Resolution of Financial Crises*

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Abstract

Strategic default eliminates the negative dynamic amplification effects on collateral prices in models of financial crises. However, when default is costly, borrowers sometimes prefer not to default and keep assets used as collateral even if their market value is below the value of outstanding debt. This wedge between assets' inside and outside value sets the stage for bargaining over debt repayment. We study how these technological factors, together with institutions that determine the bargaining power of borrowers and lenders, affect the amplification and persistence of aggregate shocks. We find strong amplification effects for moderate shocks, while for larger shocks debt renegotiation dampens amplification. The set of shocks for which amplification is strong is increasing in default costs and lenders' bargaining power. Asymmetric information about default costs leads to "V-shaped" recoveries. These results are consistent with observed real estate prices, and their macroeconomic implications, in the United States during the Great Recession.

KEYWORDS: Financial crises; balance sheet recessions; default renegotiation.
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1 Introduction

Financial crises are often preceded by asset price booms and excessive borrowing, typically against appreciating assets. The path to recovery from these crises depends on how the economy splits financial losses between creditors and debtors. This, in turn, is determined by the underlying institutional environment. In economies with strong creditor rights and high default costs, such as Japan, the adjustment typically entails a long-drawn process of corporate debt repayments with few breakdowns or restructurings of large business firms. By contrast, in economies with weak creditor right and/or low default costs, such as South East Asian countries in the late 1990s, the economy suffers an initial period of large output losses, numerous renegotiations of private debts, and large-scale transfers of property and control. The initial turmoil is usually followed by swift growth, a phenomenon sometimes denoted as a “V-shaped” recovery.[1]

In this paper, we build a model to study the ex post resolution of financial crises when the repayment of large masses of private business debts is put into question. In particular, we focus on the relative bargaining power of borrowers and lenders in case of default, determined by institutional and technological factors, and its consequences for the amplification and persistence of aggregate shocks. Our analysis is based on the celebrated model of Kiyotaki and Moore (1997) - henceforth, KM - enriched to allow for the ex post renegotiation of pre-existing debts.[2] In that way, we can capture in broad terms the two polar scenarios described above and study conditions that would make an economy go one way or another.

Severe recessions create incentives for widespread contract breaches. Thus, care should be given to borrowers' costs and benefits from default, and to how the legal system processes these situations. Bankruptcy legislation can modulate the effect of an aggregate shock through its impact on the allocation of control rights over assets after a debtor defaults. Thus, two elements are fundamental for the resolution of crises: the distribution of bargaining power between borrowers and lenders, and default costs, such as the setup costs of starting a new firm.[3] We say an economy is characterized by ex post lender-biased control rights when default costs are high and borrowers have little bargaining power. Conversely, the opposite characterizes an economy with borrower-biased control rights.

Our model implies that the distribution of control rights is key for the amplification and persistence of macroeconomic shocks. With lender-biased control rights, firms are reluctant to default. As a result, they honor existing debts, significantly depressing the

[1]For an analysis of the contrasting cases of Japan and other South East Asian countries in the 1990s see Chang (2006).

[2]KM allows for renegotiation at the interim stage, before borrowers exert effort into production. Since it is assumed that borrowers have all the interim bargaining power, this restricts borrowing to the market value of assets.

[3]Regarding the distribution of bargaining power, in the United States there is considerable variation across states in the homestead exemption in the personal bankruptcy procedures under Chapter 7. States also differ in whether foreclosed sales should take place through courts or not. Besides physical costs associated to default, it may take some time for a recently initiated project to reach its potential level of productivity.

demand for capital and leading to a collapse in asset prices. The response of the economy is thus the same as in the original KM model. By contrast, with borrower-biased control rights, for large enough shocks default becomes credible. Thus, lenders' threat to seize collateral is weak. As a result, firms extract large haircuts, cutting their financial losses. This cushions the reduction in capital demand, significantly dampening the decrease in asset prices. The range of shocks for which amplification is strong is increasing in the degree with which control rights are lender biased.

Financial crises are sometimes modelled as arising from preference shocks that affect the supply of credit and have a direct impact on asset prices. Thus, in addition to a temporary productivity shock (considered in KM), we allow for a temporary shock that reduces lenders' discount factor, which in equilibrium increases the interest rate. This has two effects on firms. By reducing the required downpayment, it allows firms to increase leverage. But, by lowering asset prices, it generates capital losses for firms. We show that the net effect depends on whether or not firms have an incentive to default. When default is credible, renegotiation dampens firms' capital losses, increasing their asset demand. Otherwise, their demand falls. Thus, our model suggests that the macroeconomic effects of financial shocks crucially depend on the type of ex post control rights that characterize the economic environment.

A limitation of the model is that default is never observed in equilibrium, which is counterfactual with episodes such as the aftermath of the East Asian financial crisis, or real estate markets in the U.S. during the Great Recession. To capture this, we build an extension of the model where borrowers are heterogeneous in their default cost, which is private information. In this context, lenders find it optimal to make debt-relief offers (i.e., haircuts) that some entrepreneurs, those with low default costs, will prefer to reject. We show that, due to default costs, an economy with asymmetric information experiences an initial sharper drop in output, but it recovers faster due to the higher net-worth of the entrepreneurial sector as whole, thus formalizing the idea of "V-shaped" recoveries.

An important ingredient of the model is that the ex post resolution of debt crises does not affect the ex ante behavior of agents. We believe this is a reasonable approximation of behavior in credit markets with respect to rare events such as financial crises. For example, in the credit boom before the Great Recession, lenders paid little attention to borrowers' repayment capacity. Mian et al. (2015) show that in the late 1990s and early 2000s lenders did not differentiate lending based on states' foreclosure requirements.[4] In commercial real estate markets debt was often issued with minimum covenants, and commercial real estate had low risk premia relative to other assets. These facts point to lenders assigning a very low probability to states of the world in which these debt contracts would be in default.

Our model can rationalize developments in real estate markets in the United States during the Great Recession, and their implications for the rest of the economy. The housing market experienced a 35% drop in prices between 2007 and 2009, with part of this decline the result of the amplification of the initial shock due to an increase in housing

[4]They show such a differential lending behavior was seen in the early 1990s, consistent with findings by Pence (2006), who documents 3% to 7% smaller mortgages in states with a judicial foreclosure requirement in the mid 1990s, as expected given the higher foreclosure costs in these states.

supply due to foreclosures.

Mian et al. (2015) use state judicial requirement as an instrument for foreclosures and establish causal effects from foreclosures to home prices, residential investment and consumer demand. They also find that foreclosures increase the net supply of houses in the market. In the context of our model, judicial foreclosure requirements shift control rights towards borrowers and the equilibrium thus should feature more renegotiation, less house sales (foreclosures), and lower decreases in asset prices. Moreover, with asymmetric information recovery should be faster in states that had more output losses associated to default (costs due to foreclosure proceedings), which are those with no judicial foreclosure requirement. These predictions of the model are supported by Mian et al. (2015).

Further evidence supporting the model's predictions is provided by Agarwal et al.'s (2017) study of the impact of the 2009 Home Affordable Modification Program. This program gave intermediaries incentives to renegotiate mortgages. Intermediaries had heterogeneous pre-program experience with loan renegotiations, which lead high-experience servicers to perform significantly more modifications. If high-experience servicers are better at screening borrowers' default costs, our model predicts that regions with more experienced servicers will have more renegotiations, and thus fewer fire sales. Agarwal et al.'s (2017) findings are consistent with this prediction.

Our work is related to the theoretical literature on the macroeconomic implications of financial imperfections.[5] Agency costs in the form of costly state verification were introduced by Bernanke and Gertler (1989), and used in dynamic stochastic general equilibrium models by Carlstrom and Fuerst (1997), where the agency arises from asymmetric information in the creation of new capital, and Bernanke et al. (1999), where the agency costs lead to an external finance premium that generates the financial accelerator. These last two models allow for default, but assume it to be exogenous and not strategic. More recent research has considered the implications of shocks to the net worth of households and intermediaries in the presence of financial frictions. Regarding the former, Eggertsson and Krugman (2012) show that credit constrained households' deleveraging in the event of a shock can lead to a liquidity trap when the zero lower bound constraint on monetary policy is binding. Similar results are found in Guerrieri and Lorenzoni (2017) in a heterogeneous-agent incomplete-market model, even with flexible prices. Gertler and Kiyotaki (2011), focus on the role of intermediaries' balance sheets and use agency problems to limit lending to banks and to impair the working of interbank markets. They find that a negative shock to bank assets affects banks' loan supply of funds and therefore the real economy. Other prominent examples of the implications of shocks to intermediaries' net worth are He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014).

Our model is also related to the literature on the consequences of the limited enforceability of debt contracts, thus allowing for strategic default. Cooley et al. (2004) assume lending can take the form of long term state contingent debt contracts, borrowers can divert capital, and default is costly. They solve for the optimal dynamic contract that is self-enforceable and find that the equilibrium features amplification. Jermann and Quadrini (2012) also allow borrowers to default and derive borrowing constraints by as-

[5]For a survey, see Brunnermeier et al. (2013). For early criticism of the ability of these models to generate large amplifications see Kocherlakota (2000), and Cordoba and Ripoll (2004).

suming that lenders can recover the collateral with an exogenous probability (otherwise recovery is zero). They interpret this time-varying probability as "financial shocks" and find that they can explain a large share of observed dynamics of real and financial variables. These two papers abstract from the effect of (endogenous) asset prices on borrowing constraints, while in our model, as in KM, it is precisely this variable that drives results. Furthermore, we also allow for financial shocks as we consider a temporary increase in the discount factor of lenders (and thus in the equilibrium interest rate).[6]

The reminder of the paper is organized as follows. Section 2 presents the basic framework, which introduces default costs and renegotiation into KM's model. Section 3 develops an extension with asymmetric information about default costs that lead to "V-shaped" recoveries. In section 4 we use the model to interpret developments in several real estate markets, and their implications for the real economy, in the United States during the Great Recession. Section 5 concludes.

2 Amplification and default costs

2.1 Basic setup

We build on KM's setup. There are two types of producers, entrepreneurs and lenders, each with measure one. The entrepreneurs will turn out to be the (constrained) borrowers in equilibrium. Both sets of agents are risk neutral and maximize their expected utility given by

$$E_t \sum_{s=0}^{\infty} \beta^s x_{t+s} \qquad \text{and} \qquad E_t \sum_{s=0}^{\infty} \prod_{j=0}^{s} \beta'_j x'_{t+s},$$

where $0 < \beta < \beta'_t < 1$ are their respective discount factors, and x_{t+s} and x'_{t+s} are their consumptions in period $t+s$ of a perishable good.

There is a fixed aggregate endowment of a productive asset, or capital, $\overline{K}$. Capital is the only factor of production. Entrepreneurs have access to a linear production technology. Their output is divided into two parts. A subset of it (with productivity a) is "tradable", meaning that it can be used for market transactions; by contrast, "non-tradable" output (c per unit of input) can only be consumed by the entrepreneur. This assumption makes consumption positive in every period even though entrepreneurs choose to apply all the tradable output to acquire capital. Lenders have access to a standard production technology with decreasing returns:

$$y'_{t+1} = G(k'_t), \text{ where } G' > 0,\, G'' < 0,\, G''' \geq 0,\, G'(0) > aR_t > G'\left(\overline{K}\right),$$

where R_t is the gross interest rate (equal in equilibrium to the inverse of β'_{t+1}).We further assume $G''' \geq 0$ to limit the number of equilibria in the model to at most three.[7]

[6]Other recent contributions of the effect of financial shocks are Liu et al. (2013), Christiano et al. (2014), and Del Negro et al. (2017).

[7]It is well know that, in models that feature a demand for assets that is increasing in their price, multiple equilibria might exist. Kiyotaki and Moore (1997) limited their attention to one of these, the

KM's key assumption is a constraint that limits entrepreneurs' ability to borrow to the collateral they can provide, given by the anticipated value of their capital holdings. Specifically, if at date t an entrepreneur has assets k_t, then she can borrow b_t as long as the promised repayment does not exceed the market value of her assets at date $t+1$:

$$R_t b_t \leq q_{t+1} k_t,$$

where q is the price of capital.

This credit constraint is rationalized through the impossibility of the borrower to precommit to making productive use of the firm's assets. In KM this leads to amplification since they make the assumption that an enforcement technology exists which precludes default *after* the productive process has started (see footnote 13 in KM). Large multiplier effects emerge as a lower demand for capital by levered entrepreneurs reduces asset prices, and forces them to allocate more output to debt servicing, which causes further drops in prices: a *transitory* reduction in borrowing firms' net worth brings about a fall of the same order of magnitude in asset prices and aggregate output, and the economy takes a substantial time to return to the steady state. In contrast, if repayments were ex post bounded by the collateral of the debtor firm, so that the whole volume of current output remains available to borrowers, a transitory decline in productivity only has second order effects on asset prices and total production. Figure 1 shows the impulse response functions of output after a temporary negative productivity shock for the cases of full repayment and costless default.

We extend the basic KM model in three directions. First, we allow borrowers to default ex post at a cost. We assume this cost is proportional to the value of assets and that it affects entrepreneurs' net worth. Hence, if a borrower defaults, she must suffer a loss of tradable output, which is larger the higher her default costs. This implies that the entrepreneur has incentives to avoid outright default. Second, we allow lenders to negotiate a haircut $\varphi \geq 0$ in the contractual value of debt to prevent entrepreneurs from defaulting. We assume lenders and entrepreneurs split the surplus (given prices) according to Nash bargaining. Third, we will consider two types of shocks: a temporary productivity shock for entrepreneurs, and a temporary preference shock for lenders that reduces asset prices. Henceforth, we follow the convention that upper-case letters refer to aggregate quantities while lower-case letters denote individual quantities.

We assume entrepreneurs' operate a linear technology, such that given our assumptions on default, output is given by,[8]

$$y_{t+1} = F(k_t) = \begin{cases} (a_t + c)k_t, & \text{if there is no default in period } t+1, \\ (a_t + c)k_t - \alpha q_t k_t, & \text{if there is default in period } t+1, \end{cases}$$

where $0 \leq \alpha \leq 1$ is a measure of default costs.

one that features the highest asset level in the hands of entrepreneurs. Default and renegotiation have implications that affect the other expectational equilibria. Studying them is particularly useful when shocks are so severe that the original KM equilibrium ceases to exist.

[8]Note that we assume default costs are proportional to assets valued at their purchase price, instead of the current market price. This assumption simplifies the derivation of our main results. We provide intuition for changes in setup and outcomes if default costs were given by $-\alpha q_{t+1} k_t$.

Figure 1: Strategic default dampens the amplification mechanism.

We are interested in studying the response of the economy to unexpected transitory shocks that hit the economy at $t = 0$. As said, we consider two types of shocks. First, a productivity shock that makes tradable production, a_0, shift from a to $a(1-\delta)$ with $\delta > 0$. Second, a preference shock that makes lenders more impatient for one period, reducing $\beta'_1 = \tilde{\beta}$ to $\beta'_1 = \tilde{\beta}(1-\epsilon)$.[9] We assume there is perfect foresight from period 1 onwards.

To ensure that the economy converges to the steady state, we need the following assumption,[10]

Assumption 1.

$$c > \left(\frac{1}{\beta} - 1\right) a.$$

Suppose lenders offer borrowers a haircut of φ. Then, the flow-of-funds-constraint of a borrower in t would be:

$$q_t k_t + I_t^{ND}\left((1-\varphi)R_{t-1}b_{t-1} - q_t k_{t-1}\right) + x_t - c k_{t-1} = (a_t - \alpha(1 - I_t^{ND})q_{t-1})k_{t-1} + b_t\,, \quad (1)$$

where I_t^{ND} is an indicator variable that takes the value of 1 for $t \neq 0$, and for $t = 0$ if there is no default.

[9] Note that $\epsilon < \bar{\epsilon}$, with $\bar{\epsilon}$ given by $\tilde{\beta}(1-\bar{\epsilon}) = \beta$. Otherwise lenders would not have an incentive to lend.

[10] This is just KM's Assumption 2.

We denote steady-state quantities by $*$. We conjecture, and later verify, that in equilibrium $\{K_0\}$ is an increasing sequence that converges to K^*. Let $R = \tilde{\beta}^{-1}$ denote the steady state interest rate. Thus, $R_0 = \frac{R}{1-\epsilon}$ and, since lenders are risk neutral and unconstrained, their Euler equation yields $q_t = \frac{G'\left(\bar{K}-K_0\right)+q_{t+1}}{R_t}$.[11] Iterating forward and imposing a no-bubble condition yields

$$q_0 = (1-\epsilon)\left\{u(K_0) + \sum_{s=1}^{\infty}\frac{1}{R^s}u\left(K_s\right)\right\}, \tag{2}$$

where $u(K_0) \equiv \frac{1}{R}G'(\bar{K}-K_0)$ is the equilibrium downpayment when $R_t = R$. Under our conjecture, $\{q_t\}$ is an increasing sequence that converges to $q^* = \left(\frac{R}{R-1}\right)a$. This, together with assumption 1, ensures that investing as much as possible ($x_t = ck_{t-1}$, $R_t b_t = q_{t+1}k_t$) is an optimal strategy for entrepreneurs. In the period of the shock,[12]

$$\begin{aligned} 1+\hat{k}_0^R(\varphi) &= \frac{a}{u\left(K_0\right)(1-\epsilon)}\left(1-\delta+\frac{R}{R-1}\left(\hat{q}_0+\varphi\right)\right) \\ 1+\hat{k}_0^D &= \frac{a}{u\left(K_0\right)(1-\epsilon)}\left(1-\delta-\frac{R}{R-1}\alpha\right), \end{aligned} \tag{3}$$

where $\hat{k}_0^i = \frac{k_0^i-k^*}{k^*}$, $\hat{q}_0 = \frac{q_0-q^*}{q^*}$. Since there is perfect foresight from $t=1$ onwards,

$$1+\hat{k}_t^i = \frac{a}{u\left(K_t\right)}\left(1+\hat{k}_{t-1}^i\right). \tag{4}$$

Since all entrepreneurs are identical, default entails output losses, and there is no information asymmetry, in equilibrium there is no default, i.e. $K_0 = k_t^R$.[13] Thus, we can find K_{t+i} as a function of K_{t+i-1} from (4). Iterating backwards we obtain $K_{t+i} = f_i\left(K_t\right)$ with $f_0(K) \equiv K$. Using this relation, it can be shown that provided $K_t \leq K^*$, $\{K_t\}$ is, as conjectured, an increasing sequence that converges to K^*.[14]

Next, we compute the implied entrepreneurs' utilities of default and renegotiation given the shocks, aggregate capital K_t, and the proposed haircut φ,

$$U^i(\varphi;K_0) = cK^* + \beta ck_1^i + \beta^2 ck_2^i + ... + \lim_{t\to\infty}\beta^t ck_{t-1}^i.$$

Using our previous results, we obtain[15]

$$\begin{aligned} \frac{U^R(\varphi;K_0)-U^D(K_0)}{\beta cK^*} &= \frac{a}{u(K_0)}\frac{1}{(1-\epsilon)}\frac{R}{R-1}\left[(\hat{q}_0+\varphi)+\alpha\right] \\ &\quad \cdot\sum_{t=0}^{\infty}\beta^t\left(\prod_{s=1}^{t}\frac{a}{u\left(f_s(K_0)\right)}\right). \end{aligned} \tag{5}$$

[11] This equation equates the current asset price to the discounted "dividends" and future asset price.

[12] If default cost were proportional to the current value of assets, the last term in (3) would be $-\frac{R}{R-1}\alpha(1+\hat{q}_t)$.

[13] This follows from Coase's theorem (Coase, 1960).

[14] To see this result, first note that since $u(K)K$ is increasing in K, so $u\left(K_{t+1}\right)K_{t+1} < aK_t < aK^*$. Thus, $K_{t+1} < K^*$ and, by induction, $K_{t+i} < K^*$ $\forall i$. This, in turn, implies that $u\left(K_{t+i}\right) < a$ $\forall i$ and, since $u\left(K_{t+i}\right)K_{t+i} = aK_{t+1-1}$, $K_{t+i} > K_{t+i-1}$.

[15] This follows since $U^i(\varphi;K_t) = cK^* + (1+\widehat{k}_t^i)\beta cK^*\left[\sum_{s=0}^{\infty}\beta^s\left(\prod_{i=0}^{s}\frac{a}{u(f_i(K_t))}\right)\right]$.

By renegotiating, a borrower saves on the default costs, α and, in exchange, accepts to keep a share of the (negative) capital gains, $\hat{q}_0 + \varphi \leq 0$, which translates in a uniformly lower level of capital, both initially and in subsequent periods.[16] From (5), it follows that when $\varphi \geq -\hat{q}_0 - \alpha$ the entrepreneur does not default. Inspecting equation (5) we see that, conditional on a haircut level φ, K_0 has a negative effect on the utility difference, $U^R - U^D$. This effect follows from the fact that a higher K_0 implies higher current and future capital prices, leading to a lower impact of a given change in period t's net worth on capital demand, and a slower convergence to steady state (see (3)). Thus, the difference $\widehat{k}_0^R - \widehat{k}_0^D$ is reduced when K_0 increases. Note that beyond their impact on K_0 and $\hat{q}_0$, the only direct effect of the shocks is that a larger preference shock amplifies the utility difference.

Renegotiation gives an entrepreneur surplus $U^R(\varphi; K_0) - U^D(K_0)$ while a lender gets surplus $(1-\varphi)\, q^* K^* - q_0 K^* = -\left(\hat{q}_0 + \varphi\right) q^* K^*$. We assume these surpluses are divided according to Nash-bargaining. In other words, φ solves

$$\max_{\varphi \in [0,1]} \left(-\hat{q}_0 - \varphi\right)^{\theta} \left(U^R(\varphi; K_0) - U^D(K_0)\right)^{1-\theta}$$

where $\theta \in [0,1]$ is the bargaining power of lenders. The objective function is concave so we can use the FOC to characterize the solution. Taking logarithms, this gives[17]

$$\varphi = \max\left[-\hat{q}_0 - \frac{\theta}{1-\theta} \frac{U^R(\varphi; K_0) - U^D(K_0)}{\frac{d(U^R(\varphi;K_0) - U^D(K_0))}{d\varphi}}, 0\right] = \max\left[-\hat{q}_0 - \theta\alpha, 0\right].$$

The equilibrium haircut depends on the proportional effect that φ has on the entrepreneurs' surplus as captured by the term $\frac{U^R(\varphi;K_0)-U^D(K_0)}{\frac{d(U^R(\varphi;K_0)-U^D(K_0))}{d\varphi}}$. This term is independent of the shocks. Let $\bar{q} \equiv -\theta\alpha$. When $\hat{q}_0 \geq \bar{q}$, the price of capital is sufficiently high that the threat of default is not credible even if $\varphi = 0$. Hence, entrepreneurs bear all the capital losses in this region. In contrast, when $\hat{q}_0 < \bar{q}$, entrepreneurs are able to bargain a positive haircut. Finally, note that when entrepreneurs are very powerful, or default costs are low, $\theta\alpha \approx 0$, lenders bear most of the losses, $\varphi \approx -\hat{q}_0$. In contrast, when lenders have more bargaining power or default is more costly, they manage to extract some surplus from the entrepreneurs and $\varphi < -\hat{q}_0$. Note, however, that there is a limit to doing so since they can never increase the burden of debt. Hence, once $\varphi = 0$ increasing θ further has no effect.

Using our previous results it follows that the equilibrium is characterized by the fol-

[16] If $\frac{\beta a}{u(0)} > 1$, the last term does not converge when $K = 0$. In that case the only offer the entrepreneur will accept is $\varphi = -\hat{q}_t$

[17] If default cost were proportional to the current value of assets, then we would get $\varphi = \max\left[-\hat{q}_0 - \theta\alpha(1 + \hat{q}_0), 0\right]$.

Figure 2: Renegotiation.

lowing equations,[18]

$$[\text{net worth}] \quad : \quad \frac{u(K_0)K_0}{K^*} = \frac{a}{1-\epsilon}\left(1-\delta+\frac{R}{R-1}\max\left[-\theta\alpha, \hat{q}_0\right]\right) \tag{6}$$

$$[\text{asset pricing}] \quad : \quad \hat{q}_0 = \hat{q}(K_0,\epsilon) = \frac{R-1}{R}\frac{(1-\epsilon)}{a}\left\{u(K_0)+\sum_{t=1}^{\infty}\frac{1}{R^t}u\left(K_s\right)\right\}-1 \tag{7}$$

The first equation is the "net worth" relation, which links the size of the capital losses faced by the entrepreneur with the amount of capital she can retain the first period. When $\hat{q}_0 \geq \bar{q}$, the threat of default is not credible and the entrepreneur bears all the capital losses. Since the entrepreneur is borrowing constrained, her demand of capital is increasing in its price in this region. In contrast, when $\hat{q}_0 < \bar{q}$, entrepreneurs' capital losses are independent of the shock (lenders absorb any further increases in capital losses). The second equation is the standard "asset pricing" relation, which states that the price of capital is the discounted sum of future dividends.

The next proposition characterizes equilibria.

Proposition 1. (i) An equilibrium exists.
(ii) There exists a threshold $\bar{\Delta}(\epsilon)$ (with $\frac{d\bar{\Delta}}{d\epsilon} \leq 0$) such that for $\delta < \Delta(\epsilon)$ there exists an

[18] If $K_t = 0$ and $\frac{\beta a}{u(0)} > 1$, then $\hat{q}_t = (1-\epsilon)\frac{u(0)}{a} - 1$ is the solution to the second equation.

equilibrium $\{K(\delta,\epsilon), \hat{q}(\delta,\epsilon)\}$ with no renegotiation, i.e. $\varphi = 0$. The equilibrium capital and prices are continuous in ϵ and δ and strictly decreasing in both arguments.
(iii) There exists a threshold $\underline{\Delta}(\epsilon) \leq \bar{\Delta}(\epsilon)$ (with $\frac{d\underline{\Delta}}{d\epsilon} \leq 0$) such that for $\delta > \underline{\Delta}(\epsilon)$ there exists an equilibrium $\{K(\delta,\epsilon), \hat{q}(\delta,\epsilon)\}$ with renegotiation, i.e. $\varphi > 0$. The equilibrium capital and prices are continuous in ϵ and δ, and strictly decreasing in δ. Equilibrium capital is strictly increasing, an prices ambiguous, in ϵ. The haircut φ is increasing in δ and ambiguous with respect to ϵ.
(iv) When $\bar{\Delta}(\epsilon) = \underline{\Delta}(\epsilon) \equiv \Delta^*(\epsilon)$, the equilibrium is unique and $\frac{d\Delta^*}{d\epsilon} \leq 0$.

Proof. See Appendix 6.1. □

Figure XX illustrates equilibria graphically. The net-worth curve has a kink at the point in which haircuts start to be positive. When $\hat{q}_0$ is above this point, defaulting is sufficiently unattractive that the threat of default is not credible. Hence, in this region, entrepreneurs bear all the losses and capital demand increases with its price. When $\hat{q}_0$ is below the threshold, default is credible, there is renegotiation and creditors share the burden of capital losses with debtors. The figure shows the case in which there are three equilibria.[19] In this case equilibrium (ii) is the one with the highest level of capital while equilibrium (iii) is the one with the lowest level.[20] As δ increases the net-worth curve moves up and to the left. For both equilibria of type (ii) and (iii) capital and prices decrease. Since for equilibrium (iii) entrepreneurs' capital gains are independent of δ (i.e. $\varphi + \hat{q}_0$ is constant), this implies φ has to increase substantially to compensate the change in $\hat{q}_0$.

In contrast, preference shocks have two opposite effects on entrepreneurs' capital demand. First, there is a positive effect since the shock reduces the required downpayment and thus allows an increase in leverage. Second, there is a negative effect since lower asset prices imply capital losses, and therefore a reduction in demand. Lemma A0 in Appendix 6.1 shows that preferences shocks shift both curves downwards (and the vertical part of the net worth to the right), but the effect is stronger on the asset pricing curve. Thus preference shocks reduce capital demand and asset prices for equilibrium (ii). In contrast, for equilibrium (iii) capital demand increases with preferences shocks. This happens because renegotiation puts a lower bound on capital losses, which eliminates the negative effect of a preference shock on demand, leaving only the positive effect from an increase in leverage. The effect of preference shocks is negative for equilibrium (ii) and ambiguous for equilibrium (iii). Since for equilibrium (iii) $\varphi + \hat{q}_0$ is constant, preference shocks have also ambiguous effects on haircuts. The effect of preference shocks on prices is ambiguous for equilibrium (iii) (since for equilibrium (iii) $\varphi + \hat{q}_0$ is constant, preference shocks have also ambiguous effects on haircuts).

For the equilibrium to be unique, the NW curve has to be steeper than the AP curve at the point in which it has the kink. In this case $\bar{\Delta}(\epsilon) = \underline{\Delta}(\epsilon) \equiv \Delta^*(\epsilon)$, and the threshold is such that the first, and only, intersection between the asset-pricing and net-worth curves

[19] There are no more than three because we assumed $G''' > 0$, which guarantees that the non-vertical branch of the NW and the AP intersect at most twice.

[20] An equilibrium with an intermediate level of capital might exist. This would be unstable and we will ignore it from our analysis.

is exactly at the kink of the latter. Then, the economy behaves like an equilibrium of type (ii) for $\delta < \Delta^*(\epsilon)$ and like an equilibrium of type (iii) for $\delta > \Delta^*(\epsilon)$.

The following proposition characterizes how the equilibrium with renegotiation is affected by default costs and bargaining power.

Proposition 2. For the equilibrium characterized in Proposition 1 (iii)
(i) The threshold $\Delta^*(\epsilon)$ is increasing in θ and α
(ii) When $\delta > \Delta^*(\epsilon)$, equilibrium capital and prices are strictly decreasing in θ and α. The effect on the haircut is ambiguous.

Proof. See Appendix 6.1 □

Figure 3: theta alpha.

Figure XX illustrates changes in default costs or bargaining power (their effects enter symmetrically in equilibrium equations (6) and (7)). Note that the only curve that moves in this case is the vertical renegotiation branch of the net-worth curve. More specifically, when lenders' bargaining power increases, or entrepreneurs' default cost is higher, $\bar{q}$ decreases and the kink in the net-worth curve shifts to the left. As long as we stay within the region where $\varphi > 0$, this implies that both K_0 and $\hat{q}_0$ decrease with θ and α. In other words, higher bargaining power by lenders and higher default costs reduce the stabilizing effect of renegotiation and move us closer to the KM world, with significant amplification. Since $\hat{q}_0$ is decreasing the effect on φ is ambiguous. To understand this result, first note that keeping K_0 constant at the original equilibrium value, the shift in the renegotiation branch of the net-worth curve measures the size of the "partial equilibrium" effect. For example, if α decreases, then entrepreneurs' threat of default is more credible and they are able to extract a larger surplus from creditors. This partial equilibrium effect triggers a general equilibrium force in the opposite direction: As entrepreneurs become wealthier (higher $\hat{q}_0 + \varphi$), they are able to buy more capital, raising prices and lowering the threat of default. The size of this general equilibrium effect is related to the steepness of the asset-pricing function. If it is very steep, a small additional amount of capital in the hands of entrepreneurs may raise prices enough such that the initial partial equilibrium effect is overturned and haircuts decrease.[21]

Finally, note that an economy with $\theta = 1$ generates the same behavior of capital and prices as an economy in which renegotiation is not allowed and entrepreneurs default. The reason is that lenders extract all the surplus when $\theta = 1$ and, hence, entrepreneurs' net-worth is unaffected by renegotiation. The only difference between both scenarios is that, in an economy with default, output will decrease further due to the default costs.

3 Asymmetric Information about Default Costs

The aftermath of a financial crisis is often characterized not only by debt restructuring negotiations but also by outright default and bankrupcies. In this section, we extend our model to allow for default in equilibrium. In order to do so, we postulate that the entrepreneurs' default costs are private information.[22] More specifically, we allow for heterogeneity in the size of the default cost α_i faced by each entrepreneur i, which is known by the entrepreneur but unknown to the lender, who only knows the cumulative distribution function $F(\alpha) \in C^2$.

Since lenders ignore the type of entrepreneurs they have lent to, they face a tradeoff in the event of an unforeseen negative shock: a higher level of debt relief makes more borrowers willing to accept, but the rent extracted from each entrepreneur gets smaller. Lenders will balance the two effects, recognizing that the willingness of borrowers to accept a certain deal will be weaker for those with low default costs. For simplicity we now assume that lenders have all the ex post bargaining power, i.e. $\theta = 1$. We keep the timing of the previous section: First lenders make an offer and then entrepreneurs decide whether to accept it or not.[23]

We solve the problem by backward induction. First, an entrepreneur must decide whether to accept or decline debt reduction φ, taking as given the dynamics of aggregate capital and prices. From equation (5), we know that entrepreneurs will only accept a haircut offer if $\alpha_i \geq -(\hat{q}_0 + \varphi)$. Taking this into account lenders minimize expected losses. For a given debt offer φ, a lender incurs in a cost (in percentage terms) given by $-\hat{q}_0$ on the fraction $F(-(\hat{q}_0 + \varphi))$ of the borrowers who default and deliver their collateral, whereas he loses φ (in percentage terms) on the complementary fraction $1 - F(-(\hat{q}_0 + \varphi))$ of credits that are renegotiated. Since individual lenders take prices, $\hat{q}_0$, as given, we can

[21]In numerical simulations we found that the effect on φ in either direction is quantitatively very small.

[22]It may be noted that, since all agents are assumed to treat ex ante the likelihood of a shock as strictly negligible, informational asymmetries about features that become relevant only in the event of a disturbance have no influence on the contracts outstanding when the shock arrives.

[23]Given that lenders are risk neutral, we proceed as if each one of them faces a continuum of entrepreneurs. This makes the number of borrowers who default for a given debt reduction offer a deterministic quantity from the point of view of a single lender, and not only at an aggregate level.

write their problem as,

$$\min_{\varphi \geq 0} \ (\hat{q}_0 + \varphi)\left(1 - F(-(\hat{q}_0 + \varphi))\right).$$

The first order condition yields,

$$1 - F\left(-\left(\hat{q}_0 + \varphi\right)\right) + f\left(-\left(\hat{q}_0 + \varphi\right)\right)\left(\hat{q}_0 + \varphi\right) \geq 0, \quad \text{with equality if } \varphi > 0. \tag{8}$$

We make assumptions to guarantee a unique solution for the lenders problems. Either $-\alpha(1 - F(\alpha))$ is strictly convex, or the Mills ratio, $\frac{1-F}{f}$, is weakly decreasing with $\lim_{\alpha \to 1} \frac{1-F}{f} < 1$.[24]

Lemma 1. Let μ denote the share of defaulting entrepreneurs. If $K_0 < K^*$, then $\hat{q}_0 + \varphi < 0$ and $\mu > 0$. In other words, there is default in equilibrium.

Proof. Note that $\hat{q}_t + \varphi = 0$ would imply the first term of (8) is positive so $\hat{q}_t = \hat{q}_t + \varphi = 0$. This is a contradiction since $K_t < K^*$ implies $\hat{q}_t < 0$. Thus $\hat{q}_t + \varphi < 0$. From (5), it follows that entrepreneurs with $\alpha_i < -\left(\hat{q}_t + \varphi\right)$ would default, and thus $\mu = F(-\left(\hat{q}_t + \varphi\right)) > 0$. □

Let $\bar{\alpha}$ denote the solution of (8) ignoring the non-negativity constraint, which is constant. The solution to lenders' problem can then be written as

$$\varphi^{AI} = \max[-\hat{q}_0 - \bar{\alpha}, 0],$$

where $\hat{q}_0$ is still given by (2). Note the symmetry with the derivation of the equilibrium haircut in the previous section, with $-\bar{q}$ replaced by $\bar{\alpha}$. This simplifies the derivation of most of our remaining results.

The net-worth relation now needs to take into account that there is default in equilibrium. Recall that entrepreneurs with $\alpha < \min[\bar{\alpha}, -\hat{q}_0]$ default, while agents with $\alpha \geq \min[\bar{\alpha}, -\hat{q}_0]$ renegotiate. Then, the net-worth relationship for an entrepreneur of type i yields

$$\frac{u(K_0)k_t(\alpha_i)}{aK^*} = \frac{1}{1-\epsilon}(1 - \delta - \frac{R}{R-1}\min[\alpha_i, \min[\bar{\alpha}, -\hat{q}_0]])$$

Integrating individual capital holdings yields

$$\begin{aligned} \frac{u(K_0^{AI})K_0^{AI}}{aK^*} \quad &= \quad \frac{1}{1-\epsilon}\Bigg(1 - \delta - \frac{R}{R-1}\min[\bar{\alpha}, -\hat{q}_0] \\ &\quad - \frac{R}{R-1}\int_0^{\min[\bar{\alpha}, -\hat{q}_0]} (\alpha - \min[\bar{\alpha}, -\hat{q}_0])dF\left(\alpha\right)\Bigg) \end{aligned} \tag{9}$$

[24]For example, a uniform distribution would satisfy either requirement. Monotone increasing probability functions satisfy the first requirement, and a symmetric Beta distribution with parameter higher than one satisfies the second one.

Note that the new net-worth relationship still describes an upward relationship between K_0 and $\hat{q}_0$ for $\hat{q}_0 \geq -\bar{\alpha}$ and a vertical line when $\hat{q}_0 < -\bar{\alpha}$. The model is closed by the same asset-pricing relationship as before.

With asymmetric information, after a preference shock entrepreneurs with low default costs take advantage of the positive leverage effect of a decrease in asset prices, with minor effects on their net worth since they default. Depending on parameters this effect might dominate the overall negative effect that the preference shock has on non-defaulting entrepreneurs. We make the following assumption to rule out this case.[25]

Assumption 2.

$$\frac{R-1}{R} < 1 - F(1 - \beta R).$$

The following proposition characterizes the equilibrium,

Proposition 3. Under assumption 2, the following holds:

(i) There exists a threshold $\bar{\Delta}^{AI}(\epsilon)$ such that for $\delta < \bar{\Delta}^{AI}(\epsilon)$ there exists an equilibrium $\left\{K^{AI}(\delta, \epsilon), \hat{q}^{AI}(\delta, \epsilon)\right\}$ with no renegotiation, i.e. $\varphi^{AI} = 0$. The equilibrium capital and prices are continuous in ϵ and δ and strictly decreasing in both arguments. The share of defaulting entrepreneurs $F(\alpha)$ increases with δ and ϵ.
(ii) There exists a threshold $\underline{\Delta}^{AI}(\epsilon) \leq \bar{\Delta}^{AI}(\epsilon)$ such that for $\delta > \underline{\Delta}^{AI}(\epsilon)$ there exists an equilibrium $\left\{K^{AI}(\delta, \epsilon), \hat{q}^{AI}(\delta, \epsilon)\right\}$ with renegotiation, i.e. $\varphi^{AI} > 0$. The equilibrium capital and prices are continuous in ϵ and δ, strictly increasing in ϵ and strictly decreasing in δ. The haircut φ^{AI} is increasing in δ and ambiguous with respect to ϵ. The share of defaulting entrepreneurs is constant at $F(\bar{\alpha})$.
(iii) When a set of conditions specified in Appendix XX are satisfied, $\Delta^{AI*}(\epsilon) \equiv \bar{\Delta}^{AI}(\epsilon) = \underline{\Delta}^{AI}(\epsilon)$. The equilibrium is unique and $\frac{d\Delta^{AI*}}{d\epsilon} < 0$.

Proof. See Appendix 6.2. □

Proposition XX shows that the features we analyzed in the previous section are robust to introducing asymmetric information. Furthermore, it states that the share of defaulting entrepreneurs increases while renegotiation is redundant (i.e. $\varphi^{AI} = 0$) and stays constant when renegotiation is triggered.[26] To understand the differential effects of asymmetric information on the equilibrium, we compare the solution to the case of perfect information. In that case, lenders can tailor the offered haircut to each entrepreneur, offering $\varphi_i^{PI} = \max[-\hat{q}_0 - \alpha_i, 0]$. Thus, the individual net-worth relation yields

$$\frac{u(K)k_0(\alpha_i)}{aK^*} = \frac{1}{1-\epsilon}(1 - \delta - \frac{R}{R-1}\min[\alpha_i, -\hat{q}_0]),$$

[25] Under assumption 2 we have $\frac{\partial \hat{q}^{NW,AI}(K^*,\delta,\epsilon)}{\partial \epsilon} = \frac{\frac{\partial \hat{q}^{NW}(K^*,\delta,\epsilon)}{\partial \epsilon}}{1-F(\hat{q}^{NW,AI})} = -\frac{R-1}{R}\frac{\epsilon}{1-F(\hat{q}^{NW,AI})} \geq -\epsilon = \frac{\partial \hat{q}(K^*,\epsilon)}{\partial \epsilon}$. Where the inequality follows since $\epsilon < \bar{\epsilon} = 1 - \beta R$. Thus K_0 cannot be above K^*.

[26] The fact that it is exactly constant relies on the assumption that there is no feedback between asset prices and default costs. Depending on other details of the model, renegotiation may slow down the pace (or even reverse) at which the share of defaulting entrepreneurs increases with the size of the shock.

which integrating across individual capital holdings yields

$$\frac{u(K_0^{PI})K_0^{PI}}{aK^*} = \frac{1}{1-\epsilon}\left(1 - \delta + \frac{R}{R-1}\hat{q}_0 - \frac{R}{R-1}\int_0^{-\hat{q}_0}(\alpha + \hat{q}_0)dF(\alpha)\right) \tag{10}$$

Comparing (9) and (10), we see that asymmetric information transfers wealth from creditors to debtors when shocks are large enough to trigger haircuts. Since lenders are unable to discriminate, they are forced to give large haircuts to high-default cost agents, which in turn boosts their wealth and dampens the response of asset prices. On the other hand, asymmetric information leads to a positive share of defaulting entrepreneurs, which generates output losses associated with default costs. Because there are no costs associated to default in subsequent periods, output will recover faster when there is asymmetric information. We collect these observations in the following proposition,

Proposition 4. In an economy with asymmetric information, a shock has the following effects, relative to an equivalent economy with perfect information:
(i) When $\hat{q}_0 \geq -\bar{\alpha}$, $\varphi^{AI} = 0$, $K_0^{AI} = K_0^{PI}$, and output is lower.
(ii) When $\hat{q}_0 < -\bar{\alpha}$, $\varphi^{AI} > 0$, $K_0^{AI} > K_0^{PI}$, and the effect on output is ambiguous.
(iii) For both (i) and (ii) output recovers faster after the initial shock.

Proof. See Appendix 6.2. □

4 The Great Recession and Real Estate Markets

Real estate prices collapsed during the Great Recession, with peak to trough decreases of around 31% in residential, and 35% in commercial real estate (Duca and Ling, 2015). The collapse in house prices led to a sharp decrease in consumer demand as mortgage holders deleveraged (Mian et al., 2013), and employment fell, particularly in nontradable sectors (Mian and Sufi, 2014). Besides household balance sheets, intermediaries' net worth, which was negatively affected by their real estate exposure, affected the scale of this crisis. Chodorow-Reich (2014) shows that borrowers whose main bank was in bad health at the time of the recession were less likely to get a loan (or favorably renegotiate one), and that this had negative effects on employment, particularly for small and medium-sized firms.[27] Finally, there is also evidence that firms' balance sheet were instrumental in the transmission of demand shocks during the Great Recession. Giroud and Mueller (2017) show that firms with higher leverage suffered higher employment losses in response to the decrease in consumer demand.

In the United States, residential mortgage loans and secured commercial property loans are typically nonrecourse.[28] Thus, we expect our model to be particularly relevant for asset price dynamics in these markets, and their implications for the real economy. Heterogeneity in default costs, or bargaining power, across borrowers or lenders should lead to different responses of property prices, and therefore in macroeconomic outcomes, to a given shock.[29]

Since we model the Great Recession as an "MIT shock", we first document that in these markets the probability of such an event was perceived to be very low. Benmelech et al. (2005) show that in the 1990s, commercial loan contracts reflected flexibility in zoning regulations that affected the use of property. Importantly, they show that zoning restrictions have a larger effect on contractual terms in states with non-judicial foreclosures.[30] Similarly, Pence (2006) finds that individual home mortgages are between 3% and 7% smaller in judicial foreclosure states in 1994 and 1995. But, Mian et al. (2015) show that in the years prior to the Great Recession, when house prices were rising, the size of mortgages did not significantly differ across states with or without judicial foreclosure requirement. And Duca and Ling (2015) estimate that a significant portion of the decline in commercial real estate premia during the boom was associated with banks

[27]Greenstone et al. (2014) use predicted county level lending, constructed using heterogeneity in banks' pre-crisis county market share and in their national change in lending, to estimate the real economy impact of the crisis.

[28]Nonrecourse debt is secured by collateral. If the borrower defaults, the issuer can seize the collateral but cannot seek out the borrower for any further compensation. In contrast, a recourse loan allows a lender to go after the debtor's assets that were not used as loan collateral in case of default.

[29]We should also observe different effects of heterogeneity in default costs, or bargaining power, on intermediaries' health, as lenders more exposed to low default cost borrowers, or operating in environments with institutions favoring borrowers' bargaining power, should suffer more, and through contractions of credit differentially affect the real economy. Finally, as long as firms use commercial real estate for collateral, those firms with lower default costs and having more bargaining power would be better able to hoard labor in the face of temporary shocks.

[30]States that require that a foreclosed sale take place through the courts (judicial states) impose substantial costs and time on lenders seeking to foreclose on a house.

taking more risks in the face of weak regulatory capital requirements. They argue that market participants likely inferred that commercial real estate's risk had declined, relative to other assets. These two findings suggest that during the boom years, market participants assigned a very low probability to a large nationwide decrease in property prices.

Given the nonrecourse nature of mortgages, and the significant drop in home prices, there was an unprecedented increase in delinquency rates and foreclosures since prices started to decline in 2006. Using data for Massachusetts between 1987 and 2009, Campbell et al. (2011) show that foreclosures induced fire sales, i.e. they had a negative price effect on nearby houses that came to the market later, and this effect is larger during foreclosure waves.[31] Since Mian et al. (2015) find evidence that foreclosures increase the number of new for-sale house listings, we thus interpret foreclosures to be a proxy of supply in real estate markets.

Our model implies that differences in default costs and bargaining power should result in differences in delinquency and sale (foreclosure) rates, with differential effects on property prices and other measures of economic activity. In regions where borrowers have more bargaining power, we should see that lenders are more willing to renegotiate loans resulting in lower foreclosure rates, higher property prices and lower output losses. Similar results should follow in regions where borrowers have lower default costs, or where the distribution of default costs is skewed to lower values. Finally, in regions with higher output losses associated to default, e.g. due to costs associated to foreclosures, we should see that output recovers faster. We now turn to two recent papers whose findings are consistent with these model predictions.

Mian et al. (2015) use differences in state foreclosure requirements to estimate the effects of foreclosures on economic outcomes. First, they show that non-judicial states had significant larger foreclosure rates than judicial states. Using state foreclosure laws as an instrument they find that foreclosures have a strong effect on house prices, with 8% lower house price growth between 2007 and 2009 in states in the 90^{th} percentile of the foreclosure rate distribution relative to the median. Using the same empirical strategy they document that foreclosures led to a decline in residential investment and auto sales. They find evidence of stronger recovery, for home prices, residential investment, and auto sales, in non-judicial states in 2012. If we associate a judicial requirement to lower bargaining power for lenders (lower θ), these results are consistent with our model's predictions.

Agarwal et al. (2017) study the impact of the 2009 Home Affordable Modification Program (HAMP). This program gave intermediaries incentives to renegotiate mortgages.[32] First, they use investor-owned properties (which were initially not eligible) as control for the effect of HAMP on renegotiations. They find that the program led to an net increase in the annual rate of permanent modifications of about 0.57 percentage points.[33] Sec-

[31]Massachusetts had an earlier foreclosure wave in the early 1990s following a local property price bust.

[32]There is some debate on why servicers were so reluctant to renegotiate mortgages during the Great Recession. Piskorski et al. (2010) argue that this is due to securitization, while Adelino et al. (2013) argue that asymmetric information is more important.

[33]At the same time, the program reduced the foreclosure rate of eligible properties by 0.37 percentage

ond, the authors exploit differences in intermediaries' ability to implement renegotiations (servicers with high preprogram renegotiation experience perform significantly more permanent HAMP modifications), and regional heterogeneity in the share of loans that are serviced by high-experience servicers, to evaluate the impact of the program on house prices and other economic outcomes. They find that regions with a higher share of experienced servicers had lower house price declines, lower consumer debt delinquency rates, and a modest increase in auto sales. We interpret that experienced servicers are better at screening borrowers, and thus more likely to identify their true default costs. Thus, according to proposition 4, in regions with more experienced servicers renegotiation is more likely and amplification is dampened. Thus, Agarwal et al.'s (2017) findings are consistent with our model's predictions.

We conclude this section by briefly discussing hotel business during the Great Depression. This sector was particularly affected by economic conditions, as it provides lodging accommodation in a spot market, with revenue earned per room falling by almost 17% in 2009.[34] The largest publicly traded hotel chains in the United States saw their stock prices fall by around 80% between July 2007 and March 2009.[35] Given the large initial shock in this sector, and the generous contracts with minimal covenants that characterized lending during the boom, our model predicts that we should observe that all borrowers either secured debt renegotiations or defaulted.

If we associate hotel group size with borrower bargaining power (or if default costs, or their distribution, were inversely related to hotel size), our model predicts that the probability of renegotiation is positively related to hotel group size.[36] A prominent example of renegotiation was the deal that Blackstone secured for Hilton's debts in April 2010.[37] Debt was restructured from \$20 to \$16 billion and maturity extended by two years. The restructuring included the repurchase of \$1.8 billion of secured debt with a 54% discount. Other large groups that restructured their debts were MGM Mirage in April 2009, and Harrah's in March 2010. Not all hotel firms managed to renegotiate their debts: Sunstone Hotel Investors defaulted on \$300 million of debt in June 2009 and had 13 hotels seized by its bank. Just days later, and signalling its low default costs, the firm announced its intention to buy hotels at a discount.

5 Conclusions

A representation of deep macroeconomic crises requires attention to the ways in which parties in financial contracts, and the legal system itself, process situations of widespread

points. The foreclosure rate is roughly 12% lower than in the control group during the program period.

[34]Source: The Economist (2010).

[35]Starwood Hotels and Resorts Worldwide, Wyndham Worldwide Corporation, and Marriott International stock prices fell 85%, 84%, and 71% respectively in this period.

[36]Strictly speaking we can make this statement only for the model with no asymmetric information, since the model with asymmetric information imposes that lenders have all the bargaining power.

[37]Blackstone acquired Hilton Hotels Corporation in July 2007 through a leveraged buyout funding the transaction with 78.5% debt and 21.5% equity. See Phalippou (2014), who also describes lending in the 2005-2007 credit boom years as "cov-lite".

broken promises. We extend the existing literature by assuming that, when the value of assets posted as collateral is below the value of debt, the parties in financial contracts may engage in negotiations to re-define payments. This corresponds to the existence of bankruptcy procedures that limit the ability of creditors to collect debts, and the observed fact that renegotiations are common in moments of large macroeconomic disturbances. The resulting analysis highlights the connection between the repercussions of a shock and the ex post bargaining power of borrowers and lenders. The outcome of debt renegotiations, and thus the macroeconomic consequences of the shock, are seen to be influenced by institutional factors and by structural and technological features which determine default costs.

In order to study the implications of the costs of strategic default on the ex post resolution of crises, we introduced a number of changes to the Kiyotaki and Moore (1997)'s model. We found that there is always a threshold level for the initial shock above which the amplification effects through the reallocation of capital to lower productivity uses and through lower asset prices is dampened as lenders are led to accept some debt reductions in order to avoid default; that threshold is larger for firms with higher default costs and in environments with weaker borrower protection. When these default costs are assumed to be private information, the model suggests that economies with more severe asymmetries would experience sharper "V-shaped" recoveries after a crisis.

The model suggests that credit-related recessions may be of two types, according to the institutional and technological costs of default. In some instances, either through government intervention or bankruptcy procedures that preserve financially troubled firms, constraints on the access to resources for production would get somehow relieved (even if this happens after a period of turmoil). Other "balance sheet crises" would show long periods of deleveraging where firms hold to their capital but investment and production are financially restricted and asset prices stay depressed, as argued by Koo (2003) for Japan's "lost decade" and the Great Depression.

The model that we have considered is useful in interpreting developments in real estate markets in the United States during the Great Recession. In particular, it suggests exceptional intervention in debt markets, such as HAMP, have significant macroeconomic effects. The model is also apt to be extended in several ways. The treatment of the shock as a zero probability bolt from the blue can be a useful analytical device, and suitable to our purpose of studying the ex post resolution of large aggregate disturbances. But it certainly leaves aside very relevant analytical and practical issues.

Since most debt obligations are defined as non-contingent amounts of goods, various kinds of behavior can modify the results of the model if agents contemplate the potential of disturbances, especially by moderating the impacts of small shocks if, for instance, debtors find it convenient to leave a slack in the collateral constraint in normal times for precautionary reasons according to their default costs (Brunnermeier et al. (2013) find that firms with higher default costs borrow less, and thus have lower financial risk), or the supply of credit incorporates the likelihood of debt reductions, depending on the characteristics of the institutions that deal with defaulted debts. Then, the macroeconomic responses may show two types of non-linearities: one, an increase in the financial multipliers with the size of the shock (evocative of a corridor effect; see Leijonhufvud (1973)) as the financial buffer stocks are exhausted and, in the other extreme, a moderation of impacts as debts are renegotiated in the event of a very strong shock.

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6 Appendix

6.1 Proof of proposition 1

We start with the following lemmas

Lemma A0. We can rewrite the net worth condition, (6), as $\hat{q}^{NW}(K_0,\delta,\epsilon)$ given by

$$\hat{q}^{NW}(K_0,\delta,\epsilon) = \max\left[\frac{R-1}{R}\left(\frac{u(K_0)K_0}{aK^*}(1-\epsilon)-(1-\delta)\right), -\theta\alpha\right].$$

It follows that if K_0 is an equilibrium such that $\hat{q}^{NW}(K_0,\delta,\epsilon) = \hat{q}(K_0,\epsilon)$, then $\frac{\partial \hat{q}(K_0,\epsilon)}{\partial\ \epsilon} \leq \frac{\partial \hat{q}^{NW}(K_0,\delta,\epsilon)}{\partial\ \epsilon} \leq 0$.

Proof. When $\hat{q}^{NW}(K_0,\delta,\epsilon) = -\theta\alpha$, $\frac{\partial \hat{q}^{NW}}{\partial\ \epsilon} = 0$ XXXX PORQUE ESTO ACA, VER PRUEBA OTROS LEMMAS (2?) (note that in this case $\frac{dK_0}{d\epsilon} > 0$). XXX Else $\frac{\partial \hat{q}^{NW}}{\partial\ \epsilon} = -\frac{\hat{q}^{NW}+\frac{R-1}{R}(1-\delta)}{1-\epsilon}$, and $\frac{\partial \hat{q}}{\partial\ \epsilon} = -\frac{\hat{q}+1}{1-\epsilon}$. The result then follows when $\hat{q}^{NW} = \hat{q}$ since $\frac{R-1}{R}(1-\delta) \leq 1$. QED

Lemma A1. At the steady state, and when there is no renegotiation, $\hat{q}^{NW}(K_0,\delta,\epsilon)$ is steeper than $\hat{q}(K_0,\epsilon)$.

Proof. The slopes of $\hat{q}^{NW}(K_0,\delta,\epsilon)$ and $\hat{q}(K_0,\epsilon)$ when $K_0 = K^*$ are given by[38]

$$\begin{aligned}
\frac{d\hat{q}^{NW}}{dK_0}|_{K_0=K^*} &= (1-\epsilon)\frac{R-1}{R}\frac{1}{K^*}\left(\frac{1}{\eta}+1\right), \\
\frac{d\hat{q}}{dK_0}|_{K_0=K^*} &= (1-\epsilon)\frac{R-1}{R}\frac{1}{K^*}\left(\frac{1}{\eta}+\frac{1}{\eta}\frac{\frac{\eta}{R(\eta+1)}}{1-\frac{1}{R}\frac{\eta}{\eta+1}}\right).
\end{aligned}$$

Since $R > 1$, it is always the case that the former is steeper. QED

Lemma A2. Let $K^{\varphi}(\delta,\epsilon)$ and $\hat{q}^{\varphi}(\delta,\epsilon)$ be the implicit solutions to

$$\begin{aligned}
-\theta\alpha - \hat{q}^{NW}\left(K^{\varphi}(\delta,\epsilon),\delta,\epsilon\right) &= 0 \\
\hat{q}^{\varphi}(\delta,\epsilon) - \hat{q}\left(K^{\varphi}(\delta,\epsilon),\epsilon\right) &= 0.
\end{aligned}$$

Then, $K^{\varphi}(\delta,\epsilon)$ and $\hat{q}^{\varphi}(\delta,\epsilon)$ are well-defined and continuous functions, with $\frac{dK^{\varphi}}{d\delta} < 0$, $\frac{d\hat{q}^{\varphi}}{d\delta} < 0$, and $\frac{dK^{\varphi}}{d\epsilon} > 0$.

Proof. By definition, the pair $(K^{\varphi},\hat{q}^{\varphi})$ denotes the intersection between the asset pricing relation (7) with a vertical line at capital demand consistent with the onset of renegotiation according to (6). This would be an equilibrium of the model if $-\theta\alpha \geq \hat{q}(K^{\varphi},\epsilon)$ (and provided $K^{\varphi} \geq 0$). Note that since $-\theta\alpha$ is unaffected by δ while $\hat{q}^{NW}$ is strictly increasing in δ, there is a unique solution to this system of equations. Furthermore, since both curves are continuous, by the implicit function theorem, K^{φ} and $\hat{q}^{\varphi}$ are continuous functions. Applying again the implicit function theorem, the derivative of K^{φ} with respect to the productivity shock is given by,

$$\frac{dK^{\varphi}}{d\delta} = -\frac{\frac{\partial \hat{q}^{NW}}{\partial\delta}}{\frac{\partial \hat{q}^{NW}}{\partial K}} < 0.$$

[38]Here $\frac{1}{\eta}$ is, as in KM, $\frac{d\log u(K)}{d\log K_0}|_{K=K^*}$.

Furthermore, it is immediate that $\frac{d\hat{q}^{\varphi}}{d\delta}<0$. A similar analysis for preference shocks shows that $\frac{dK^{\varphi}}{d\epsilon}>0$ (since $\frac{\partial\hat{q}^{NW}}{\partial\epsilon}<0$). The sign of $\frac{d\hat{q}^{\varphi}}{d\epsilon}$ is ambiguous. XXX PORQUE ES AMBIGUO? ACA Y EN OTRAS PARTES TENGO DUDAS XXX QED

Lemma A3. Consider the model without renegotiation. Then, for any $K<K^{*}$ and ϵ, there exists at most one δ such that $\hat{q}^{NW}(K,\delta,\epsilon)=\hat{q}(K,\epsilon)$.

Proof. First, note that $\frac{d\hat{q}^{NW}}{dK}$ is independent of δ. By the fundamental theorem of calculus, we can always write

$$\hat{q}^{NW}(K,\delta,\epsilon)=\hat{q}^{NW}(K^{*},\delta,\epsilon)-\int_{K}^{K^{*}}\frac{d\hat{q}^{NW}}{dK}(s)\,ds.$$

Since - given K and ϵ - $\hat{q}(K,\epsilon)$ is a constant and $\hat{q}^{NW}(K^{*},\delta,\epsilon)$ is strictly increasing in δ, there is at most one δ that solves $\hat{q}^{NW}(K,\delta,\epsilon)=\hat{q}(K,\epsilon)$. QED

(i) First note that $\hat{q}^{NW}(K^{*},\delta,\epsilon)=\max[\frac{R-1}{R}(\delta-\epsilon),\bar{q}]$, and that $\hat{q}(K^{*},\epsilon)=-\epsilon$, such that $\hat{q}^{NW}(K^{*},\delta,\epsilon)\geq\hat{q}(K^{*},\epsilon)$. Next, consider two cases. First if $\hat{q}(K^{\varphi}(\delta,\epsilon),\epsilon)\leq\max[-\frac{R-1}{R}(1-\delta),\bar{q}]$, then $(K^{\varphi}(\delta,\epsilon),\hat{q}(K^{\varphi}(\delta,\epsilon),\epsilon))$ is an equilibrium. If $\hat{q}(K^{\varphi}(\delta,\epsilon),\epsilon)>\max[-\frac{R-1}{R}(1-\delta),\bar{q}]$, then $\hat{q}(K^{\varphi}(\delta,\epsilon),\epsilon)\geq\hat{q}^{NW}(K^{\varphi}(\delta,\epsilon),\delta)$. Since both $\hat{q}$ and $\hat{q}^{NW}$ are continuous in K,[39] it follows that there exists $0<K^{eq}\leq K^{*}$ such that $\hat{q}(K^{eq},\epsilon)=\hat{q}^{NW}(K^{eq},\delta,\epsilon)$.

(ii) When $\delta=\epsilon=0$, $\hat{q}^{NW}(K^{*},0,0)=\hat{q}(K^{*},0)=0$. Since $\hat{q}^{NW}$ is continuous in K, δ, and ϵ, and $\hat{q}$ is continuous in K and ϵ, we can define the function $K^{KM}(\delta,\epsilon)$ implicitly from the equation $\hat{q}^{NW}(K,\delta,\epsilon)-\hat{q}(K,\epsilon)=0$ in a neighborhood of $\delta=\epsilon=0$. By the implicit function theorem $K^{KM}(\delta,\epsilon)$ is continuous in both δ and ϵ. Clearly, $\hat{q}^{KM}(\delta,\epsilon)=\hat{q}^{NW}\left(K^{KM}(\delta,\epsilon),\delta\right)$ is also continuous in δ and ϵ. For the productivity shock

$$\frac{dK^{KM}}{d\delta}=-\frac{\frac{\partial\hat{q}^{NW}}{\partial\delta}}{\frac{\partial\hat{q}^{NW}}{\partial K}-\frac{\partial\hat{q}}{\partial K}}<0,$$

where the inequality follows since $G'''\geq0$ and lemma A1 guarantee that $\frac{\partial\hat{q}^{NW}}{\partial K}>\frac{\partial\hat{q}}{\partial K}>0$. A similar analysis yields $\frac{dK^{KM}}{d\epsilon}<0$, since from lemma A0, $\frac{\partial\hat{q}^{NW}}{\partial\epsilon}-\frac{\partial\hat{q}}{\partial\epsilon}\geq0$. Since productivity shocks do not affect the asset pricing curve, $\frac{d\hat{q}^{KM}}{d\delta}<0$. The sign of $\frac{d\hat{q}^{KM}}{d\epsilon}$ is ambiguous. XXX DE NUEVO XXXX

The threshold $\bar{\Delta}(\epsilon)$ is defined as the $\min[\delta^{KM}(\epsilon),\delta^{REN}(\epsilon)]$,where $\delta^{KM}(\epsilon)$ is the highest productivity shock for which the solution of $\hat{q}^{NW}(K,\delta,\epsilon)-\hat{q}(K,\epsilon)=0$ in a neighborhood of $\delta=\epsilon=0$ is continuous in δ for given ϵ. And $\delta^{REN}(\epsilon)$ is determined by $\hat{q}^{KM}(\delta^{DEF}(\epsilon),\epsilon)=-\theta\alpha$, i.e. when the first intersection between the net worth and asset pricing curves occurs at the kink of the former. By the implicit function theorem, $\frac{d\bar{\Delta}(\epsilon)}{d\epsilon}\leq0$.

(iii) The threshold $\underline{\Delta}(\epsilon)$ is determined by $\hat{q}^{NW}(K^{\varphi}(\underline{\Delta}(\epsilon),\epsilon),\underline{\Delta}(\epsilon),\epsilon)=\hat{q}(K^{\varphi}(\underline{\Delta}(\epsilon),\epsilon),\epsilon)=-\theta\alpha$, i.e. it is the productivity shock at which the net worth and asset pricing curves intersect at the kink of the former. Since the asset pricing is not affected by the productivity

[39] XXX PONER ESTA FOOTNOTE ANTES? XXX More rigorously, $\hat{q}^{NW}$ is upperhemicontinuous. Henceforth, we ignore this technicality which is inconsequential for our analysis.

shock, while this shifts the net worth upwards, an equilibrium with renegotiation, as characterized in lemma A2 exists for all $\delta \geq \underline{\Delta}(\epsilon)$ (as long as $K^{\varphi}(\delta,\epsilon) \geq 0$). From lemma A0 we have $\frac{d\underline{\Delta}(\epsilon)}{d\epsilon} \leq 0$. Comparative statics for the equilibrium with renegotiation with respect to the shocks follow from lemma A2. Since when there is renegotiation $\varphi + \hat{q}_0$ is constant, the effect of shocks on the haircut is the opposite of the effect on prices. Thus $\frac{d\varphi}{d\delta} > 0$, while the sign of $\frac{d\varphi}{d\epsilon}$ is ambiguous. But, if at the original equilibrium the $\hat{q}^{NW}$ curve is steeper than the $\hat{q}_t$ curve, then φ is increasing in ϵ.

(iv) For $\bar{\Delta}(\epsilon) = \underline{\Delta}(\epsilon) \equiv \Delta^*(\epsilon)$ it must be that $\bar{\Delta}(\epsilon) = \delta^{REN}(\epsilon)$, i.e. the first intersection between the net worth and asset pricing curves occurs at the kink of the former. This implies that $K^{KM}(\Delta^*(\epsilon),\epsilon) = K^{\varphi}(\Delta^*(\epsilon),\epsilon)$. Since $\frac{dK^{KM}}{d\epsilon} < 0$, $\frac{dK^{\varphi}}{d\epsilon} > 0$, and

$$\frac{dK^{KM}}{d\delta} = -\frac{\frac{\partial \hat{q}^{NW}}{\partial \delta}}{\frac{\partial \hat{q}^{NW}}{\partial K} - \frac{\partial \hat{q}}{\partial K}} > -\frac{\frac{\partial \hat{q}^{NW}}{\partial \delta}}{\frac{\partial \hat{q}^{NW}}{\partial K}} = \frac{dK^{\varphi}}{d\delta},$$

it must be the case that $\frac{d\Delta^*(\epsilon)}{d\epsilon} \leq 0$.

We now want to prove equilibrium uniqueness. We start considering the case $\delta < \Delta^*(\epsilon)$. Clearly, there can be no equilibrium with renegotiation. $G''' \geq 0$ and Lemma A3 guarantee that $K^{KM}(\delta,\epsilon)$ is the unique equilibrium. Next, suppose $\delta \geq \Delta^*(\epsilon)$. For $K > K^{\varphi}(\Delta^*(\epsilon),\epsilon)$ there can be no other equilibrium from Lemma A3. For $K < K^{\varphi}(\Delta^*(\epsilon),\epsilon)$, we know that $-\theta\alpha > \hat{q}(K,\epsilon)$. Hence, the renegotiation equilibrium is unique.

6.1.1 Proposition 3

(i) This follows from the fact that $\Delta^*(\epsilon)$ is defined by the intersection of the $\hat{q}_t^{NW}$ and $\hat{q}_t$ curves when the first one has a kink, i.e. when $\hat{q}_t^{NW} = -\theta\alpha$. Since the upward sloping part of the former and the latter are unaffected by change in θ or in α while the vertical branch of the former shifts to the left with an increase in α or θ, $\Delta^*(\epsilon)$ is an increasing function of α or θ (at the kink, since $G''' \geq 0$, the net worth curve is steeper than the asset pricing curve).

(ii) A decrease in α or θ shifts the vertical branch of the net worth curve to the right, thus increasing the equilibrium K^{φ} and prices. The effect on haircuts is ambiguous. This depends on the slope of the $\hat{q}_t$ and $\hat{q}_t^{NW}$ curves at the original equilibrium K^{φ}. If the former is steeper (which since $G''' \geq 0$ happens when $\delta > \Delta(\epsilon)$) then the decrease in α or θ will result in lower haircuts.

6.2 Proofs asymmetric information

6.2.1 Proposition 4

(i) The proof parallels that of proposition 1 (i). Assumption 2 guarantees that $\hat{q}^{NW,AI}(K^*,\delta,\epsilon) \geq \hat{q}(K^*,\epsilon)$. We need to define capital holdings if every entrepreneur defaults, $\tilde{K}^{AI}(\delta,\epsilon)$, as $(1-\epsilon)u(\tilde{K}^{AI}(\delta,\epsilon))\tilde{K}^{AI}(\delta,\epsilon) = \left(1-\delta-\frac{R}{R-1}E[\alpha]\right)aK^*$ (note that if every entrepreneur is defaulting, from (9) it follows that $-\hat{q}^{NW,AI} \geq 1$). If $\hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) \leq \underline{\kappa} = -1$, then $(\max[\tilde{K}^{AI}(\delta,\epsilon),0], \hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon)$ is an equilibrium. If $\hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) >$

$\underline{\kappa}$, then $\hat{q}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) \geq \hat{q}^{NW,AI}(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\delta,\epsilon)$. Since both $\hat{q}$ and $\hat{q}^{NW,AI}$ are continuous, then it follows that there exists $\max[\tilde{K}^{AI}(\delta,\epsilon),0] < K^{eq} \leq K^*$ such that $\hat{q}(K^{eq},\epsilon) = \hat{q}^{NW,AI}(K^{eq},\delta,\epsilon)$.

(ii) This follows the proof of Proposition 1 (ii). When $\delta = \epsilon = 0$, $\hat{q}^{NW,AI}(K^*,0,0) = \hat{q}(K^*,0) = 0$. Since $\hat{q}^{NW,AI}$ is continuous in K, δ, and ϵ, and $\hat{q}$ is continuous in K and ϵ, we can define the function $K^{KM,AI}(\delta,\epsilon)$ implicitly from the equation $\hat{q}^{NW,AI}(K,\delta,\epsilon) - \hat{q}(K,\epsilon) = 0$ in a neighborhood of $\delta = \epsilon = 0$. By the implicit function theorem $K^{KM}(\delta,\epsilon)$ is continuous and decreasing in δ, since

$$\frac{dK^{KM,AI}}{d\delta} = -\frac{\frac{\partial \hat{q}^{NW,AI}}{\partial \delta}}{\frac{\partial \hat{q}^{NW,AI}}{\partial K} - \frac{\partial \hat{q}}{\partial K}} < 0,$$

where the inequality follows since $G''' \geq 0$, assumption 2, and lemma A1 guarantee that $\frac{\partial \hat{q}^{NW,AI}}{\partial K} > \frac{\partial \hat{q}}{\partial K} > 0$ for all $\delta < \Delta^{AI}(\epsilon)$, and $\frac{\partial \hat{q}^{NW,AI}}{\partial \delta} > 0$. A similar analysis yields $\frac{dK^{KM,AI}}{d\epsilon} < 0$, since from assumption 2, $\frac{\partial \hat{q}^{NW,AI}}{\partial \epsilon} - \frac{\partial \hat{q}}{\partial \epsilon} \geq 0$.

(iii) Since creditors have all the capital when $K = 0$, $\hat{q}(0,\epsilon) = (1-\epsilon)\frac{R}{R-1}\left(\frac{u(0)-a}{a}\right)$. If $\hat{q}(0,\epsilon) > \underline{\kappa}$, this can only be an equilibrium when $\delta = 1$, i.e. when all entrepreneurs have their net worth wiped out. When $\delta < 1$ default moderates the effect on net worth and it is no longer the case that $K = 0$ is an expectational equilibrium in the neighborhood of $\delta = 1$, since entrepreneurs with very small default costs have positive net worth. An equilibrium with $\mu = 1$ requires that the net worth relation becomes vertical at $\tilde{K}^{AI}(\delta,\epsilon)$. In this case prices are given by $\hat{q}_t(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon)$. If $\hat{q}_t(\max[\tilde{K}^{AI}(\delta,\epsilon),0],\epsilon) \leq \underline{\kappa}$, then indeed every type of entrepreneur is better off defaulting and this is an equilibrium.

6.2.2 Proposition 5

We prove first this preliminary result

Lemma A4. Let $K^{\varphi,AI}(\delta,\epsilon)$ and $\hat{q}^{\varphi,AI}(\delta,\epsilon)$ be the implicit solutions to

$$\begin{aligned} -\bar{\alpha} - \hat{q}^{NW,AI}\left(K^{\varphi,AI}(\delta,\epsilon),\delta,\epsilon\right) &= 0 \\ \hat{q}^{\varphi,AI}(\delta,\epsilon) - \hat{q}\left(K^{\varphi,AI}(\delta,\epsilon),\epsilon\right) &= 0 \end{aligned}$$

Then, $K^{\varphi,AI}(\delta,\epsilon)$ and $\hat{q}^{\varphi,AI}(\delta,\epsilon)$ are well-defined and continuous functions, with $\frac{dK^{\varphi,AI}}{d\delta} < 0$, $\frac{d\hat{q}^{\varphi,AI}}{d\delta} < 0$, and $\frac{dK^{\varphi,AI}}{d\epsilon} > 0$.

Proof. This parallels the proof of Lemma A2. QED

(i) This follows the proof of Proposition 4 (i), considering two cases. First note that if $K^{\varphi,AI}(\delta,\epsilon) < 0$, then $\hat{q}^{NW,AI}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) \leq -1$. If $\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) \leq \max[\underline{\kappa},-\bar{\alpha}]$, then $\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right)\right)$ is an equilibrium. If $\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) > \max[\underline{\kappa},-\bar{\alpha}]$, then $\hat{q}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right) > \hat{q}^{NW,AI}\left(\max[K^{\varphi,AI}(\delta,\epsilon),0],\epsilon\right)$ Since both $\hat{q}$ and $\hat{q}^{NW,AI}$ are continuous in K, it follows that there exists $\max[K^{\varphi,AI}(\delta,\epsilon),0] < K^{eq} \leq K^*$ such that $\hat{q}(K^{eq},\epsilon) = \hat{q}^{NW,AI}(K^{eq},\delta,\epsilon)$.

(ii) Next, we want to prove that under Assumption 3, there exists some $\bar{\Delta}^{AI}(\epsilon) \in \left[0,\Delta^{AI}(\epsilon)\right)$ such that $\{K^{eq}(\delta,\epsilon),\hat{q}^{eq}(\delta,\epsilon)\}$ defined by

$$\{K^{eq}(\delta,\epsilon),\hat{q}^{eq}(\delta,\epsilon)\} = \left\{ \begin{array}{c} \left\{K^{KM,AI}(\delta,\epsilon),\hat{q}^{KM,AI}(\delta,\epsilon)\right\} \text{ if } \delta < \bar{\Delta}^{AI}(\epsilon) \\ \left\{\max[K^{\varphi,AI}(\delta,\epsilon),0],\max[\hat{q}^{\varphi,AI}(\delta,\epsilon),\hat{q}(0,\epsilon)]\right\} \text{ if } \delta \geq \bar{\Delta}^{AI}(\epsilon) \end{array} \right\}$$

is a set of equilibria in which the resulting capital schedule $K^{eq}(\delta,\epsilon)$ and asset prices $\hat{q}^{eq}(\delta,\epsilon)$ are continuous functions of δ and ϵ. Note that, as in proposition 2, large shocks might wipe out entrepreneurs' net worth, i.e. $K^{\varphi,AI}(\delta,\epsilon) < 0$. Since this leads to the trivial equilibrium with $K_0 = 0$, henceforth we will assume that $K^{\varphi,AI}(\delta,\epsilon) \geq 0$.

Assumption 3.

$$\begin{aligned} &\frac{R}{R-1}\left(-\bar{\alpha}\left(1-F\left(\bar{\alpha}\right)\right)-\int_0^{\bar{\alpha}}\alpha dF(\alpha)\right) \\ &> (1-\epsilon)\frac{u\left(K^{KM,AI}(\Delta^{AI}(\epsilon),\epsilon)\right)K^{KM,AI}(\Delta^{AI}(\epsilon),\epsilon)}{aK^*} - \left(1-\Delta^{AI}(\epsilon)\right). \end{aligned}$$

Under Assumption 3 we know that $-\bar{\alpha} > \hat{q}\left(K^{KM,AI}(\Delta^{AI}(\epsilon),\epsilon),\epsilon\right)$. This implies that when $\delta = \Delta^{AI}(\epsilon)$, the equilibrium features renegotiation, $\varphi > 0$, and thus $K^{KM,AI}\left(\Delta^{AI}(\epsilon),\epsilon\right) < K^{\varphi,AI}\left(\Delta^{AI}(\epsilon),\epsilon\right)$. From Lemma A4 we know that as we decrease δ, K_0 increases. Since $K^{KM,AI}(0,\epsilon) > K^{\varphi,AI}(0,\epsilon)$ (where the latter might require a negative haircut), this implies that there exists a $\bar{\Delta}^{AI}(\epsilon) < \Delta^{AI}(\epsilon)$ for which the equilibrium corresponds to $\left(K^{KM,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right),\hat{q}^{KM,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right)\right)$, i.e. the net worth and asset pricing curves intersects at the point when the former has a kink.

To prove equilibrium uniqueness, we start considering the case $\delta < \bar{\Delta}^{AI}(\epsilon)$. There can be no equilibrium with renegotiation. $G''' \geq 0$ and Lemma A3 adapted to $q^{NW,AI}$ guarantee that $K^{KM,AI}(\delta,\epsilon)$ is the unique equilibrium. Next, suppose $\delta \geq \bar{\Delta}^{AI}(\epsilon)$. For $K > K^{\varphi,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right)$ there can be no other equilibrium from Lemma A3 adapted to $q^{NW,AI}$. For $K < K^{\varphi,AI}\left(\bar{\Delta}^{AI}(\epsilon),\epsilon\right)$, we know that $-\bar{\alpha} > \hat{q}(K,\epsilon)$. Hence, the renegotiation equilibrium is unique. Concerning the response of haircuts, we have seen in the proof of Proposition 4 (ii) that $K^{KM,AI}(\cdot)$ and $\hat{q}^{KM,AI}(\cdot)$ are decreasing in δ and ϵ. We start with productivity shocks. From Lemma A4 we know that $K^{\varphi,AI}(\cdot)$ is decreasing in δ. This implies that prices are also decreasing in δ, and since $\hat{q}_t + \varphi^{AI} = -\bar{\alpha}$, this implies that the haircut, φ^{AI}, is increasing in δ. For preference shocks, Lemma A4 tells us that $K^{\varphi,AI}(\cdot)$ is increasing in ϵ. Since the shock shifts the $\hat{q}_t$ curve downwards the effect of the shock on prices is ambiguous as the direct effect of ϵ is negative but there is a positive effect from the increase in K. As a result the effect on φ^{AI} is ambiguous. But, if at the original equilibrium the $\hat{q}^{NW,AI}$ curve is steeper than the $\hat{q}_t$ curve, then φ^{AI} is increasing in ϵ.

(iii) This parallels the proof of proposition 2 (iii).

6.2.3 Proposition 6

We derive comparative statics results for the following distribution function

$$F(x) = \frac{x^\gamma}{x^\gamma + (1-x)^\gamma},$$

with $\gamma \geq 1$. Its density is given by

$$f(x) = \frac{\gamma x^{\gamma-1}(1-x)^{\gamma-1}}{[x^\gamma + (1-x)^\gamma]^2}.$$

(i) We show first that $\mu = 1 - F(\overline{\alpha}) = 1 - \frac{\overline{\alpha}}{\gamma}$.

From first order condition (8) holding with equality we have

$$\frac{\overline{\alpha}}{\gamma}(\overline{\alpha}^\gamma + (1-\overline{\alpha})\gamma) = (1-\overline{\alpha})^\gamma,$$

using this condition to replace the denominator in the cumulative distribution function

$$F(\overline{\alpha}) = \frac{\overline{\alpha}^{\gamma+1}}{\gamma(1-\overline{\alpha})^\gamma}.$$

First order condition (8) holding with equality can also be rewritten as

$$\overline{\alpha}^{\gamma+1} = (1-\overline{\alpha})^\gamma(\gamma - \overline{\alpha}),$$

Replacing in the cumulative function gives

$$F(\overline{\alpha}) = 1 - \frac{\overline{\alpha}}{\gamma}.$$

(ii) From the previous result it follows that to show that the more severe is the asymmetry of information (indicated by a lower γ), the more default arises in equilibrium requires proving that $\frac{\gamma}{\overline{\alpha}}\frac{d\overline{\alpha}}{d\gamma} < 1$. This follows from the application of the implicit function theorem on the following expression of (8) holding with equality

$$\ln(\gamma - \overline{\alpha}) + \gamma(1-\overline{\alpha}) - (\gamma+1)\overline{\alpha} = 0.$$

The implicit derivative is given by

$$\frac{d\overline{\alpha}}{d\gamma} = -\frac{\ln(\frac{\overline{\alpha}}{1-\overline{\alpha}}) - \frac{1}{\gamma-\overline{\alpha}}}{\frac{\gamma+1}{\overline{\alpha}} + \frac{\gamma}{1-\overline{\alpha}} + \frac{1}{\gamma-\overline{\alpha}}}.$$

Thus the question of whether $\frac{\gamma}{\overline{\alpha}}\frac{d\overline{\alpha}}{d\gamma} < 1$ boils down to whether

$$-\gamma \ln(\frac{\overline{\alpha}}{1-\overline{\alpha}}) + \frac{\gamma}{\gamma-\overline{\alpha}} < \gamma + 1 + \frac{\gamma\overline{\alpha}}{1-\overline{\alpha}} + \frac{\overline{\alpha}}{\gamma-\overline{\alpha}},$$

which can be rewritten as

$$-\gamma \ln\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right) < \gamma + \frac{\gamma\overline{\alpha}}{1-\overline{\alpha}}.$$

This condition is satisfied as long as $\overline{\alpha} \geq \frac{1}{2}$, which can be verified from the following expression of (8) holding with equality

$$\left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right)^{\gamma+1} = 1 + \frac{\gamma-1}{1-\overline{\alpha}}.$$