do not hold and the politico-economic equivalence result is undermined.

5.1.1. Basic environment

We extend the setup described in Section 2 to an infinite horizon. The economy is inhabited by two-period lived overlapping generations that are homogeneous within cohorts. Workers inelastically supply labor, $l_t = 1$, and accumulate capital; production is neoclassical.

In the social security regime, a proportional labor income tax τ_t funds transfers to retirees, $\nu_t w_t \tau_t$. The admissibility restrictions $\mathcal{P}_t = \{\tau_t | \tau_t \in \mathbb{R}_+\}$ rule out transfers from retirees to workers. In the debt regime, the government repays maturing debt at the rate z'_t, issues an exogenous stock of short-term debt $\bar{b}'_{t+1} > 0$ per retiree and levies a proportional labor income tax τ'_t.[23] The admissibility restrictions $\mathcal{P}'_t = \{(\tau'_t, z'_t)|(\tau'_t, z'_t) \in \mathbb{R} \times \mathbb{R}_+\}$ again rule out transfers from retirees to workers.[24]

In both the social security and the debt regime, the state includes the capital stock and the exogenous demographic shock. In addition, the state in the debt regime also includes the exogenous value $\bar{b}'_t$. For notational convenience, we suppress the exogenous state variables in the following. In the basic environment, we thus have $\mu_t = k_t$ and $\mu'_t = k'_t$.

Economic equivalence requires identical initial capital stocks, identical government cash flows in each period, and identical present values of net tax payments for each cohort. These cross-regime restrictions reduce to

$$\begin{aligned}
k'_t &= k_t, \\
z'_s &= \tau_s \nu_s w_s / \bar{b}'_s \quad \text{for all } s \geq t, \\
\tau'_s &= \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1} \nu_{s+1} w_{s+1}}{w_s} \quad \text{for all } s \geq t.
\end{aligned}$$

To assess politico-economic equivalence consider first the state spaces $\mathcal{M}_t$ and $\mathcal{M}'_t$. Suppose that the capital stock in the initial period may take any positive value, $\mathcal{M}_0 = \mathcal{M}'_0 = [0, \infty)$. The set of states in period t that can be attained by feasible policies then ranges from zero (the capital stock subject to confiscatory taxation) to a maximum value, $\bar{k}_t(k_0)$ and $\bar{k}'_t(k'_0)$. Since the maximum values result in the absence of any taxation (assuming that consumption is a normal good) we have $\bar{k}_t(k_0) = \bar{k}'_t(k'_0)$ as long as $k_0 = k'_0$. As a consequence, $\max_{k_0} \bar{k}_t(k_0) = \max_{k'_0} \bar{k}'_t(k'_0)$, and $\mathcal{M}_t(\mathcal{M}_0, \mathcal{P}) = \mathcal{M}'_t(\mathcal{M}'_0, \mathcal{P}')$.

For any state $k_t \in \mathcal{M}_t$ in the social security regime a strongly associated state under the debt regime is given by $k'_t = k_t$ because for any $(\tau'^{t-1}, z'^{t-1}) \in \mathcal{P}'^{t-1}(k'_t)$ there exists a $\tau^{t-1} \in \mathcal{P}^{t-1}$ such that (k_t, τ^{t-1}) and $(k'_t, (\tau'^{t-1}, z'^{t-1}))$ are economically equivalent when $k'_t = k_t$. In fact, k'_t is the unique strongly associated state. The reverse statement holds as well: For any state $k'_t \in \mathcal{M}'_t$ in the debt regime the unique, strongly associated state under the social security regime is given by $k_t = k'_t$. Condition 1 therefore is satisfied. We conclude from Proposition 2 that politico-economic equivalence is guaranteed as long as $k'_0 = k_0$, and in fact it is guaranteed for *any* political aggregation mechanism. Moreover, politico-economic equivalence also robustly holds with symmetric commitment.[25]

[23] The quantity of debt can be normalized without loss of generality. To see this, note that a competitive equilibrium pins down the market value of newly issued debt as well as total debt repayment. Multiplying debt prices and repayment rates by a constant and dividing the total stock of debt by the same constant therefore does not affect the competitive equilibrium conditions. Moreover, adopting the proposed normalization does not constrain the effective choice set of political decision makers—with $\bar{b}_t > 0$, the amount of resources transferred to bond holders can fully be controlled by the choice of repayment rate—nor does it constrain the ownership structure of government debt and thus, the relative exposure of different groups to public debt.

[24] To streamline notation, we do not distinguish between debt repayment in periods $t \geq 1$ and "debt repayment" to retirees in the initial period who simply receive a transfer.

[25] With one-period, symmetric commitment the state in the social security regime is given by $\mu_t = (k_t, \tau_t)$ and in the debt regime by $\mu'_t = (k'_t, z'_t)$. The economic equivalence relations continue to hold, with the exception that τ_t and z'_t are part of the respective states rather than the continuation policy sequences from period $t-1$ onwards. With this qualification, and as long as $k'_0 = k_0$ and $z'_0 = \tau_0 \nu_0 w_0 / \bar{b}'_0$, all arguments from the case without commitment extend.

Forni (2005) analyzes the baseline setup under the assumption that a median voter is politically decisive. He shows that, for some parameter constellations, multiple equilibria with self-fulfilling expectations may exist in which strictly positive social security tax rates are sustained. Contemporaneous political decision makers support strictly positive taxes if they expect future social security benefits to be a decreasing function of the capital stock.[26] From the above discussion, we immediately conclude that the social security regime in Forni's (2005) model is politico-economically equivalent (conditional on some initial capital stock) to a debt regime.

Boldrin and Rustichini (2000) analyze the baseline setup augmented by a trigger strategy under the assumption that a young median voter is politically decisive. They assume that political decision makers are confronted with a "suggested" social security tax rate as determined by their predecessors, and that political decision makers choose an updated suggestion for their successors in addition to the actual social security tax rate. Equilibrium policy choices depend on whether the policy choice in the preceding period conformed with the suggestion or not. Boldrin and Rustichini (2000) show that this trigger strategy provides sufficiently strong incentives for political decision makers to support equilibria with strictly positive social security transfers.

As already mentioned, Proposition 2 (and accordingly, Proposition 3) extends to environments with trigger strategies if a corresponding trigger strategy in the new regime can be specified. In Boldrin and Rustichini's (2000) environment, such a corresponding trigger strategy could be based on a comparison of suggested and actually chosen debt repayment rates. Conditional on this corresponding trigger strategy and appropriate initial states, the social security regime in Boldrin and Rustichini's (2000) model therefore is politico-economically equivalent to a debt regime.

5.1.2. Elastic labor supply and distorting taxes

With elastic labor supply, economic equivalence also requires that marginal tax rates be identical across regimes; under the assumption of proportional labor income taxes, this warrants a second tax instrument. Let θ_t and θ'_t denote a second proportional tax in the social security and debt regime, respectively, whose proceeds are refunded lump sum to workers. Ruling out lump-sum taxes on workers implies non-negative values for these tax rates, and ruling out lump-sum taxes on retirees implies non-negative social security benefits or debt repayment rates. The admissibility restrictions in the two regimes therefore are given by $\mathcal{P}_t = \{(\tau_t, \theta_t) | (\tau_t, \theta_t) \in \mathbb{R}^2_+\}$ and $\mathcal{P}'_t = \{(\tau'_t, \theta'_t, z'_t) | (\tau'_t, \theta'_t, z'_t) \in \mathbb{R} \times \mathbb{R}^2_+\}$. The state is $\mu_t = k_t$ in the social security regime and $\mu'_t = k'_t$ in the debt regime. Economic equivalence requires

$$\left.\begin{aligned} k'_t &= k_t, \\ z'_s &= \tau_s \nu_s w_s l_s / \bar{b}'_s \quad \text{for all } s \geq t, \\ \tau'_s &= \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1} \nu_{s+1} w_{s+1} l_{s+1}}{w_s l_s} \quad \text{for all } s \geq t, \\ \theta'_s &= \theta_s + \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1} \nu_{s+1} w_{s+1} l_{s+1}}{w_s l_s} \quad \text{for all } s \geq t \end{aligned}\right\} \tag{2}$$

such that marginal tax rates are identical across regimes, $\tau'_s + \theta'_s = \tau_s + \theta_s$.

Following a parallel reasoning as in the discussion of the baseline environment, we conclude that $\mathcal{M}_t(\mathcal{M}_0, \mathcal{P}) = \mathcal{M}'_t(\mathcal{M}'_0, \mathcal{P}')$. Moreover, (2) implies for $k_t = k'_t$ that for every admissible policy sequence $\tau_s, \theta_s \geq 0$ in the social security regime (and thus, for the equilibrium policy sequence under any political aggregation mechanism), the equivalent policy sequence in the debt regime is admissible as well. When the initial regime is the social security regime, Condition 2 therefore is satisfied and Condition 3 is satisfied as well if $k_0 = k'_0$. However, Condition 4 may be violated and Condition 1 therefore does not hold: There exist admissible and feasible policy sequences in the debt regime such that the equivalent policy sequences in the social security regime are not admissible. In particular, if $z'_{t+1} > 0$, then one feasible policy choice p'_t involves zero contemporaneous taxes, $\tau'_t + \theta'_t = 0$, but strictly positive debt repayment, $z'_t > 0$, which can be financed out of new debt issues. The economically equivalent policy in the social security regime, which satisfies $\theta_t = \tau'_t + \theta'_t - z'_t \bar{b}'_t / (\nu_t w'_t l'_t) < 0$, is not admissible in this case. Since Condition 1 is violated, politico-economic equivalence must be assessed on a case by case basis.

Gonzalez-Eiras and Niepelt (2008) analyze a social security regime in the model with endogenous labor supply under the assumption that preferences are aggregated through probabilistic voting. They show that strictly positive social security transfers are sustained in politico-economic equilibrium. In a debt regime, these transfers correspond with a positive debt repayment rate. Since this implies a violation of Condition 4, as detailed above, politico-economic equivalence cannot be guaranteed. In fact, politico-economic equivalence fails: Gonzalez-Eiras and Niepelt (2008) find that the equilibrium tax rate θ_t in the social security regime sometimes is in a corner. Relaxing the non-negativity constraint $\theta_t \geq 0$ therefore would result in a different equilibrium allocation.

[26] Forni (2005) considers the case where the capital stock evolves within a certain range of parameter-dependent values. See Gonzalez-Eiras (2011) for a general characterization of equilibrium.

But the opposite conclusion follows in the limit of the finite horizon economy under the assumption that a young median voter is politically decisive. Equilibrium social security transfers then equal zero in all periods and as a consequence, politico-economic equivalence is guaranteed under this alternative political aggregation mechanism.[27]

Gonzalez-Eiras and Niepelt's (2008) model with probabilistic voting can also be used to demonstrate how the third strategy sketched at the beginning of Section 5, based on a modified use of Proposition 3, may be exploited to characterize the politico-economic equilibrium in the debt regime although Condition 4 does not hold under the original admissibility restrictions. In the first step, we relax the admissibility restrictions in the social security regime to $\hat{\mathcal{P}} = \{(\tau_t, \theta_t) | (\tau_t, \theta_t) \in \mathbb{R}_+ \times \mathbb{R}\}$, i.e. we allow the θ instrument to take negative values. Second, we deduce the policy functions in the equilibrium subject to the relaxed admissibility restrictions; these functions are nearly identical to the policy functions in the initial equilibrium.[28] Finally, we verify the equivalence conditions.

To verify Condition 2, we need to show that conditional on $k_t = k'_t$ the policy sequence in the debt regime that replicates the allocation under the equilibrium policy in the relaxed social security regime, conforms with the admissibility restrictions $\theta'_s, z'_s \geq 0$ for all $s \geq t$. From (2), this can only be guaranteed if the equilibrium sequence $\hat{\theta}_s$ in the relaxed social security regime is not "too negative" for all $s \geq t$. For the demographic shocks considered by Gonzalez-Eiras and Niepelt (2008) this is the case.[29] Conditional on $k_0 = k'_0$, Condition 3 is satisfied as well. Finally, Condition 4 (which was violated when considering the original social security regime $\mathcal{P}$) now is satisfied as well, due to the relaxation to $\hat{\mathcal{P}}$, since all admissible and feasible one-period deviations in the debt regime can be replicated in the relaxed social security regime. We have thus characterized the politico-economic equilibrium in the debt regime without constructing the equilibrium from scratch and although the conditions for Propositions 2 and 3 were not met.

5.1.3. Richer household heterogeneity

If household heterogeneity is reflected in a non-trivial debt ownership structure, and absent commitment to the repayment rate, politico-economic equivalence generally cannot be guaranteed. To see this, consider an environment with debt where households within a cohort are non-representative or households live for more than two periods. The debt ownership structure then is endogenous (in contrast to a setup with homogeneous, two-period lived households) and without commitment, it constitutes a state variable that determines the extent to which a change in the repayment rate affects the wealth distribution. The set of implementable policies in the debt regime then varies with an endogenous state variable that is not present in the social security regime. Evidently, this discrepancy would tend to undermine Condition 4. More fundamentally, it undermines Condition 2 as we now demonstrate in the context of a minimal extension of the baseline setup.[30]

The admissibility restrictions are as in Section 5.1.1. In the social security regime a state $\mu_t = \{a^i_t\}_{i \in \mathcal{I}_t}$ is given by the cross section of private asset holdings.[31] In the debt regime the state $\mu'_t = \{a^{i'}_t, b^{i'}_t\}_{i \in \mathcal{I}_t}$ includes the cross section of private and public debt holdings (which must satisfy $\int_{i \in \mathcal{I}_t} b^{i'}_t \, di = \bar{b}'_t$). Economic equivalence requires identical initial capital stocks, identical initial wealth levels of retirees, identical government cash flows in every period, and identical present values of net tax payments for each young household in each cohort:

$$\begin{aligned}
\int_{i \in \mathcal{I}_t} a^i_t \, di &= \int_{i \in \mathcal{I}_t} a^{i'}_t \, di, \\
a^{i'}_t + b^{i'}_t z'_t &= a^i_t + \nu_t \tau_t w_t \quad \text{for all } i \in \mathcal{I}_t \cap \mathcal{I}_{t-1}, \\
z'_s &= \tau_s \nu_s w_s / \bar{b}'_s \quad \text{for all } s \geq t, \\
\tau'_s &= \tau_s - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1} \nu_{s+1} w_{s+1}}{w_s} \quad \text{for all } s \geq t.
\end{aligned}$$

To see how an endogenous, non-trivial debt ownership structure undermines Condition 2 in the absence of commitment, consider two distinct states in the initial regime, μ_t and $\hat{\mu}_t$, that involve the same capital stock but give rise to different

[27] A continuation policy sequence in the social security regime with tax rates equal to zero is economically equivalent to a continuation policy sequence in the debt regime with repayment rates of zero. But if future debt repayment rates equal zero, any feasible policy in the debt regime must finance contemporaneous debt repayment out of current taxes, $z'_t = \tau'_t \nu_t w'_t l'_t / \bar{b}'_t$. The admissibility restriction $z'_t \geq 0$ then implies $\tau'_t \geq 0$. As a consequence, the economically equivalent policy choice in the social security regime satisfies $\tau_t = \tau'_t \geq 0$ and $\theta_t = \theta'_t \geq 0$ which satisfies all admissibility restrictions.

[28] Subject to the original admissibility restrictions $\mathcal{P}$, Gonzalez-Eiras and Niepelt (2008) derive the policy functions $\tau_t(\mu_t) = \max[\omega^1_t, 0]$, $\theta_t(\mu_t) = \max[\omega^2_t, 0]$, where ω^1_t and ω^2_t denote known functions of exogenous state variables (not of the capital stock). Subject to the relaxed admissibility restrictions $\hat{\mathcal{P}}$, the policy functions are given by $\hat{\tau}_t(\mu_t) = \max[\omega^1_t, 0]$, $\hat{\theta}_t(\mu_t) = \omega^2_t$ as is straightforward to show.

[29] More precisely, Condition 2 is satisfied when the state space of exogenous demographic shocks is restricted to only include values for these shocks that are actually realized during the simulation period.

[30] For simplicity, we abstract from fundamental heterogeneity within a cohort. If households differed with respect to their labor productivity a social security system with proportional tax rates and lump sum benefits would redistribute within cohorts. Economic equivalence then would require additional tax instruments in the debt regime.

[31] The state μ_t does not separately include the capital stock since the latter equals aggregate private asset holdings.

competitive equilibria (in particular $\tau_t(\mu_t) \neq \tau_t(\hat{\mu}_t)$).[32] We show that a state μ'_t in the new regime can then be constructed such that μ'_t is associated both with μ_t and $\hat{\mu}_t$. This is the case if

$$\left.\begin{aligned} a_t^{i'} + b_t^{i'} z'_t &= a_t^i + \nu_t \tau_t(\mu_t) w_t \ \text{ for all } i \in \mathcal{I}_t \cap \mathcal{I}_{t-1}, \\ z'_s &= \tau_s(\mu_t) \nu_s w_s / \bar{b}'_s \ \text{ for all } s \geq t, \\ \tau'_s &= \tau_s(\mu_t) - \frac{r_{t,s+1}}{r_{t,s}} \frac{\tau_{s+1}(\mu_t) \nu_{s+1} w_{s+1}}{w_s} \ \text{ for all } s \geq t \end{aligned}\right\} \tag{3}$$

and

$$\left.\begin{aligned} a_t^{i'} + b_t^{i'} \hat{z}'_t &= \hat{a}_t^i + \nu_t \tau_t(\hat{\mu}_t) w_t \ \text{ for all } i \in \mathcal{I}_t \cap \mathcal{I}_{t-1}, \\ \hat{z}'_s &= \tau_s(\hat{\mu}_t) \nu_s \hat{w}_s / \bar{b}'_s \ \text{ for all } s \geq t, \\ \hat{\tau}'_s &= \tau_s(\hat{\mu}_t) - \frac{\hat{r}_{t,s+1}}{\hat{r}_{t,s}} \frac{\tau_{s+1}(\hat{\mu}_t) \nu_{s+1} \hat{w}_{s+1}}{\hat{w}_s} \ \text{ for all } s \geq t. \end{aligned}\right\} \tag{4}$$

Here, $\tau_s(\mu_t)$ denotes the period-s element of the policy sequence $p^{t-1}(\mu_t)$ and we use the fact that $w_t = \hat{w}_t$ because of identical capital stocks under μ_t and $\hat{\mu}_t$.

The policy sequences p'^{t-1} and $\hat{p}'^{t-1}$ in (3)–(4) are admissible because the tax rates implemented by the continuation policies $p^{t-1}(\mu_t)$ and $p^{t-1}(\hat{\mu}_t)$ are positive. Moreover, a state μ'_t satisfying (3)–(4) exists and is characterized by the conditions

$$a_t^{i'} = \frac{a_t^i \tau_t(\hat{\mu}_t) - \hat{a}_t^i \tau_t(\mu_t)}{\tau_t(\hat{\mu}_t) - \tau_t(\mu_t)} \ \text{ for all } i \in \mathcal{I}_t \cap \mathcal{I}_{t-1},$$

$$\frac{b_t^{i'}}{\bar{b}_t} = 1 + \frac{\hat{a}_t^i - a_t^i}{\nu_t w_t (\tau_t(\hat{\mu}_t) - \tau_t(\mu_t))} \ \text{ for all } i \in \mathcal{I}_t \cap \mathcal{I}_{t-1}.$$

(Note that $\int_{i \in \mathcal{I}_t} a_t^i \, di = \int_{i \in \mathcal{I}_t} a_t^{i'} \, di$.) The state μ'_t therefore is associated both with μ_t and $\hat{\mu}_t$, and Condition 2 is violated. This negative result extends to settings with representative households that live for more than two periods.

The possibility of an endogenous, non-trivial debt ownership structure arises in the environment considered by Tabellini (2000) who analyzes a two-period lived overlapping generations economy with heterogeneous time endowments among young households.[33] It also arises in the environment considered by Cooley and Soares (1999) who analyze a four-period lived overlapping generations economy.[34] To satisfy Condition 2 in these environments and possibly guarantee equivalence, debt holdings could be restricted to be symmetric across retirees (in the former model) or targeted to workers in their last period before retirement (in the latter). But even if debt could be issued in accordance with these restrictions, secondary markets could easily compromise those efforts.[35]

With one-period commitment to the repayment and tax rates, debt holdings are not an element of the state (in addition to households' financial assets) and richer household heterogeneity does not undermine the equivalence result. As argued in Section 5.1.1 Condition 1 holds and politico-economic equivalence is robust. Cukierman and Meltzer (1989) analyze a model with social security and debt where political decision makers can commit to policy instruments one period in advance. They argue that voters are indifferent between social security and debt policies.[36]

[32] This requires that the distribution of private asset holdings affects the equilibrium policy in the social security regime, a natural outcome when the wealth distribution enters $\Omega(\cdot)$.

[33] Tabellini (2000) shows that, in a median voter framework with weak intergenerational altruism, a coalition of poor young and old households may sustain a social security system whose size increases with the degree of inequality, but decreases with the rate of population growth. Tabellini (2000) assumes proportional taxes and lump-sum benefits such that lifetime taxes of a household are an affine function of income during young age. Replicating households' budget sets in a debt regime without old-age benefits thus requires an affine tax function.

[34] Cooley and Soares (1999) assume that the median voter in the initial period chooses a tax rate that serves as time-invariant suggested social security tax rate in all subsequent periods whereas successive median voters only choose between implementing the proposed tax rate or dismantling the social security system forever. Numerically solving a calibrated version of their model, Cooley and Soares (1999) find that the median voter is of age two and sustains positive intergenerational transfers.

[35] In a different setting where the repayment rate on debt can vary across investors, Broner et al. (2010) show that debt may be reallocated on secondary markets to the politically most influential investors.

[36] The fact that heterogeneity undermines politico-economic equivalence is used in other contexts to render politico-economic equilibria determinate. See, for example, Bassetto (2008).

5.2. Tax smoothing policies

When taxes distort incentives, their timing affects the equilibrium allocation. An important economic equivalence result of Bassetto and Kocherlakota (2004) states that, nevertheless, policies that differ with respect to the timing (but not the present value) of distorting tax collections can be economically equivalent. It is natural to ask whether this economic equivalence result extends to the political sphere. We find that this is only the case for regimes with commitment, as for example in the case of Ramsey policies.

Bassetto and Kocherlakota (2004) show that variations in the timing of distorting tax collections need not alter the equilibrium allocation if taxes on lagged labor income are admissible. Consider for example the case where labor income in period t might either be taxed at rate $\tau_{t,t}$ in period t or both at rate $\tau'_{t,t}$ in period t and at rate $\tau'_{t,t+1}$ in period $t+1$. If $\tau_{t,t} = \tau'_{t,t} + r_{t,t+1}\tau'_{t,t+1}$, switching from the former to the latter tax policy changes the timing of tax collections and the level of debt but does not alter effective marginal or average tax rates on period t labor income. A policy change of this kind therefore preserves households' budget sets and the equilibrium allocation. In general, economic equivalence requires

$$\left.\begin{array}{l} \tau_{s,s} = \tau'_{s,s} + \dfrac{r_{t,s+1}}{r_{t,s}}\tau'_{s,s+1} \\ z_t b^i_t = z'_t b^{i'}_t - \tau'_{t-1,t} l^{i'}_{t-1} \\ z_s \text{ satisfies government DBC} \end{array}\right\} \text{for all } i \in \mathcal{I}_s,\ s \geq t.$$

Consider now the implications for politico-economic equivalence. Without commitment, the admissibility restrictions in the policy regime without and with taxes on lagged income, respectively, are given by $\mathcal{P}_t = \{(\tau_{t,t}, z_t) | (\tau_{t,t}, z_t) \in \mathbb{R} \times \mathbb{R}_+\}$ and $\mathcal{P}'_t = \{(\tau'_{t,t}, \tau'_{t-1,t}, z'_t) | (\tau'_{t,t}, \tau'_{t-1,t}, z'_t) \in \mathbb{R}^2 \times \mathbb{R}_+\}$. In the former regime, the state is $\mu_t = \{b^i_t\}_{i\in\mathcal{I}_t}$, and in the latter it is $\mu'_t = \{b^{i'}_t, l^{i'}_{t-1}\}_{i\in\mathcal{I}_t}$. Condition 2 fails in this environment because it is possible that two different states in the former regime are associated with one and the same state in the latter regime as can be shown by following a similar strategy as in Section 5.1.3.[37] Politico-economic equivalence therefore is not guaranteed. Intuitively, Condition 2 is violated because the tax on lagged income in the latter regime is both non-distorting at the time it is levied and a function of a tax base that varies across households. This generates substitutability between debt holdings and the tax base which lies at the root of the condition's violation. Absent household heterogeneity, no such substitutability is present and Condition 2 is satisfied.

With commitment (as for example in the case of Ramsey policies) politico-economic equivalence follows trivially if $b^i_0/\bar{b}_0 = b^{i'}_0/\bar{b}'_0$ and $l^{i'}_{-1} = 0$ for all $i \in \mathcal{I}_0$. The latter restriction rules out the possibility that the tax on lagged income gives rise to a lump sum tax that is only available in one regime. Even if political decisions are taken sequentially, politico-economic equivalence may still hold as long as there is one period commitment to $\tau'_{t-1,t}$.[38]

Battaglini and Coate (2008) present a politico-economic model with sequential choice of fiscal policy in an environment with tax distortions. Our results indicate that the equilibrium in their model cannot be re-interpreted as equilibrium in a model where taxes are additionally raised on lagged income. Yared (2010) presents another model with sequential choice and tax distortions. In his model, there is only one type of household holding debt, and Condition 2 therefore is satisfied. In contrast, Condition 4 is violated since in the regime with taxes on lagged income, negative net transfers (beyond non-payment on outstanding debt) can be implemented while this is not admissible in the initial regime. Politico-economic equivalence therefore cannot be guaranteed.

5.3. Corrective taxes versus cap-and-trade

In economies with externalities, various mechanisms may help induce agents to internalize the social consequences of their actions. In the debate about global warming and the appropriate policy responses to it, corrective taxes and cap-and-trade policies feature prominently among these mechanisms. This final application considers their equivalence properties.

Consider an economy with a dynamic production externality. Output is produced according to a neoclassical production function $y(\cdot)$, using capital k_t (and potentially other factors, in fixed supply) as input. For simplicity, capital is assumed to fully depreciate after one period. Total factor productivity depends on lagged aggregate output, for example because of a climate externality. Output of household i in period t thus equals $y^i_t = y(k^i_{t-1}, y_{t-1})$.

---

[37] Consider two states in the new regime, $\mu'^1_t = \{b^{i'1}_t, l^{i'1}_{t-1}\}_{i\in\mathcal{I}_t}$ and $\mu'^2_t = \{b^{i'2}_t, l^{i'2}_{t-1}\}_{i\in\mathcal{I}_t}$, with associated states μ_t and $\hat{\mu}_t$ in the initial regime supporting different competitive equilibria. A state $\mu'_t = \{b^{i'}_t, l^{i'}_{t-1}\}_{i\in\mathcal{I}_t}$ satisfying

$$\left.\begin{array}{l} z_t(\mu_t) b^i_t = z'^1_t b^{i'1}_t - \tau'^1_{t-1,t} l^{i'1}_{t-1} = z'^1_t b^{i'}_t - \tau'^1_{t-1,t} l^{i'}_{t-1} \\ z_t(\hat{\mu}_t) \hat{b}^i_t = z'^2_t b^{i'2}_t - \tau'^2_{t-1,t} l^{i'2}_{t-1} = z'^2_t b^{i'}_t - \tau'^2_{t-1,t} l^{i'}_{t-1} \end{array}\right\} \text{for all } i \in \mathcal{I}_s, s \geq t,$$

has both μ_t and $\hat{\mu}_t$ as associated states since each household's financial wealth net of lump sum taxes under $(\mu'_t, p'^{1,t-1})$ and $(\mu'^1_t, p'^{1,t-1})$ coincides (and thus $(\mu'_t, p'^{1,t-1})$ is economically equivalent to $(\mu_t, p^{t-1}(\mu_t))$) and the same holds true for financial wealth net of lump sum taxes under $(\mu'_t, p'^{2,t-1})$ and $(\mu'^2_t, p'^{2,t-1})$ (and thus $(\mu'_t, p'^{2,t-1})$ is economically equivalent to $(\hat{\mu}_t, p^{t-1}(\hat{\mu}_t))$). Such a state μ'_t generically can be found since the system of equations has as many equations as unknowns.

[38] For brevity, we omit the proof.

A Pigovian tax regime features a corrective tax τ_t on capital inputs k^i_{t+1} whose revenue is distributed across households according to a sharing rule, $\{\sigma^i_t\}_{i\in\mathcal{I}_t}$, with $\sum_{i\in\mathcal{I}_t}\sigma^i_t=1$. In contrast, a cap-and-trade regime features caps on each agent's capital input level, $\{\bar{k}^i_{t+1}\}_{i\in\mathcal{I}_t}$, and the permission to trade capital input permits in competitive markets. For now, we assume that policy is chosen under commitment. Letting $\#X$ denote the cardinality of the set X, the admissibility restrictions in the tax regime are given by $\mathcal{P}_0=\mathcal{P}=\{(\tau,\{\sigma^i\}_{i\in\mathcal{I}})|(\tau_t,\{\sigma^i_t\}_{i\in\mathcal{I}_t})\in\mathbb{R}^{\#\mathcal{I}_t+1}_+,\sum_{i\in\mathcal{I}_t}\sigma^i_t=1$ for all $t\geq 0\}$, and the restrictions in the cap-and-trade regime are given by $\mathcal{P}'_0=\mathcal{P}'=\{\{\bar{k}^{i'}\}_{i\in\mathcal{I}}|\{\bar{k}^{i'}_{t+1}\}_{i\in\mathcal{I}_t}\in\mathbb{R}^{\#\mathcal{I}_t}_+$ for all $t\geq 0\}$. The state under either regime is given by lagged output, which affects current productivity, and the cross section of installed capital stocks that determine current production, $\mu_0=(\{k^i_0\}_{i\in\mathcal{I}_0},y_{-1})$ and $\mu'_0=(\{k^{i'}_0\}_{i\in\mathcal{I}_0},y'_{-1})$.

With a Pigovian tax, an agent investing k^i_{t+1} incurs a net tax $\tau_t(k^i_{t+1}-\sigma^i_t\sum_{j\in\mathcal{I}_t}k^j_{t+1})$.[39] With a cap-and-trade system, this agent incurs net outlays $(k^{i'}_{t+1}-\bar{k}^{i'}_{t+1})\rho'_t$ as a consequence of the cap where ρ'_t denotes the market price of a permit. A pair (μ_0,p^{-1}) is economically equivalent to the pair (μ'_0,p'^{-1}) if the budget set of each household is the same under the two pairs (conditional on a given sequence for aggregate output) and other conditions are satisfied.[40] Equality of budget sets requires $y(k^i_0,y_{-1})+\sum_{s=0}^\infty r_{0,s}\tau_s(\sigma^i_s\sum_{j\in\mathcal{I}_s}k^j_{s+1}-k^i_{s+1})=y(k^{i'}_0,y'_{-1})+\sum_{s=0}^\infty r_{0,s}\rho'_s(\bar{k}^{i'}_{s+1}-k^i_{s+1})$ for all $k^i_{s+1}\in\mathbb{R}_+$, $s\geq 0$, and for all $i\in\mathcal{I}$. This restriction implies $\rho'_s=\tau_s$ for all $s\geq 0$ and thus, $\sum_{s=0}^\infty r_{0,s}\tau_s\left(\sigma^i_s\sum_{j\in\mathcal{I}_s}k^j_{s+1}-\bar{k}^{i'}_{s+1}\right)=y(k^{i'}_0,y'_{-1})-y(k^i_0,y_{-1})$ for all $i\in\mathcal{I}_0$.

Note that μ_0 and $\mu'_0=\mu_0$ are strongly associated. To see this, observe that $\mu_0=\mu'_0$ implies $y(k^i_0,y_{-1})=y(k^{i'}_0,y'_{-1})$ for all $i\in\mathcal{I}_0$. Economic equivalence therefore requires $\rho'_s=\tau_s$ and $\sum_{i\in\mathcal{I}_s}(k^i_{s+1}-\bar{k}^{i'}_{s+1})=0$ for all $s\geq 0$ as well as $\sum_{s=0}^\infty r_{0,s}\tau_s\sigma^i_s\sum_{j\in\mathcal{I}_s}k^j_{s+1}=\sum_{s=0}^\infty r_{0,s}\tau_s\bar{k}^{i'}_{s+1}$. For every admissible policy in the tax regime, an admissible policy in the cap-and-trade regime satisfies these restrictions, and vice versa. The states therefore are strongly associated.

Note next that μ_0 and μ'_0 are *not* strongly associated if $\mu'_0\neq\mu_0$. To see this observe that $\mu_0\neq\mu'_0$ implies $y(k^i_0,y_{-1})\neq y(k^{i'}_0,y'_{-1})$ for some $i\in\mathcal{I}_0$. Without loss of generality, assume that $y(k^i_0,y_{-1})<y(k^{i'}_0,y'_{-1})$ for some $i\in\mathcal{I}_0$. Consider the admissible tax policy $\sigma^i_s=0$ for all $s\geq 0$. The economically equivalent policy in the cap-and-trade regime must satisfy $\bar{k}^{i'}_{s+1}<0$ for some $s\geq 0$, which is not admissible.

Since each $\mu_0\in\mathcal{M}_0$ is uniquely strongly associated with $\mu'_0=\mu_0\in\mathcal{M}'_0$ and vice versa, Condition 1 is satisfied. As a consequence, politico-economic equivalence of the two policy regimes is guaranteed, regardless of the political aggregation mechanism in place, if $\mu_0=\mu'_0$.

With sequential policy choice the state is given by $\mu_t=(\{k^i_t,a^i_t\}_{i\in\mathcal{I}_t},y_{t-1})$ or $\mu'_t=(\{k^{i'}_t,a^{i'}_t\}_{i\in\mathcal{I}_t},y'_{t-1})$ where a^i_t or $a^{i'}_t$ denotes financial asset holdings of household i between periods $t-1$ and t. Again, the pair (μ_t,p^{t-1}) is economically equivalent to the pair (μ'_t,p'^{t-1}) if the budget set of each household is the same under the two pairs (conditional on a given sequence for aggregate output) and other conditions are satisfied.[41] Equality of budget sets requires $y(k^i_t,y_{t-1})+a^i_t+\sum_{s=t}^\infty r_{t,s}\tau_s(\sigma^i_s\sum_{j\in\mathcal{I}_s}k^j_{s+1}-k^i_{s+1})=y(k^{i'}_t,y'_{t-1})+a^{i'}_t+\sum_{s=t}^\infty r_{t,s}\rho'_s(\bar{k}^{i'}_{s+1}-k^i_{s+1})$ for all $k^i_{s+1}\in\mathbb{R}_+$, $s\geq t$, and for all $i\in\mathcal{I}$. For every period t, there exists a policy sequence in either regime such that a maximum level of total output results in that period, $\bar{y}_t$ and $\bar{y}'_t$ say. Letting $\mathcal{M}_0=\mathcal{M}'_0=\mathbb{R}^{2\#\mathcal{I}_0+1}_+$, economic equivalence implies that $\bar{y}_t=\bar{y}'_t$. It follows that $\mathcal{M}_t(\mathcal{M}_0)=\{(\{k^i_t,a^i_t\}_{i\in\mathcal{I}_t},y_{t-1})\in\mathbb{R}^{2\#\mathcal{I}_t+1}_+|\sum_{i\in\mathcal{I}_t}k^i_t\leq\bar{y}_{t-1},\ y_{t-1}\leq\bar{y}_{t-1},\ \sum_{i\in\mathcal{I}_t}a^i_t=0\}$ and $\mathcal{M}'_t(\mathcal{M}'_0)=\mathcal{M}_t(\mathcal{M}_0)$. A parallel logic to the one above applies, and Condition 1 is satisfied. Politico-economic equivalence of the two policy regimes is guaranteed, regardless of the political aggregation mechanism in place, as long as $\mu_0=\mu'_0$.

Golosov et al. (2011) analyze optimal corrective taxation of carbon emissions under commitment. Their model features a representative infinitely lived agent, two sectors of production and a carbon cycle.[42] Our results show that a cap-and-trade regime would be politico-economically equivalent in the environment studied by Golosov et al. (2011), as argued by the authors. Our results also show that the equivalence result would continue to hold if policy were chosen sequentially, and similarly if household heterogeneity were introduced provided that emission permits could be allocated among households in a way that replicates the distributive implications of the Pigovian tax and transfer scheme.[43]

6. Conclusions

We have derived sufficient conditions for politico-economic equivalence of policy regimes (conditional on initial states). These conditions apply in general dynamic environments with endogenous policy choice—either sequential or under

[39] Since agents are atomistic, they take $\sum_{j\in\mathcal{I}_t}k^j_{t+1}$ as given when making their investment decisions.

[40] The other conditions are $\sum_{i\in\mathcal{I}_s}(k^i_{s+1}-\bar{k}^{i'}_{s+1})=0$, $s\geq 0$, and $\sum_{i\in\mathcal{I}_0}(k^i_0-k^{i'}_0)=0$.

[41] The other conditions are $\sum_{i\in\mathcal{I}_s}(k^i_{s+1}-\bar{k}^{i'}_{s+1})=0$, $s\geq t$, and $\sum_{i\in\mathcal{I}_t}(k^i_t-k^{i'}_t)=0$.

[42] While capturing the same type of dynamic externality as the simple example considered above, Golosov et al.'s (2011) model is considerably more complex because the authors aim at estimating the optimal corrective tax—equal to the discounted value of marginal external damages—in a plausibly calibrated quantitative framework.

[43] This might require, for example, that permits are allocated to agents who do not generate externalities.

commitment—and they rely on an intuitive comparison of choice sets. They provide a useful tool to characterize politico-economic equilibrium in "new" policy regimes and allow to assess the consequences of institutional change.

We have exemplified the usefulness of the equivalence results in the context of several applications, relating to social security reform, tax-smoothing policies and measures to correct externalities. As far as social security reform is concerned, the analysis identifies a class of environments—characterized by minimal household heterogeneity and non-distorting taxes—where politico-economic equivalence holds independently of the political aggregation mechanism. But it also makes clear that sufficient heterogeneity among households or differentially tight admissibility restrictions across regimes may undermine equivalence.

It is frequently argued that pre-funding of social security may improve outcomes by reducing labor supply distortions. This argument relies on the assumption, often implicit, that certain competitive equilibria may be implemented by admissible debt policies but not by admissible social security policies, that is, it presupposes violations of *economic* equivalence.[44] Our conclusion regarding the failure of *politico-economic* equivalence differs from that standard argument but is related. According to our conclusion, political decision makers in a debt regime have larger choice sets than in a social security regime. This may generate political support for institutional change towards pre-funding.

When applied to environments with taxes on contemporaneous and lagged incomes, our results make clear that economic equivalence generally does not extend to the political sphere. From a narrow economic point of view only the net present value of distorting tax functions matters for the equilibrium allocation. In politico-economic equilibrium, in contrast, timing is crucial because it determines the elasticity of the tax base. Politico-economic equivalence therefore only holds if policy is chosen once and for all, as for example with Ramsey policies.

Regarding measures to correct externalities, we have found that regimes with Pigovian taxes and cap-and-trade schemes are politico-economically equivalent, provided that permits can be allocated among households in a way that replicates the distributive implications of the Pigovian tax and transfer scheme. This result holds both when policy is chosen sequentially and once and for all.

Naturally, the applicability of our equivalence conditions extends beyond the particular environments we have considered and is not confined to the realm of fiscal and regulatory policy. Against the background of an appropriately defined equivalence class of policies—be they fiscal, monetary or other—the conditions may be applied to any model featuring an endogenous choice of such policies.

Appendix A. Formal definitions and proofs

A.1. Competitive equilibrium allocation

Following Stokey (1991), let $x^{n,t-1} \equiv (x_t^n, x^{n,t}) \in \mathcal{X}^{n,t-1}$ denote the sequence of actions from period t onward of a household or firm of type $n \in \mathcal{N}$, and let $x^{t-1} \equiv (x_t, x^t) \in \mathcal{X}^{t-1}$ denote the profile of such action sequences across all household and firm types.[45]

Aggregate feasibility as of period t is represented by a correspondence, $\mathcal{F}_t : \mathcal{M}_t \to \mathcal{P}^{t-1} \times \mathcal{X}^{t-1}$.[46] That is, $(p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t)$ if the sequences (p^{t-1}, x^{t-1}) are feasible conditional on μ_t. Let $\mathcal{C}_t \subseteq \mathcal{M}_t \times \mathcal{P}_t \times \mathcal{X}_t$ denote the set of first elements in feasible sequences,

$$\mathcal{C}_t = \{(\mu_t, p_t, x_t) \in \mathcal{M}_t \times \mathcal{P}_t \times \mathcal{X}_t : (p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t) \text{ for some } (p^t, x^t) \in \mathcal{P}^t \times \mathcal{X}^t\}.$$

The aggregate law of motion for the state is defined on the set $\mathcal{C}_t$, $\mathcal{L}_t : \mathcal{C}_t \to \mathcal{M}_{t+1}$. Consistency requirements imply conditions on $\mathcal{F}_t$ and $\mathcal{L}_t$.[47]

Feasibility on the level of an individual household or firm $i \in \mathcal{I}$ of type $n \in \mathcal{N}$ as of period t is represented by another correspondence, $\mathcal{F}_t^n : \mathcal{M}_t \times \mathcal{P}^{t-1} \times \mathcal{X}^{t-1} \times \mathcal{M}_t^n \to \mathcal{X}^{n,t-1}$. That is, $x^{i,t-1} \in \mathcal{F}_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^i)$ if the sequence $x^{i,t-1}$ is feasible conditional on μ_t, p^{t-1}, x^{t-1} and μ_t^i. Feasibility restrictions may incorporate budget constraints since prices can be linked to marginal rates of transformation or substitution which in turn are related to aggregate quantities. Consistency implies an aggregation restriction.[48] Let $\mathcal{C}_t^n \subseteq \mathcal{C}_t \times \mathcal{M}_t^n \times \mathcal{X}_t^n$ denote the set of first elements in individually feasible sequences for a household or firm $i \in \mathcal{I}$ of type $n \in \mathcal{N}$,

$$\begin{aligned}\mathcal{C}_t^n = \{&(\mu_t, p_t, x_t, \mu_t^i, x_t^i) \in \mathcal{C}_t \times \mathcal{M}_t^n \times \mathcal{X}_t^n : (p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t) \text{ and} \\ &x^{i,t-1} \in \mathcal{F}_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^i) \text{ for some } (p^t, x^t, x^{i,t}) \in \mathcal{P}^t \times \mathcal{X}^t \times \mathcal{X}^{n,t}\}.\end{aligned}$$

44 See Feldstein and Liebman (2002) for an overview over the literature and Rangel (1997) for a critique.

45 Throughout, we partition a sequence x^{t-1} say that runs from period t to the end of the planning horizon into its period t element, x_t, and the remainder of the sequence, x^t.

46 Note the difference between the feasibility of *policy-/action-sequence* combinations discussed here and the feasibility of *policy sequences* discussed in Section 3.

47 First, $(p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t)$ implies $(p^t, x^t) \in \mathcal{F}_{t+1}(\mathcal{L}_t(\mu_t, p_t, x_t))$. Second, $(\mu_t, p_t, x_t) \in \mathcal{C}_t$ and $(p^t, x^t) \in \mathcal{F}_{t+1}(\mathcal{L}_t(\mu_t, p_t, x_t))$ imply $(p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t)$.

48 In particular, $(p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t)$ implies $x^{n,t-1} \in \mathcal{F}_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^n)$ for all $n \in \mathcal{N}$.

From the consistency requirement imposed on $\mathcal{F}_t^n$, it follows that $(\mu_t, p_t, x_t) \in \mathcal{C}_t$ implies $(\mu_t, p_t, x_t, \mu_t^n, x_t^n) \in \mathcal{C}_t^n$ for all $n \in \mathcal{N}$. The law of motion for the state of an individual household or firm $i \in \mathcal{I}$ of type $n \in \mathcal{N}$ is defined on the set $\mathcal{C}_t^n$, $\mathcal{L}_t^n : \mathcal{C}_t^n \to \mathcal{M}_{t+1}^n$. Consistency requirements imply conditions on $\mathcal{F}_t^n$ and $\mathcal{L}_t^n$.[49]

The objective function of a household $i \in \mathcal{I}$ of type $n \in \mathcal{N}$ as of period t is given by $\Omega_t^n : \mathcal{M}_t \times \mathcal{P}^{t-1} \times \mathcal{X}^{t-1} \times \mathcal{M}_t^n \times \mathcal{X}^{n,t-1} \to \mathbb{R}$. We assume that this function is bounded. The objective function of a firm active in period t is to maximize static profits. For notational simplicity, we denote the objective of a firm of type $n \in \mathcal{N}$ also by Ω_t^n.

The set $\mathcal{E}_t(\mu_t) \subseteq \mathcal{F}_t(\mu_t)$ comprises the policy as well as household and firm action sequences that are associated with competitive equilibria. The equilibrium requirement implies that x^{t-1} is a best response that is,

$$\mathcal{E}_t(\mu_t) = \{(p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t) : x^{n,t-1} = \arg\max_{x^{i,t-1}} \Omega_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^n, x^{i,t-1}) \text{ for all } n \in \mathcal{N}\}$$

subject to $x^{i,t-1} \in \mathcal{F}_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^n)$. From the consistency requirements imposed earlier, it follows that $(p^{t-1}, x^{t-1}) \in \mathcal{E}_t(\mu_t)$ implies $(p^t, x^t) \in \mathcal{E}_{t+1}(\mu_{t+1})$ for $\mu_{t+1} = \mathcal{L}_t(\mu_t, p_t, x_t)$.

Based on the previous objects introduced in Stokey (1991), we define the competitive-equilibrium-allocation correspondence, $\text{CE}_t : \mathcal{M}_t \times \mathcal{P}^{t-1} \to \mathcal{X}^{t-1}$. That is, $x^{t-1} \in \text{CE}_t(\mu_t, p^{t-1})$ if $(p^{t-1}, x^{t-1}) \in \mathcal{E}_t(\mu_t)$, and $\text{CE}_t(\mu_t, p^{t-1}) = \emptyset$ if (μ_t, p^{t-1}) does not implement any competitive equilibrium allocation. (Even if x^{t-1} does not directly constitute an allocation, the link between the two objects is immediate. From now on, we therefore refer to equilibrium allocations rather than sequences of household and firm actions.) In the main text, we drop the time subscript t of the correspondence for notational simplicity.

As usual, the marginal rates of substitution and transformation that are implied by a competitive equilibrium allocation (or a set thereof) pin down equilibrium prices (or a set thereof).

A.2. Proposition 1

In the context of Proposition 1, we compare two choice sets of a household $i \in \mathcal{I}$ of type $n \in \mathcal{N}$ as of period t. These choice sets encompass all restrictions imposed by the dynamic and intertemporal budget constraints as well as other constraints on the level of the household, for instance the consumption set or quotas instituted by policy. The first choice set is defined by

$$\mathcal{F}_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^i) \tag{5}$$

where x^{t-1} is the equilibrium sequence of aggregate private sector actions in a competitive equilibrium with allocation $\text{CE}_t(\mu_t, p^{t-1})$ and prices q_t. The second choice set is given by

$$\mathcal{F}_t^n(\mu'_t, p'^{t-1}, x^{t-1}, \mu'^i_t); \tag{6}$$

it is characterized by potentially modified aggregate and individual states, μ'_t and μ'^i_t; a potentially modified policy sequence, p'^{t-1}; but unchanged private sector actions. A formal statement and proof of Proposition 1 follows:

Proposition 1. *Consider a state and policy sequence, (μ_t, p^{t-1}), that implement a competitive equilibrium allocation as of period t (or a set thereof), $\text{CE}_t(\mu_t, p^{t-1})$, with corresponding prices q_t. Consider a new state and policy sequence, (μ'_t, p'^{t-1}), that satisfy the following conditions:*

i. *state variables that determine production possibilities are identical across μ_t and μ'_t;*
ii. *restrictions on inputs and/or outputs of firms are identical across p^{t-1} and p'^{t-1};*
iii. *choice sets defined by (5) and (6) are identical for each household $i \in \mathcal{I}$ of type $n \in \mathcal{N}$ that is "alive" in or after period t[50];*
iv. *at the equilibrium allocation and prices (or at each allocation and prices in the set of equilibrium allocations and prices), (μ'_t, p'^{t-1}) satisfy the government budget constraints.*

Then, (μ'_t, p'^{t-1}) are economically equivalent to (μ_t, p^{t-1}).

Proof. Conjecture that the pair (μ'_t, p'^{t-1}) indeed implements the same competitive equilibrium allocation, $\text{CE}_t(\mu_t, p^{t-1})$, and the same equilibrium prices, q_t.

With household choice sets unchanged and preferences not directly dependent on policy, household actions are unaltered. With firm production possibilities unaffected by policy and policy restrictions on inputs and/or outputs as well as prices unchanged, firm actions are unaltered. With production unchanged, (new and old) household and firm actions are

[49] First, the profile of type specific laws of motion across all types, $\{\mathcal{L}_t^n(\mu_t, p_t, x_t, \mu_t^n, x_t^n)\}_{n\in\mathcal{N}}$, coincides with the aggregate law of motion, $\mathcal{L}_t(\mu_t, p_t, x_t)$, for all $(\mu_t, p_t, x_t) \in \mathcal{C}_t$. Second, $(p^{t-1}, x^{t-1}) \in \mathcal{F}_t(\mu_t)$ and $x^{i,t-1} \in \mathcal{F}_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^i)$ for some $n \in \mathcal{N}$ implies $x^{i,t} \in \mathcal{F}_{t+1}^n(\mu_{t+1}, p^t, x^t, \mu_{t+1}^i)$ for $\mu_{t+1} = \mathcal{L}_t(\mu_t, p_t, x_t)$ and $\mu_{t+1}^i = \mathcal{L}_t^n(\mu_t, p_t, x_t, \mu_t^i, x_t^i)$. Third, if $(\mu_t, p_t, x_t) \in \mathcal{C}_t$, $(p^t, x^t) \in \mathcal{F}_{t+1}(\mu_{t+1})$, $(\mu_t, p_t, x_t, \mu_t^i, x_t^i) \in \mathcal{C}_t^n$ and $x^{i,t} \in \mathcal{F}_{t+1}^n(\mu_{t+1}, p^t, x^t, \mu_{t+1}^i)$ for some $n \in \mathcal{N}$ then $x^{i,t-1} \in \mathcal{F}_t^n(\mu_t, p^{t-1}, x^{t-1}, \mu_t^i)$.

[50] For a household not yet active in period t, her state is empty.

feasible. Moreover, household choices and the government's new policy sequence satisfy the relevant budget constraints. The pair (μ'_t, p'^{t-1}) therefore implements the same competitive equilibrium allocation, $\text{CE}_t(\mu_t, p^{t-1})$, verifying the conjecture. □

A.3. Law of motion (1)

Let the set $\mathcal{E}_t(\mu_t; p^t(\cdot)) \subseteq \mathcal{E}_t(\mu_t)$ comprise the policy as well as household and firm action sequences that are associated with competitive equilibria and are consistent with a given continuation policy function $p^t(\cdot)$, that is

$$\mathcal{E}_t(\mu_t; p^t(\cdot)) = \{(p^{t-1}, x^{t-1}) \in \mathcal{E}_t(\mu_t) : p^{t-1} = (p_t, p^t(\mathcal{L}_t(\mu_t, p_t, x_t)))\}.$$

Let $\mathcal{P}_t(\mu_t; p^t(\cdot)) \subseteq \mathcal{P}_t$ denote the set of first elements in equilibrium policy sequences conditional on the state μ_t that are consistent with the given continuation policy function $p^t(\cdot)$,

$$\mathcal{P}_t(\mu_t; p^t(\cdot)) = \{p_t \in \mathcal{P}_t : (p^{t-1}, x^{t-1}) \in \mathcal{E}_t(\mu_t; p^t(\cdot)) \text{ for some } (p^t, x^{t-1}) \in \mathcal{P}^t \times \mathcal{X}^{t-1}\}.$$

That is, $p_t \in \mathcal{P}_t(\mu_t; p^t(\cdot))$ if $((p_t, p^t(\mathcal{L}_t(\mu_t, p_t, x_t))), x^{t-1}) \in \mathcal{E}_t(\mu_t; p^t(\cdot))$ for some $x^{t-1} \in \mathcal{X}^{t-1}$.

Equation (1) in the text gives the law of motion of the state,

$$\mu_{s+1} = g_s(\mu_s, p_s; p^s(\cdot)), s \geq t.$$

The first argument of this function takes values from the set $\mathcal{M}_s, s \geq t$. For any $\mu_s \in \mathcal{M}_s$ the second argument of the function takes values from the set $\mathcal{P}_s(\mu_s; p^s(\cdot))$.

A.4. Proposition 2

To simplify notation, we write $\mu_{t+1}(p_t)$ instead of $g_t(\mu_t, p_t; p^t(\cdot))$ for the state in the initial regime that is implied by a particular policy choice, leaving the current state and the continuation policy function implicit. For the state in the new regime that is implied by a competitive equilibrium allocation $\text{CE}_t(\mu'_t, p'^{t-1})$, we write $\mu'_{t+1}(\mu'_t, p'^{t-1})$.

Denote the (unknown) policy function in the initial regime by $p_t(\mu_t)$ and the continuation policy function by $p^t(\mu_{t+1})$. From Condition 1, there exists a unique μ'_t and an admissible p'^{t-1} such that $(\mu_t, (p_t(\mu_t), p^t(\mu_{t+1}(p_t))))$ is economically equivalent to (μ'_t, p'^{t-1}). The policy sequence p'^{t-1} can be split into a period t component, p'_t, and a continuation policy, p'^t. If (μ_t, p^{t-1}) implements a competitive equilibrium allocation in the initial regime then $(\mu_{t+1}(p_t), p^t)$ implements the continuation competitive equilibrium allocation with $p^{t-1} = (p_t, p^t)$. Similarly, if (μ'_t, p'^{t-1}) implements a competitive equilibrium allocation in the new regime then $(\mu'_{t+1}(\mu'_t, p'^{t-1}), p'^t)$ implements the continuation competitive equilibrium allocation with $p'^{t-1} = (p'_t, p'^t)$. Economic equivalence of (μ_t, p^{t-1}) and (μ'_t, p'^{t-1}) therefore implies economic equivalence of $(\mu_{t+1}(p_t), p^t)$ and $(\mu'_{t+1}(\mu'_t, p'^{t-1}), p'^t)$. Since strong association implies a one-to-one relation between states we conclude that $\mu_{t+1}(p_t)$ and $\mu'_{t+1}(\mu'_t, p'^{t-1})$ are strongly associated. Using the policy sequences just characterized, we thus can define a policy function and continuation policy function in the new regime, $p'_t(\cdot)$ and $p'^t(\cdot)$ respectively, that satisfy $p'_t(\mu'_t) = p'_t$ and $p'^t(\mu'_{t+1}(\mu'_t, p'^{t-1})) = p'^t$ and are defined over the domains $\mathcal{M}'_t$ and $\mathcal{M}'_{t+1}$ respectively. Based on the continuation policy functions $p'^t(\cdot)$ we can also define a law of motion of the state in the new regime,

$$g'_s(\mu'_s, p'_s; p'^s(\cdot)) \equiv \mu'_{s+1}(\mu'_s, p'^{s-1}), s \geq t.$$

To simplify notation, we also write $\mu'_{t+1}(p'_t)$ instead of $g'_t(\mu'_t, p'_t; p'^t(\cdot))$.

Conjecture that in the new regime in period t, future policy choices are expected to be determined according to the continuation policy function $p'^t(\cdot)$. We claim that the policy function in the new regime then is given by $p'_t(\cdot)$. To verify the claim by contradiction, suppose that the policy function is given by another function, $\pi'_t(\cdot)$ say, such that for some $\mu'_t \in \mathcal{M}'_t$ the allocation $\text{CE}_t(\mu'_t, (\pi'_t(\mu'_t), p'^t(\mu'_{t+1}(\pi'_t(\mu'_t)))))$ is strictly preferred to $\text{CE}_t(\mu'_t, (p'_t(\mu'_t), p'^t(\mu'_{t+1}(p'_t(\mu'_t)))))$ and $\pi'_t(\mu'_t) \in \mathcal{P}'_t$. From Condition 1, there exists a μ_t strongly associated with μ'_t and a $\pi_t \in \mathcal{P}_t$ in the initial regime such that $(\mu'_t, (\pi'_t(\mu'_t), p'^t(\mu'_{t+1}(\pi'_t(\mu'_t)))))$ is economically equivalent to $(\mu_t, (\pi_t, p^t(\mu_{t+1}(\pi_t))))$. By definition of the policy function, $\text{CE}_t(\mu_t, p^{t-1}(\mu_t))$ is preferred (at least weakly) to $\text{CE}_t(\mu_t, (\pi_t, p^t(\mu_{t+1}(\pi_t))))$. Political decision makers in the new regime share this preference and can implement the former equilibrium allocation by choosing $p'_t(\mu'_t)$ rather than $\pi'_t(\mu'_t)$. This establishes the desired contradiction and thus, verifies the claim.

We conclude that political decision makers in the new regime implement policy choices according to the policy function $p'_t(\cdot)$ if the continuation policy function $p'^t(\cdot)$ is expected. But by induction, these expectations are consistent with equilibrium. The functions $p'_t(\cdot)$ and $p'^t(\cdot)$ therefore satisfy the conditions of politico-economic equilibrium.

From Condition 1 and the previous argument, political decision makers in period $t = 0$ in the new regime can implement the equilibrium allocation in the initial regime and wish to do so. The same holds true in subsequent periods; for starting from strongly associated states, the subsequent states in the continuation competitive equilibrium allocations are strongly associated as well.

Finally, economic equivalence of $(\mu'_0, (p'_0(\mu'_0), p'^0(\mu'_1(p'_0(\mu'_0)))))$ and $(\mu_0, (p_0(\mu_0), p^0(\mu_1(p_0(\mu_0)))))$ implies that the equilibrium policy choices in the new policy regime implement the same competitive equilibrium allocation as in the initial policy regime. The result then follows.

A.5. Proposition 3

We show that there exists a politico-economic equilibrium in the new regime with policy and continuation policy functions $\{\tilde{p}'_t(\cdot), \tilde{p}'^{t-1}(\cdot)\}_{t\geq 0}$, policy choices $p'^{\star -1} \equiv \tilde{p}'^{-1}(\mu'_0)$, and the same competitive equilibrium allocation as in PEE$(\mu_0, \mathcal{P})$.

Conjecture that in the new regime in period t, future policy choices are expected to be determined according to the equivalent continuation policy function $\tilde{p}'^t(\cdot)$. (From Condition 2, this function is well defined over the domain $\mathcal{M}'_{t+1}$.) We claim that the policy function in the new regime then is given by $\tilde{p}'_t(\cdot)$. To verify the claim by contradiction, suppose that the policy function is given by another function, $\pi'_t(\cdot)$ say, such that for some $\mu'_t \in \mathcal{M}'_t$ the allocation $\text{CE}_t(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1}(\pi'_t(\mu'_t)))))$ is strictly preferred to $\text{CE}_t(\mu'_t, (\tilde{p}'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1}(\tilde{p}'_t(\mu'_t)))))$ and $\pi'_t(\mu'_t) \in \mathcal{P}'_t$. Let μ_t be associated with μ'_t. (From Condition 2, such a μ_t exists.) From Condition 4, there exists an admissible policy choice $\pi_t \in \mathcal{P}_t$ in the initial regime such that $(\mu'_t, (\pi'_t(\mu'_t), \tilde{p}'^t(\mu'_{t+1}(\pi'_t(\mu'_t)))))$ is economically equivalent to $(\mu_t, (\pi_t, p^t(\mu_{t+1}(\pi_t))))$. By definition of the policy function, $\text{CE}_t(\mu_t, p^{t-1}(\mu_t))$ is preferred (at least weakly) to $\text{CE}_t(\mu_t, (\pi_t, p^t(\mu_{t+1}(\pi_t))))$. Political decision makers in the new regime share this preference and, from Condition 2, can implement the former equilibrium by choosing $\tilde{p}'_t(\mu'_t)$ rather than $\pi'_t(\mu'_t)$. This establishes the desired contradiction and thus, verifies the claim.

We conclude that political decision makers in the new regime implement policy choices according to the policy function $\tilde{p}'_t(\cdot)$ if the continuation policy function $\tilde{p}'^t(\cdot)$ is expected. But by induction, these expectations are consistent with equilibrium. The functions $\tilde{p}'_t(\cdot)$ and $\tilde{p}'^t(\cdot)$ therefore satisfy the conditions of politico-economic equilibrium. Based on these functions, the law of motion of the state in the new regime may be defined.

From Condition 3 and the previous argument, political decision makers in period $t = 0$ in the new regime can implement the equilibrium allocation in the initial regime and wish to do so. The same holds true in subsequent periods. For if μ_t is associated with μ'_t and if $(\mu_t, (p_t, p^t(\mu_{t+1}(p_t))))$ is economically equivalent to $(\mu'_t, (p'_t, \tilde{p}'^t(\mu'_{t+1}(p'_t))))$, then $\mu_{t+1}(p_t)$ is associated with $\mu'_{t+1}(p'_t)$ as well (because $\mu'_{t+1}(p'_t) \in \mathcal{M}'_{t+1}(\mathcal{M}'_0, \mathcal{P}')$ and from Condition 2).

Finally, economic equivalence of $(\mu'_0, (\tilde{p}'_0(\mu'_0), \tilde{p}'^0(\mu'_1(\tilde{p}'_0(\mu'_0)))))$ and $(\mu_0, (p_0(\mu_0), p^0(\mu_1(p_0(\mu_0)))))$ implies that the equilibrium policy choices in the new policy regime implement the same competitive equilibrium allocation as in the initial policy regime. The result then follows.

References

Barro, R.J., 1974. Are government bonds net wealth? Journal of Political Economy 82 (6), 1095–1117.
Bassetto, M., 2008. Political economy of taxation in an overlapping-generations economy. Review of Economic Dynamics 11 (1), 18–43.
Bassetto, M., Kocherlakota, N., 2004. On the irrelevance of government debt when taxes are distortionary. Journal of Monetary Economics 51 (2), 299–304.
Battaglini, M., Coate, S., 2007. Inefficiency in legislative policymaking: a dynamic analysis. The American Economic Review 97 (1), 118–149.
Battaglini, M., Coate, S., 2008. A dynamic theory of public spending, taxation, and debt. The American Economic Review 98 (1), 201–236.
Boldrin, M., Rustichini, A., 2000. Political equilibria with social security. Review of Economic Dynamics 3, 41–78.
Broner, F., Martin, A., Ventura, J., 2010. Sovereign risk and secondary markets. The American Economic Review 100 (4), 1523–1555.
Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2007. Business cycle accounting. Econometrica 75 (3), 781–836.
Coleman, W.J., 2000. Welfare and optimum dynamic taxation of consumption and income. Journal of Public Economics 76 (1), 1–39.
Cooley, T.F., Soares, J., 1999. A positive theory of social security based on reputation. Journal of Political Economy 107 (1), 135–160.
Correia, I., Nicolini, J.P., Teles, P., 2008. Optimal fiscal and monetary policy: equivalence results. Journal of Political Economy 116 (1), 141–170.
Cukierman, A., Meltzer, A.H., 1989. A political theory of government debt and deficits in a Neo-Ricardian framework. The American Economic Review 79 (4), 713–732.
Díaz-Giménez, J., Giovanetti, G., Marimon, R., Teles, P., 2008. Nominal debt as a burden on monetary policy. Review of Economic Dynamics 11, 493–514.
Farhi, E., Sleet, C., Werning, Y., Yeltekin, S., 2012. Nonlinear capital taxation without commitment. Review of Economic Studies 79 (4), 1469–1493.
Feldstein, M., Liebman, J.B., 2002. Social security. In: Auerbach, A.J., Feldstein, M. (Eds.), Handbook of Public Economics, vol. 4. Elsevier Science, Amsterdam. Chapter 32.
Forni, L., 2005. Social security as Markov equilibrium in OLG models. Review of Economic Dynamics 8, 178–194.
Ghiglino, C., Shell, K., 2000. The economic effects of restrictions on government budget deficits. Journal of Economic Theory 94, 106–137.
Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A., 2011. Optimal taxes on fossil fuel in general equilibrium. Working Paper 17348. NBER, Cambridge, Massachusetts.
Gonzalez-Eiras, M., 2011. Social security as Markov equilibrium in OLG models: a note. Review of Economic Dynamics 14 (3), 549–552.
Gonzalez-Eiras, M., Niepelt, D., 2008. The future of social security. Journal of Monetary Economics 55 (2), 197–218.
Gonzalez-Eiras, M., Niepelt, D., 2012. Economic and politico-economic equivalence. Working Paper 12.02. Study Center Gerzensee.
Krusell, P., Quadrini, V., Ríos-Rull, J.-V., 1997. Politico-economic equilibrium and economic growth. Journal of Economic Dynamics and Control 21 (1), 243–272.
Niepelt, D., 2005. Social security reform: economics and politics. In: Greulich, G., Lösch, M., Müller, C., Stier, W. (Eds.), Empirische Konjunktur- und Wachstumsforschung. Rüegger, Zürich, pp. 181–195.
Niepelt, D., 2014. Debt maturity without commitment. Journal of Monetary Economics 68 (S), 37–54.
Rangel, A., 1997. Social security reform: efficiency gains or intergenerational redistribution. Mimeo. Harvard University.
Sargent, T.J., 1987. Dynamic Macroeconomic Theory. Harvard University Press, Cambridge, Massachusetts.
Song, Z., Storesletten, K., Zilibotti, F., 2012. Rotten parents and disciplined children: a politico-economic theory of public expenditure and debt. Econometrica 80 (6), 2785–2803.
Stokey, N.L., 1991. Credible public policy. Journal of Economic Dynamics and Control 15 (4), 627–656.
Tabellini, G., 2000. A positive theory of social security. Scandinavian Journal of Economics 102 (3), 523–545.
Yared, P., 2010. Politicians, taxes and debt. Review of Economic Studies 77 (2), 806–840.