Why Might the Old Want to Honor Sovereign Debt?

Martín Gonzalez-Eiras[*]

June 1, 2017

Abstract

Survey evidence shows that, in times of high default risk, the old are often relatively in favor of sovereign debt repayment. I characterize the optimal dynamic contract between a country and foreign lenders to account for these life-cycle preferences. Efficient allocations under political constraints provide incentives not to default by promising higher future consumption to the current young. Thus, even if the direct cost of being excluded from international capital markets is lower for the elderly, they might be most affected from a default. In production economies positive shocks lead to an increase in investment. The model helps rationalize the observed weak correlations between default and output, and between default and outstanding debt.

1   Introduction

On June 26th 2015 the Greek government called a snap referendum on whether the country should accept or reject a reform and fiscal adjustment programme drawn up by the so called "troika" composed of the European Commission, the ECB, and the IMF. The referendum took place on July 5th, and given that Greece defaulted on an IMF loan on June 30th, a the time of the vote a No majority was seen as likely to lead the country to exit the eurozone and default on its loans from the European governments and the

ECB.[1] Thus, the Greek referendum, and polls conducted at the time to measure support for the No vote as well as socio-economic indicators of voters, provide information on the distribution of Greek citizens' preferences towards default along several dimensions of heterogeneity.

The No vote tallied 61.3% of the referendum's total, and a poll showed significant ideological influence as among left-leaning voters 91% voted No, while 82.8% of those reporting a moderate center-right political inclination voted Yes.[2] The poll showed that voters cared about expected economic outcomes of a potential default as well. Those reporting great financial difficulty favored the No vote by 63.1%, support that decreased to 52.3% among those reporting to be living comfortably. More interestingly, the support for the No vote varied markedly by age as can be seen in in table 1. While only 15% of those between ages 18 and 24 voted Yes, 55.1% of those aged 65 and above did so. And this in spite of the fact that the programme proposed by the troika included provisions for pension cuts. Thus, intergenerational conflict seems salient in shaping a country's attitude towards sovereign debt.

With a few exceptions, existing models of sovereign debt assume that a small open economy is populated by a representative agent and that policy is determined by a benevolent government with no commitment. To rationalize the disparity in preferences by age this paper introduces an overlapping-generations demographic structure onto the benchmark limited-commitment framework. Conflicts of interest arise because the old are less affected by the threat of exclusion from international capital markets should the country default on outstanding obligations.

More specifically I characterize the efficient consumption allocations from the perspective of a benevolent social planner that faces the constraint that policy is determined sequentially by probabilistic voting among those currently alive. With individuals that live for only two periods, and have no altruism towards future generations, a dynamic contract has limited room to provide incentives. But unless policy is geared only towards the old, there is a scope for a welfare improving long-term lending relationship.

Besides the mentioned evidence from the Greek referendum, Tomz (2004) has survey data from Argentina taken six months after the 2001 default on nearly USD 100 billion sovereign debt. In his regressions for preference for repayment the coefficient on age is positive and significant. Curtis, Jupille, and Leblang (2012) study the Iceland 2011 referendum on debt repayment. Although they do not control for age in their regressions, I use their data to show that the old have higher preferences for debt repayment.

Recent studies suggests that domestic political factors influence a country's incentive to default. Using a newly constructed historical data set, Tomz and Wright (2007) analyze

---

[1]Total outstanding foreign liabilities were approximately €340 billion.

[2]Support raised to 31.9% among right-leaning voters. These and other statistics on the breakdown of the referendum vote come from a poll made between June 29 and July 3 of 20015 by Public Issue, a Greek polling company. The sample size consisted of 2069 responses.

Table 1: A demographic breakdown of Greece's 2015 referendum

Age group	18-24	25-35	35-44	45-54	55-64	65+
Share of "Yes" vote	0.15	0.277	0.326	0.308	0.406	0.551

Source: Public Issue

169 default episodes and conclude that although defaults are more common in bad times than in good times the relation is weak. In about 39% of observations, countries avoided default even though output was below trend, and more than a third defaulted while output exceeded trend (a fifth during a boom in which output exceed trend by more than ten percentage points).

Literature review

Much of the literature attempts to clarify the role that different penalties to default play in supporting a positive level of debt. Among these are *reputational* models that assume that the only penalty available is to restrict a defaulting country's access to international capital markets. Whether reputational considerations are sufficient to support a positive level of debt, depends on the options open to a country after it defaults[3]. If the legal institutions are such that a defaulting country can be completely excluded from international capital markets, reputation can support some debt or insurance contracts. Eaton and Gersovitz (1981) pioneered this literature. See Aguiar and Amador (2014) for a recent survey.

The importance of domestic creditors as a constituency in favor of repayment was first studied by Guembel and Sussman (2009). Broner, Martin and Ventura (2010) show that with secondary markets for sovereign debt, repayment follows since the smallest expected haircut would lead residents to buy all outstanding debt which would also result in full repayment. Research recently started to ask the question of who, among residents, wants to repay. Curtis et al (2012), and Tomz (2004) are examples of empirical studies that address this question using survey data.

The closest related work is Dovis et al (2015) who use a dynamic optimal contract approach with an OLG structure to study cycles of debt crisis and changing inequality. Their interest is on intragenerational conflict as in their model the government has incentives to default on contracts to redistribute income within cohorts.

The remainder of the paper is structured as follows. Section 2 presents survey evidence on preferences for repayment by age. Section 3 develops the basic model for an endowment economy, whose equilibrium is then characterized in section 4. Section 5 extends the model to production economies. Section 6 concludes.

---

[3]See Bulow and Rogoff (1989), Worrall (1990), and Kletzer and Wright (1998).

2 Evidence on preferences for repayment by age

2.1 Greece 2015

The European sovereign debt crisis started late 2009 in the wake of the Great Recession. The crisis started in Greece and soon spread to Ireland, Portugal, Spain, and Italy. Evidence that the Greek crisis was largely due to an unsustainable fiscal situation led to the belief that fiscal mismanagement underlay the problems of these other periphery countries. It took time, and several policy measures and rescue packages from the ECB, the European Union, and the IMF to appease capital markets. And since the past few years, only Greece suffered from high borrowing rates due to its high government structural deficits and accelerating debt levels.

With hindsight, it is clear that Greece should have been allowed to restructure its debt in 2010. But the widespread thinking at the time was that allowing Greece to default on its sovereign debt would have immediate and negative effects on the rest of Europe, if not the world economy. Major French and German banks, loaded up with Greek debt, were thought to have failed in such a contingency. The markets for bonds of other Southern European countries would probably have collapsed. In short, markets believed that a Greek default would have created a second Lehman-type of panic.

In May 2010 Greece started the first of a series of fiscal adjustment measures that would bring down its primary deficit from €24.7bn (10.6% of GDP) in 2009 to a small surplus in 2014. These measures contributed to a worsening of the Greek recession, and were resented by a large fraction of the population. In December 2014 the Greek parliament called a general election that led to the left-leaning Syriza government that had campaigned by refusing to accept the terms of the current bailout agreement. After several months of playing bully, the Greek prime minister, Alexis Tsipras, called a snap referendum on June 26th after walking out of negotiations with the country's creditors. On June 30th, Greece defaulted on an IMF loan. The referendum, which took place on July 5th, asked voters if they wanted to accept a reform and fiscal adjustment programme.

On July 1st, Tsipras said a No vote would strengthen Greece's negotiating hand, not push it out of the euro. Despite this, the consensus among economic and political analysts was that if Greece failed to renew its rescue package this would increase the probability of having to default on its loans from European governments and the ECB for a total of €340 billion. When the votes where counted, the No majority was larger than expected, at 61.3% of the total. Antonis Samaras, the main opposition leader, who among other former prime ministers campaigned for the Yes vote, resigned after the vote. Given these facts it is no surprise that a survey conducted on voters' preferences and socio-economic characteristics would show that political inclination is correlated with support for the No vote.

As mentioned in the introduction, the survey also shows evidence that voters cared

about expected economic outcomes, in line with the voiced fears in the media that a No vote would probably lead Greece to exit the euro. And in this respect, a No vote revealed a preference for default on sovereign debt. In the survey those reporting greater financial difficulty favored the No vote. More surprisingly the old were relatively in favor of this program compared to the young. Table 1 shows that while only 15% of those aged 18 to 24 voted "Yes", 55.1% of those aged 65 and above accepted austerity, even in the face of projected pension cuts as detailed in the troika's proposed program.

A problem with the survey evidence from Greece is that the model predicts age to be correlated with opposition to sovereign default, but is silent on other dimensions on which there might be heterogeneity among voters (except, by construction, on ideology). It might be the case that being old is correlated with other factors that explain preferences for repayment. Thus to test this model's implication it is necessary to perform a regression analysis that controls for these other factors. Although I do not have this data for the Greek referendum, a similar dataset for Iceland is publicly available.

2.2 Iceland 2011

Iceland was hit hard by the Great Recession since 85% of its banking sector went bankrupt. The government nationalized the largest banks, including Landsbanki which had been funding part of its operations though internet-based savings accounts, called "Icesave", from European customers. This bank could not repay depositors who were subsequently bailed out by their national governments (mostly from Britain and the Netherlands). These governments then demanded reimbursement from the Icelandic treasury. Iceland accepted in principle but held two referendums on whether to accept or reject proposed repayment terms. Of these only the second is informative since at the time the first referendum took place in 2010 there was a superior offer on the table.

Curtis, Japille and Leblang (2012) conducted a survey shortly after the second referendum held on April 9 2011. In principle, citizens were not voting on whether to repay Iceland's sovereign debt, but when and how. Nevertheless, from their answers to the survey question, "How did you vote in the April 9th "Icesave" referendum? Did you vote in favor of having the repayment plan in force, did you vote to have it repealed, or did you submit a blank ballot?", preferences for repayment can be deduced. The authors find evidence that self-interest shaped voting behavior, together with partisan or ideological and education and other sociotropic effects. Table 2 shows that 39% of those between ages 18 and 24 favored repayment while almost 50% of those aged 65 and above did so. Thus the difference is less marked than in the Greek case. Nevertheless, the following regression analysis shows that age is a significant explanatory variable for repayment preferences.

Curtis et al (2012) do not control for age in their regressions.[4] Since the data is

---

[4]In footnote 47 the authors claim age to be highly collinear with both education and investment behavior with correlations of 0.78 and 0.69 respectively. Using the publicly available data I found these

Table 2: A demographic breakdown of Iceland's 2011 referendum

Age group	18-24	25-35	35-44	45-54	55-64	65+
Share of "Yes" vote	0.39	0.38	0.45	0.41	0.45	0.495

Source: Curtis el al (2012)

publicly available I can test the effect of age on repayment preferences. I perform four regressions that I report in table 3. First I regress the reported vote choice on age. The estimated coefficient is positive but not significant, which is not a surprise considering that age might be correlated with other determinants of repayment preferences. Since the model prediction that we want to test is of a pure age effect, it is necessary to control for these other determinants. The second column in table 3 reports the the results of a regression that uses all the controls employed in Curtis et al (2012).[5] We see that the coefficient on age is positive and significative. For robustness I apply a Lasso-type procedure for variable selection proposed by Belloni et al (2014) that starts to iterate from a highly saturated model. As can be seen in column three, the coefficient of interest is still positive and significative.[6] Finally I perform a probit regression with the same specification as in column four and report the marginal effect of age on vote choice. This coefficient is equivalent to an increase in eight percentage points in the preference for repayment if age increases, at the mean, by 20 years.[7]

An important caveat is that with cross section data age and birth year are perfectly correlated and the estimated positive effect of age on preferences for repayment might be due to older cohorts being more willing to repay, and growing older not having an effect on these preferences. Thus, it is possible that the difference in observed preferences for default reflect a change in the environment in which different generations of Icelanders formed their social, economic and political beliefs. These cohort effects have been observed as an important determinant of preferences. For example, Giuliano and Spilimbergo (2014) show that preferences for redistribution in the US are affected by regional macroeconomic shocks that took place when individuals where between the ages of 18 and 25 years. Survey data in Europe show that the old are more averse to immigration than the young. Calahorrano (2011) uses panel data for Germany to show that this result is driven by

---

correlations to be -0.16 and 0.18 respectively.

[5]Specifically all the controls of the first specification plus "prime minister approval" which is used as an alternative measure of domestic political cues. Similar results hold if this variable is omitted.

[6]Not surprisingly the explanatory variables that the procedure select are similar to those employed in Curtis et al (2012) regression. The fact that "prime minister approval" was selected *alongside* the other measures of domestic political cues was what motivated me to use it in the regression reported in column two.

[7]The reason for the main econometric specification to be OLS is that it is well known that when the dependent variable is binary, linear regression is about as good as non-linear models like Probit. See Angrist and Pischke (2009).

Table 3: Preference for repayment in Iceland's 2011 referendum

Model specification	1	2	3	4	5
Coefficient on age	0.0018	0.0024**	0.003**		0.004*
Robust standard errors	(0.0011)	(0.0012)	(0.0012)		(0.0016)
Coefficient on "old"				0.098***	
Robust standard errors				(0.0508)	
Coefficient on "young"				-0.0036	
Robust standard errors				(0.0636)	
R^2		0.003	0.269	0.262	0.259	0.216
N		669	628	630	630	630

Source: Curtis el al (2012)

cohort effects.

While it is not possible to disentangle age and cohort effects, as a robustness check I perform a regression of attitudes towards foreigners and claimed identity on age. The first variable captures respondents' ratings toward the UK, Netherlands and India and the latter is a variable based on whether respondents proclaim themselves European or Icelandic. Results (not reported) show that the old are more likely to feel Icelandic and have a colder attitude towards foreign countries. This results show the old as less cosmopolitan, a characteristic that is consistent with the mentioned European data on attitudes to immigration. Thus, for preferences for repayment to be driven by cohort effects, these would have to be beliefs that make older Icelanders to be more insular but at the same time more adamant on breaking an obligation with foreign governments.

2.3 Argentina 2002

Between March 1991 and January 2002 Argentina had a currency board that, by depriving it from monetary policy as a stabilization tool, exposed it to supply shocks. At the end of the nineties, Argentina experienced a combination of negative shocks as the dollar, the currency of its peg, was very strong, its main trading partner, Brazil, underwent a large devaluation in January 1999, and interest rates in the US were high (they only were reduce in the wake of the bursting of the dot com bubble in 2001). As a result of these shocks production and employment suffered for three years in a row and citizens blamed the weak coalition government of Fernando De La Rúa. When the governing coalition broke down in October 2000 measures of country risk sky rocketed and Argentina faced a virtual shutdown of capital markets.

The IMF provided a rescue package that proved to be insufficient given the government's weak political position. After street riots in December 2001, president De La Rúa resigned and after two weeks of turmoil, Parliament chose Eduardo Duhalde, a sit-

ting senator and former governor of the province of Buenos Aires, to complete the original mandate. President Duhalde devalued the currency and suspended service on nearly USD 100 billion in foreign bonds, triggering the largest sovereign default in history.

To analyze how adjust adjustment costs and reputational benefits affected preferences for repayment in Argentina in 2002, Michael Tomz conducted a survey of 442 Argentine voters in July 2002, six months after the default. His results are reported in Tomz (2004). A caveat with his sample is that it only consists of citizens of Buenos Aires' metropolitan area, whom have higher incomes and levels of education attainment than citizens from the rest of the country. Nevertheless, his survey has proxy measures of income and education.[8]

Tomz (2004) uses several controls to test for his hypotheses, including the mentioned proxies for income and education, employment status (employed in public or private sector, or unemployed), and ideology as captured in a variable measuring nationalism. He also uses age as a control, and given the other controls that he uses, it is likely that the coefficient of age captures the pure effect in which I am interested. In all his regressions of preferences for repayment, age turns out to be positive and significative. As with the case of Iceland, it is not possible to separate age from cohort effects.

3 Model

3.1 Structure and Preferences

Consider a small open endowment economy with an infinite sequence of dates, $t = 0, 1, \ldots$. The economy is subject to exogenous endowment shocks which follow a finite-state i.i.d. process.[9] Let y_t denote the period t realization of the endowment, and $\pi(y_t)$ its probability. Denote by $h^t \in H^t$ a history of the economy observed up to time t, including the sequence of endowment realizations up to period t and choices made up to period $t-1$. Denote by h_t the period t part of history h^t (i.e. the period t realization of the endowment and period $t - 1$ actions). The economy is inhabited by two-period lived overlapping generations, a sequence of governments indexed by $t = 0, 1, \ldots$, and external lenders. There is no population growth.

Preferences of individuals born in period t and history h^t are described by the utility function

$$u_t(h^t) = u(c^y(h^t)) + \beta E_t[u(c^o(h^{t+1}))],$$

where $u(\cdot)$ is a twice differentiable, increasing, strictly concave function, with $\lim_{c \to 0} u'(c) =$

---

[8]More specifically, it has information on a measure of wealth: the number and kind of cars, goods and services in the households. Although there is no variable for education attainment respondents were asked a number of questions to determine their degree of "economic knowledge". See Tomz (2004).

[9]I make this assumption for simplicity. It also highlights the history dependence of optimal allocations. Most results directly generalize to a Markov chain from some initial state y_0. When possible, additional assumptions that would be needed are noted.

∞; $\beta \in (0, 1)$ is the discount factor; expectations are conditional on observed history;[10] and $(c^y(h^t), c^o(h^{t+1}))$ denotes a consumption path over the life cycle.

We are interested in characterizing efficient allocations, and therefore need a social welfare function to rank outcomes. It is assumed that equal weights are given to all cohorts, and that time is discounted at rate δ. The utilitarian welfare function would then be

$$U(h_0) = \frac{\beta}{\delta} E_0 \left[u(c^o(h_0)) \right] + \sum_{t=0}^{\infty} \delta^t E_0 \left[u(c^y(h^t)) + \beta u(c^o(h^{t+1})) \right]. \tag{1}$$

Competitive infinitely-lived external lenders are risk neutral, discount the future at factor β', and have the ability to commit to their obligations. Lending takes place using a full set of state-contingent assets with prices $q^*(h^t)$. Denoting by x_t their consumption in period t, their preferences are thus given by

$$\sum_{t=0}^{\infty} \beta'^t E_0[x_t].$$

3.2 Government and Credibility Constraints

The government centralizes decisions in the economy. This implies that given the observed state, h^t, consumption is determined, $c^y(h^t), c^o(h^t)$, payments are made to lenders $b(h^t)$, and borrowing takes place using a portfolio of state-contingent liabilities, $b((h^t, h_{t+1}))$, (raising resources $\sum_{h^{t+1}} \frac{q^*(h^{t+1})}{q^*(h^t)} b(h^{t+1})$). Since markets are complete the resource constraint of the small open economy can be written as

$$b_0 \leq \sum_{t \geq 0, h^t} q^*(h^t) \left(y(h^t) - c^y(h^t) - c^o(h^t) \right), \tag{2}$$

where $b_0 \equiv b(h_0)$ is the initial net foreign liability position.[11] The right-hand side is the present expected discounted value of net exports. Balance of payments accounting requires that this be equal to initial net foreign liabilities.

The first-best outcome would have lenders absorbing all endowment uncertainty, and implementing a consumption profile that reflected the relative impatience of the country vis a vis the rest of the world.[12] Instead, the small open economy has limited commitment such that at each point in time its government can decide to renege on its obligations. More specifically, policy decisions are made by a sequence of governments indexed by $t = 0, 1, \ldots$. Government t cares only about agents that are currently alive and evaluates

---

[10]Although shocks are i.i.d., the potential history-dependence of allocations requires the use of conditional expectations.

[11]Alternatively this can be interpreted as the equivalent initial debt that solves bargaining between borrowers and lenders when they start a long-term lending relationship.

[12]For example, if $\delta > \beta'$ then consumption would be increasing in time.

welfare according to the utilitarian criterion,

$$\omega u(c^o(h^t)) + (1 - \omega)\left(u(c^y(h^t)) + \beta E_t[u(c^o(h^{t+1}))]\right),\tag{3}$$

with relative weights ω and $1 - \omega$ on the old and young, respectively.[13] In Appendix 7.1 I show how this utilitarian criterion can be justified if policy is decided sequentially and decisions are aggregated by probabilistic voting.

With lack of commitment, the history of the economy includes a dummy variable, e_t, that takes the value of 1 if the government adhered to the contract in period $t - 1$, and 0 if it did not. We can write $h_t = (y_t, e_t, l_t)$, where l_t is a dummy variable indicating whether foreign lenders lent to the sovereign in period $t-1$ (i.e. $l_t = 1$ if $b(h^t) \neq 0$).[14] I assume lenders observe e_t before making their lending decisions. If government t reneges on its outstanding debt payment, $b(h^t)$, then it will decide to allocate current consumption optimally, and faces an expected continuation value for the current young given by $E_t[V^{oD}((h^t, (y_{t+1}, 0, l_t)))]$.

Following the sovereign debt literature on reputational incentives, it is assumed that a deviation results in the exclusion from international capital markets. For simplicity, I assume that the exclusion is permanent such that the subsequent governments do not interact with foreign lenders. This implies that $l_t = l_{t-1}e_t$. Since there are no physical state variables, following a deviation consumption would then be a function of the current endowment realization, and the continuation value is independent of current decisions by government t, thus $V^{oD}((h^t, (y_{t+1}, 0, 0))) = V^{oD}(y_{t+1})$, and $E_t[V^{oD}((h^t, (y_{t+1}, 0, 0)))] = E[V^{oD}(y_{t+1})]$ is constant.[15] Optimal consumption under a deviation is given by the solution to the following equations:

$$
\begin{aligned}
\omega u'(c^{oD}(y_t)) &= (1 - \omega)u'(c^{yD}(y_t)),\\
c^{yD}(y_t) + c^{oD}(y_t) &= y_t.
\end{aligned}\tag{4}
$$

and $V^{oD}(y_{t+1}) = u(c^{oD}(y_{t+1}))$, since there is no mechanism to give future governments incentives to choose a different allocation than the static optimum.

The value of deviation to government t at history h^t is then summarized by $\underline{V}(y_t) = \omega u(c^{oD}(y_t)) + (1 - \omega)\left(u(c^{yD}(y_t)) + \beta E[u(c^{oD}(y_{t+1}))]\right)$. Given the characterization of optimal consumption under deviation, $\underline{V}(y_t)$ is weakly increasing with current output y_t.[16] For an allocation to be compatible with period governments' incentives to deviate, it must

---

[13]See Fahri et al. (2004) for an application of this utilitarian criterion in a different context.

[14]Note that l_t does not provide information on the portfolio of state contingent securities offered in $t - 1$. This information is not needed to characterize the history of the dynamic game.

[15]Since a deviation is followed by exclusion from capital markets, period governments only interact with themselves. Absent commitment, they are bound to choose the static autarky optimal consumption allocation. If commitment to trigger strategies were allowed, other equilibria might follow.

[16]If the endowment follows a Markov process we would need to make monotonicity assumptions to guarantee that $\underline{V}(y_t)$ is increasing in current output.

satisfy the following credibility constraints in every period t and history h^t,

$$\omega u(c^o(h^t)) + (1 - \omega)\left(u(c^y(h^t)) + \beta E_t[u(c^o(h^{t+1}))]\right) \geq \underline{V}(y_t). \tag{5}$$

3.3 Timing

At the beginning of period t the shock y_t is realized. This determines the economy's history, h^t, and its state contingent obligations, $b(h^t)$. Afterwards a candidate is elected for office as period t's government. When choosing among competing candidates, voters take into account the effects of the policy platforms on offer, in particular whether a sovereign default would take place or not. Next, the elected government implements an allocation (c_t^y, c_t^o), and afterwards foreign lenders use contingent contracts to lend $b(h^{t+1})$. Households and foreign lenders form expectations about future policy and an equilibrium results.

3.4 Policy Game and Equilibrium

Formally, the game is described as follows. The relevant history h^t at the beginning of period t consists of the sequence of past endowment realizations, and government and foreign lenders' choices as summarized in dummy variables e_t and l_t. Each possible history of the game up to any date t defines a subgame of the repeated game beginning at that date. Denote a strategy for period-t government by $(c_t^o(h^t), c_t^y(h^t))$, and for foreign lenders by $b_{t+1}(h^t, e_t)$.[17] For the government the strategy determines the consumption allocation, and for the foreign lenders the contingent contracts used to lend, if there is no default. The budget constraint determines the current (net) payment made by the country to foreign lenders.

We focus on subgame perfect equilibria. Given a history h^t, the strategies $((c^o, c^y), b)$ determine policy and debt contracts in period t, generating an updated history h^{t+1}. Starting from $t = 0$ the corresponding consumption allocations are thus determined (c_t^o, c_t^y). We are interested in characterizing the best equilibrium from the perspective of the social welfare function (1). Following Chari and Kehoe (1990) and Abreu (1988), this equilibrium can be found using the threat of reversion to a punishment subgame perfect strategy with the worst continuation value in the face of a deviation. This corresponds to a "self-enforcing" equilibrium, or in the terminology of Chari and Kehoe (1990), a "sustainable plan".

In the model, a punishment is only relevant in so far as it affects the current young, whose utility only determines in part the period payoff of a government. In general this complicates the derivation of the punishment strategies that lead to the worst outcome,

---

[17]Note that foreign lenders make their decisions after observing e_t. Thus the history of the game for them is extended to include this variable. If there is more than one foreign lender, it is assumed that they all follow the same strategy.

and the characterization of the set of sustainable allocations.[18] With permanent exclusion from capital markets, and no commitment to trigger strategies from period governments, the harshest punishment is given by $V^{oD}(y_{t+1}) = u(c^{oD}(y_{t+1}))$ characterized above.

The credibility constraint (5) ensures that the period-t government prefers the equilibrium outcome to deviating. Therefore, to characterize the equilibrium allocations we can assume that default does not occur in equilibrium and use standard optimization techniques. We now turn to this task.

4   Equilibrium

If there is no default in equilibrium, then the relevant part of histories is given by the sequence of endowment realizations. With some abuse of notation, I henceforth represent the sequence of endowment realizations up to date t by h^t. Competition among lenders implies that the period expected (risk-free) gross return is $R = \frac{1}{\beta'}$. The equilibrium international price of a unit of consumption in state h^t in units of period zero consumption is given by $q^*(h^t) = \frac{\pi(h^t)}{R^t}$, with $\pi(h^t) = \prod_{i=0}^{t} \pi(y_i)$.

With the above characterization of our small open economy and international capital markets, efficient allocations are those that maximize (1) subject to the constraints (5) and (5). The constrained-optima allocation can be characterized using Lagrangian techniques. Since the objective function is concave and the constraints are convex, the first-order conditions are necessary and sufficient for a solution. Letting μ denote the multiplier on the resource constraint (5), and $\lambda(h^t)\pi(h^t)\beta^t$ denote the multipliers on the credibility constraints (5), the first-order conditions are,[19]

$$
(\delta R)^t \left(1 + (1-\omega)\lambda(h^t) \right) u'(c^y(h^t)) = \mu,
$$
$$
(\delta R)^t \left(\frac{\beta}{\delta} + \omega\lambda(h^t) + \frac{\beta}{\delta}(1-\omega)\lambda(h^{t-1}) \right) u'(c^o(h^t)) = \mu. \tag{6}
$$

The multiplier μ is the marginal value of initial assets.

4.1   First best

Since households are risk averse and foreign lenders are risk neutral, the first best would have the latter absorbing all uncertainty. This would happen if credibility constraints

---

[18]This issue in general arises in dynamic games with an OLG structure. Farhi et al (2012) circumvent this since in their setup taxation can lead to the collapse of output, thus the worst outcome features no consumption. Dovis et al (2015) impose that deviation results in a particular allocation.

[19]The rescaling of the credibility constraint multipliers is with no loss of generality since $\pi(h^t)\delta^t$ is a positive number.

where always slack. In this case optimal allocations satisfy

$$
\begin{aligned}
u'(c^y(h^t)) &= \frac{\beta}{\delta} u'(c^o(h^t)), \\
u'(c^y(h^{t+1})) &= \delta R u'(c^y(h^t)).
\end{aligned}
\tag{7}
$$

Thus, the intratemporal allocation of consumption reflects the difference between individual and social discount factors, and the intertemporal profile reflects the difference between foreign and social discount factors. In particular, when $\beta = \delta = \beta'$, $c^y(h^t) = c^o(h^t) = \bar{c}$, and this constant level of consumption satisfies the resource constraint (5). Thus in this case the multiplier μ would be associated with this constant level of consumption: $u'(\bar{c}) = \mu$.

4.2   Constrained allocations and limit consumption behavior

If $\delta R > 1$, then (7) shows that young's consumption is increasing when the incentive constraint is not binding (old's consumption would also be increasing if the constraint did not bind in the previous period). Thus, eventually (5) would no longer bind, leading to the first best. If $\delta R < 1$, whenever the incentive constraint is slack, consumption (for both the young and old) is below the previous period's level. Consumption cannot fall indefinitely, as this would eventually violate the incentive constraint (5). When the constraint binds, consumption will increase. This leads to fluctuations in consumption, a similar result as in models with a representative agent with infinite horizon.[20]

Henceforth I will focus on the case in which the foreign and social discount factors are the same, i.e. $\delta R = 1$. In this case, the multiplier μ is associated with the level of *minimum* consumption that the young are guaranteed, $\underline{c}$, characterized by $u'(\underline{c}) = \mu$.[21] But consumption for the young can be higher than this level if the incentive constraint is binding in the current period. And (6) shows that this translates into higher consumption for them in the following period when old. Thus, consumption can be used to affect incentives in the past, but only for one period. This contrasts with models with a representative agent with an infinite horizon which feature back loading as future consumption gives incentives not to deviate in every previous period in which the participation constraint was binding. With finite lives, it is not necessary for the sovereign to be more impatient than foreign lenders to rationalize why assets are not accumulated to allow consumption to converge to its unconstrained first best.

---

[20] See Aguiar and Amador (2014).

[21] Note the difference with the case with a representative agent with an infinite horizon with $\delta R = 1$ in which the multiplier of the resource constraint is associated with the constant level of consumption once the highest endowment is realized for the first time. See Worrall (1990) and Aguiar and Amador (2014).

To further characterize the optimum let's rewrite the first-order conditions as

$$u'(c^y(h^t)) = \frac{u'(\underline{c})}{(1 + (1-\omega)\lambda(h^t))}, \tag{8}$$

$$u'(c^o(h^t)) = \frac{u'(\underline{c})}{\left(\frac{\beta}{\delta} + \omega\lambda(h^t) + \frac{\beta}{\delta}(1-\omega)\lambda(h^{t-1})\right)} = \frac{u'(c^y(h^{t-1}))\left(1 + (1-\omega)\lambda(h^{t-1})\right)}{\left(\frac{\beta}{\delta} + \omega\lambda(h^t) + \frac{\beta}{\delta}(1-\omega)\lambda(h^{t-1})\right)}, \tag{9}$$

where the second equality in (9) uses (8) in period $t-1$. If in period t, the credibility constraint is not binding, $\lambda(h^t) = 0$, and

$$c^y(h^t) = \underline{c}, \tag{10}$$

$$\frac{\beta}{\delta}u'(c^o(h^t)) = u'(c^y(h^{t-1})), \tag{11}$$

thus the young receive $\underline{c}$ while the old receive a consumption level that depends on the consumption they were allocated when young in the previous period. Therefore, when the credibility constraint is binding and the young are given a consumption above $\underline{c}$, they know that this determines the floor on their old age consumption as well. For simplicity, henceforth I will assume that $\delta = \beta$, such that when $\lambda(h^t) = 0$, $c^o(h^t) = c^y(h^{t-1})$.

4.3 Recursive Structure and Debt Dynamics

The first order conditions (8) and (9) show that history h^t can be summarized by the values of multipliers, μ, λ_{t-1}, and λ_t. Omitting the constant μ, the optimal contract will satisfy $c^y(h^t) = f^y(\lambda_t)$, $c^o(h^t) = f^o(\lambda_t, \lambda_{t-1})$. I will now show that there exists an implicit law of motion for the multiplier λ_t such that the optimal contract has a recursive structure with state variables y_t and λ_{t-1}.

When the incentive constraint is binding,

$$\omega u(c^o(h^t)) + (1-\omega)\left(u(c^y(h^t)) + \beta E_t[u(c^o(h^{t+1}))]\right) = \underline{V}(y_t). \tag{12}$$

From (9) it can be seen that an increase in λ_{t-1} implies an increase in $c^o(h^t)$ and since the right-hand side of (12) does not depend on λ_{t-1}, both $c^y(h^t)$ and $E_t[u(c^o(h^{t+1}))]$ must decrease. From (8) this implies that λ_t is reduced. Thus in general the multiplier of the credibility constraint depends on its past value and on the current output state.

In states in which the credibility constraint does not bind, consumption is a function only of λ_{t-1}, as seen in (10) and (11). When the constraint is binding, consumption is a function also of y_t, and equation (12) implicitly defines the law of motion for the state variable λ such that $\lambda_t = f^\lambda(y_t, \lambda_{t-1})$.[22] And when $\lambda_{t-1} = 0$; λ_t, $c^o(h^t)$, and $c^y(h^t)$ are

---

[22]The equation defining the law of motion for the Lagrange multiplier is

$$\omega u(f^o(\lambda_t, \lambda_{t-1})) + (1-\omega)\left(u(f^y(\lambda_t)) + \beta E_t[u(f^o(\lambda_{t+1}, \lambda_t))]\right) = \underline{V}(y_t).$$

This is a non-linear expectational difference equation, in general with no closed-form solution. Its solution

14

functions only of current output. For convenience, in what follows notation $c^i(h^t)$ instead of $c^i(y_t, \lambda_{t-1})$ will continue to be used for the optimal contract. With some abuse of notation the optimal contract has the recursive structure

$$
\begin{aligned}
c^y(h^t) &= f^y(y_t, \lambda_{t-1}), \\
c^o(h^t) &= f^o(y_t, \lambda_{t-1}), \\
\lambda_t = \lambda(h^t) &= f^\lambda(y_t, \lambda_{t-1}).
\end{aligned}
$$

I will now show that in the current setup in general there is no one-to-one correspondence between the Lagrange multiplier λ and debt. Aggregate consumption in bad states is increasing in λ_{t-1}. This follows since in those states $c^y(h^t) = \underline{c}$ and, as shown, $c^o(h^t)$ is increasing in λ_{t-1}. This leads to the standard interpretation that the higher λ_{t-1} the lower is the level of debt with which the economy enters period t in those states.

The previous analysis of (12) showed that, when $\lambda_t > 0$, an increase in λ_{t-1} increases $c^o(h^t)$ but decreases $c^y(h^t)$, thus whether aggregate consumption increases or decreases with λ_{t-1} when the credibility constraint is binding depends on parameters. Since an increase in aggregate consumption is interpreted as a reduction in state contingent debt, this implies that in general there is no monotonous relation between debt and state variables. Therefore debt cannot replace λ as a state variable as done in standard models.[23] The following proposition presents parameter restrictions such that aggregate consumption is always increasing (and thus debt is decreasing) in λ_{t-1}. In this case debt can be used as a state variable instead of the multiplier.

Proposition 1. When $\delta R = 1$, consumption will converge to an ergodic distribution and $\lim_{t \to \infty} P(\lambda(h^t) = 0) < 1$, i.e. the incentive constraint always binds with positive probability. When $\omega \leq \omega^S$ (with $\omega^S \geq \frac{1}{2}$) there is a monotone relation between debt and the multiplier.

Proof. See appendix 7.3. □

4.4 Output Shocks and Default Incentives

To study the effect of history, as summarized by λ_{t-1}, on the correlation between current output and incentives to deviate we start by defining $\bar{y}_t(\lambda_{t-1})$ as the endowment level at which the credibility constraint starts to bind. Since $\underline{V}(y_t)$ is monotonous this threshold

---

is clearly a function of λ_{t-1} and y_t. It might also be a function of future expectations of y. But these are constant in the i.i.d. case and would all be determined by y_t if the endowment followed a Markov process. Thus there is no loss of generality in writing its solution as $f^\lambda(y_t, \lambda_{t-1})$. If one is interested in a numerical solution for the law of motion of λ_t an alternative would be to use the Parameterized Expectations Approach. See den Haan and Marcet (1990) or Marcet and Marimon (1992).

[23]Debt is always a function of state variables, $b(h^t) = b(y_t, \lambda_{t-1})$, but in general it is not always true that $\frac{db(y_t, \lambda_{t-1})}{d\lambda_{t-1}} < 0$.

is well defined, and it must be the case that $\lambda_t = 0$ when $y_t \leq \bar{y}_t(\lambda_{t-1})$ and $\lambda_t > 0$ when $y_t > \bar{y}_t(\lambda_{t-1})$. Since the higher is $\bar{y}_t(\lambda_{t-1})$ the more likely it is that consumption will be smoothed, increases in this threshold can be seen as improvements in the economy's ability to share risk with the rest of the world.[24] At this threshold,

$$\omega u(c^y(h^{t-1})) + (1 - \omega)(u(\underline{c}) + \beta E_t[u(c^o(y_{t+1}))]) = \underline{V}(\bar{y}_t(\lambda_{t-1})). \tag{13}$$

Given that $c^y(h^t) = \underline{c}$ and $E_t[u(c^o(y_{t+1}))]$ cannot change with an increase in λ_{t-1}, but $c^y(h^{t-1})$ increases, then $\bar{y}_t(\lambda_{t-1})$ increases with λ_{t-1}. Thus the incentives to deviate in a given period are not only a function of current, but also of past output: When in the past output was high, consumption increased to avoid default. But for those who were young in the past, this increase was spread over their lifetime. Therefore today the old have high consumption and the overall incentive to deviate is muted.

Since there is no monotone relation between λ_{t-1} and debt repayments, incentives to deviate are not tightly related to outstanding (state-contingent) debt. It is only when $\omega \leq \omega^S$ that default incentives are positively correlated with debt. But even in this case this correlation, as well as the negative correlation of default incentives with current output, depends on preferences and political parameter ω, and therefore changes over time and across countries.

We can also study how an increase in the volatility of endowment, keeping the mean constant, affects $\bar{y}_t(\lambda_{t-1})$. In (13) this reduces the right-hand side as $E_t[u(c^{oD}(y_{t+1})]$ is negatively affected by the mean preserving spread. In the left-hand side, the term $E_t[u(c^o(y_{t+1}))]$ either is reduced by a smaller amount, or even can increase depending on the level of the consumption floor, $\underline{c}$ (there are two competing effects, the mean preserving spread tends to reduce the value of future expected utility, but consumption being stabilized for low shocks tends to increase it). The overall effect is therefore to increase $\bar{y}_t(\lambda_{t-1})$, as found in standard models of sovereign risk. Nevertheless, it might be the case that the credibility constraint binds in more states, thus the effect of volatility on incentives to deviate is ambiguous.[25]

It is illustrative to rewrite the credibility constraint (5) as

$$\omega\left(u(c^o(h^t)) - u(c^{oD}(y_t))\right) + (1-\omega)\big(u(c^y(h^t)) + \beta E_t[u(c^o(h^{t+1}))]$$
$$-(u(c^{yD}(y_t)) + \beta E[u(c^{oD}(y_{t+1}))])\big) \geq 0. \tag{14}$$

The left hand side is the weighted average of the surplus that young and old get from the contract. When the constraint is binding this average is zero, implying that one type of voters favors a default while the other is against it. In particular, whenever $c^o(h^t) > c^{oD}(y_t)$, the old will favor repayment.

---

[24]If $\bar{y}_t(\lambda_{t-1}) \geq \bar{y}$, where $\bar{y}$ is the highest possible endowment realization, then risk sharing is perfect since in no state the constraint is binding.

[25]Of course, higher volatility implies lower ex ante welfare.

4.5 Net Capital Flows

Since aggregate consumption is constant, at level $\underline{c} + c^y(h^{t-1})$, for low endowment realizations, the country is a net recipient of capital flows for bad shocks. These capital inflows are financed by capital outflows in other output states. Thus, at least one threshold income level, $\hat{y}_t(\lambda_{t-1})$, exists such that when output is (locally) above it the country experiences net capital outflows, and conversely when output is (locally) below this threshold.

In models with an infinitely lived representative agent $\hat{y}_t(\lambda_{t-1}) \leq \bar{y}_t(\lambda_{t-1})$, i.e. the credibility constraint only binds in states for which the country is expected to make a positive payment to its lenders (see Aguiar and Amador (2014)). The next proposition shows that, with commitment problems arising from the aggregation of default preferences among different generations, there might be incentives to default even when the country is a net recipient of resources from the rest of the world, i.e. $\hat{y}_t(\lambda_{t-1}) > \bar{y}_t(\lambda_{t-1})$.

Proposition 2. There exists a unique threshold income level, $\hat{y}_t(\lambda_{t-1})$, such that whenever $y_t < \hat{y}_t(\lambda_{t-1})$, the country is a net recipient of capital flows. For some parameters it is the case that $\hat{y}_t(\lambda_{t-1}) > \bar{y}_t(\lambda_{t-1})$.

Proof. See appendix 7.4. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

If $\hat{y}_t(\lambda_{t-1}) < \bar{y}_t(\lambda_{t-1})$, an increase in λ_{t-1} increases the minimum aggregate consumption and thus $\hat{y}_t(\lambda_{t-1})$ is increasing in λ_{t-1}. If $\hat{y}_t(\lambda_{t-1}) > \bar{y}_t(\lambda_{t-1})$, we saw that the effect of an increase in λ_{t-1} on aggregate consumption (and therefore on $\hat{y}_t(\lambda_{t-1})$) is indeterminate. It will be positive when $\omega \leq \omega^S$. And in general we cannot say whether the effect of λ_{t-1} is stronger on $\hat{y}_t(\lambda_{t-1})$ or on $\bar{y}_t(\lambda_{t-1})$.

4.6 Distribution of political power

The optimal contract reflects two types of pressures. First, the overall willingness to default when the country experiences good endowment shocks. Second, the tension between desired generational equity and the distribution of political power: When individual and social discount factors are the same the desired social allocation has equal consumption for young and old. But when the credibility constraint is binding the allocation gets distorted towards the autarkic one with marginal utilities reflecting the relative political power of the old, $\frac{\omega}{1-\omega}$. The next proposition summarizes the effects of a change in the distribution of political power.

Proposition 3. There exist thresholds $0 < \omega_1^P \leq \omega_2^P \leq \frac{1}{2}$ such that when $\omega \leq \omega_1^P$ or $\omega \geq \omega_2^P$, we have that $\frac{d\mathrm{V}(y_t)}{d\omega} \geq 0$, implying $\frac{d\bar{y}_t(\lambda_{t-1})}{d\omega} < 0$ and $\frac{d\hat{y}_t(\lambda_{t-1})}{d\omega} > 0$, i.e. an increase in the political power or the old results in a stronger incentives to default reducing risk sharing. With CRRA preferences with coefficient of risk aversion less than one $\omega_1^P = \omega_2^P$.

Proof. See appendix 7.5. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Proposition 3 implies that in some parameter regions an increase in the political power of the old, ω, leads to a decrease in $\bar{y}_t(\lambda_{t-1})$, i.e. a worsening of the economy's ability to share risk with the rest of the world. Since this implies the economy receives less inflows in bad states, an increase in ω leads to an increase in $\hat{y}_t(\lambda_{t-1})$. For CRRA preferences with coefficient of risk aversion less than one $\omega_1^P = \omega_2^P$, and an increase in the political power of the old always reduces the country's risk sharing possibilities.

The intuition for this result is that there are two effects of changes in ω on default incentives. First, the weighted average of autarkic consumptions is lowest when $\omega = \frac{1}{2}$. Thus, a more polarized society finds deviation more valuable. Second, the political contribution of the future expected utility under the prevailing contract, relative to expected utility under autarky, $(1 - \omega)\left(E[u(c^o(h^{t+1})] - E[u(c^{oD}(y_{t+1})]\right)$, is decreasing in ω. The incentives to deviate arising from the future expected consequences of current actions are negatively related to this term. Therefore they are always increasing with ω. For $\omega < \frac{1}{2}$ both effects are working in opposite directions, since an increase in ω results in a less polarized society. But for $\omega > \frac{1}{2}$, they work in the same direction leading to an unambiguous higher incentive to deviate from an increase in ω.

4.7 Preferences for Default by Age

We have seen that when the credibility constraint is binding, one type of voters favors a default while the other opposes it. To see what determines the olds' preferences for default, it is illustrative to consider first the case in which the relative weights of generations in the political process is the same as in the social welfare function, i.e. $\omega = \frac{1}{2}$. In this case $c^o(h^t) \geq c^y(h^t)$ with equality holding whenever $\lambda_{t-1} = 0$, and $c^o(h^t) > c^y(h^t)$ when the credibility constraint was binding in the previous period. Since in autarky $c^o(h^t) = c^y(h^t)$ in every state, we can interpret this outcome as if incentives are given in the form of (state contingent) social security transfers: whenever the credibility constraint is binding, the current young are promised higher consumption when old than the future young.[26]

When $\omega = \frac{1}{2}$, $\lambda_{t-1} = 0$ and $\lambda_t > 0$, thus $c^o(h^t) = c^y(h^t)$, and regrouping terms in (12)

$$2\left[u(c^y(h^t)) - u(\frac{y_t}{2})\right] = E_t[u(\frac{y_{t+1}}{2})] - E_t[u(c^o(h^{t+1}))] < 0 \tag{15}$$

This implies that whenever the constraint is binding in this period, but not in the previous one, net exports are positive, i.e. $\hat{y}_t(0) \leq \bar{y}_t(0)$. As mentioned before, this is a standard result from a representative agent economy. It holds in this special case because with equal weighting and $\lambda_{t-1} = 0$ the optimal allocation gives equal consumption to young and old, and under default consumptions are also equalized.[27] Another implication of

---

[26]This was the interpretation of optimal allocations in Gonzalez-Eiras (2000) derived for stationary equilibria, under the assumption that both young and old had veto power in the political process.

[27]Even with equal weights, for high enough λ_{t-1} it might be the case that $c^y(h^t) + c^o(h^t) > y_t$, i.e. $\hat{y}_t(\lambda_{t-1}) > \bar{y}_t(\lambda_{t-1})$.

(15) is that $c^o(h^t) < c^{oD}(y_t) = \frac{y_t}{2}$. Thus whenever the credibility constraint is binding, but was not binding in the past, the old are in favor of default.

The following proposition shows how the old's preferences for repayment are affected by past outcomes.

Proposition 4. When $\omega \geq \omega^D(\lambda_{t-1})$, with $\frac{d\omega^D(\lambda_{t-1})}{d\lambda_{t-1}} \geq 0$, and $\omega^D(0) \leq \frac{1}{2}$, the old prefer a default.

Proof. See appendix 7.6. □

Not surprisingly, the old favor a default when the political process is biased towards them such that they would be able to extract a larger share of resources in autarky. But this preference for defaulting is mitigated if they are entitled to high current consumption as a result of past promises. Thus the larger past endowment shocks the more likely it is that the old will oppose a default.

5 Robustness to Production Economies

We now assume that there are domestic firms that hire labor from the young and capital from the old to produce with a constant returns to scale technology that is subject to i.i.d. productivity shocks, s_t. Thus, output $y(h^t)$ is given by $s_t F(k_t(h^{t-1}), 1)$, where $k_t(h^{t-1})$ is capital, and h^{t-1} is the history up to period $t-1$ which now includes past productivity shocks.[28] The planner wants to maximize the objective function (1) subject to the intertemporal budget constraint, which now reads,[29]

$$b_0 \leq \sum_{t \geq 0, h^t} q^*(h^t) \left(s_t F(k_t(h^{t-1}), 1) - (R - 1 + \delta)k_t(h^{t-1}) - c^y(h^t) - c^o(h^t) \right),$$

where capital depreciates at rate δ. And the planner still faces a sequence of credibility constraints, now given by,

$$\omega u(c^o(h^t)) + (1 - \omega) \left(u(c^y(h^t)) + \beta E_t[u(c^o(h^{t+1}))] \right) \geq \underline{V}(s_t F(k_t(h^{t-1}), 1)).$$

Note that incentives to default are a function of the history of shocks as summarized by the capital stock.[30]

It is possible to set up the planner's problem in primal form. In general this requires adding an implementability constraint that guarantees that allocations can be implemented in a competitive equilibrium. I assume that period t governments can control consumption of workers and retirees and savings decisions of the former using lump sum

---

[28]Labor supply is exogenous and normalized to one.

[29]See Aguiar and Amador (2014).

[30]I assume that default implies autarky. Allocations in autarky, and thus default incentives, will then continue to be increasing in aggregate output.

transfers and taxes or subsidies on savings. With these policy instruments, the multiplier on the implementability constraint is zero and the first order conditions on consumption are the same as for the endowment economy.[31]

Now we must also consider the first order condition with respect to capital and how this affects future credibility constraints

$$\mu E_t \left[s_{t+1} F_k(k_{t+1}(h^t), 1) - (R - 1 - \delta) \right] = E_t[\lambda_{t+1} \underline{V}'(s_{t+1} F(k_{t+1}(h^t), 1) s_{t+1} F_k(k_{t+1}(h^t), 1)] \tag{16}$$

With no incentive constraints, it is optimal to invest up to the point in which the expected marginal product of capital equals its opportunity cost. Thus, the left hand side of (16) would be zero. But investment is lower, such that the expected marginal product of capital is above its opportunity cost, if the credibility constraint binds with positive probability in the following period. The intuition, which is the same as in representative agent models with an infinite horizon, is that when incentives might need to be provided in the future a decrease in the future capital stock reduces the incentives to deviate. Thus, it is optimal to underinvest. Note that a positive productivity shock in period t increases investment since we saw that an increase in λ_t reduces λ_{t+1}.

6   Conclusions

Recent empirical studies present evidence that is inconsistent with existing representative agent models of sovereign risk. This raises the question on how preferences for repayment are distributed among a country's residents and how they are aggregated into policy. In this paper I introduce an overlapping-generations demographic structure onto the benchmark limited-commitment framework. I then characterize the efficient consumption allocations from the perspective of a benevolent social planner that faces the constraint that policy is determined sequentially by probabilistic voting among those currently alive.

Results show that, as in representative agent models, risk sharing is constrained since the small open economy has incentives to deviate when output is high. Incentives to deviate are time varying, even when shocks are i.i.d., since when in the previous period output was high this reduces the current incentives to deviate. This cannot in general be associated to the fact that higher past output is reflected in lower outstanding debt, as there is no monotone relation between the multiplier on current incentive constraints (which measures default incentives) and state-contingent debt payments.

Contrary to the implication of models with a representative agent with an infinite horizon, there might be incentives to default even when the country is a net recipient of capital flows. Thus the model can rationalize the weak relation between defaults and

---

[31]The values of the multiplier will be different, and the time series properties of consumption and debt will be different. But the main intuitions of the results that we found for the endowment economy hold for a production economy.

output gap in historical data, and the fact that occasionally countries default when their debt-to-GDP ratio is low relative to the country's history.

The old are likely to favor repayment when they are weak in the political process or when in the past they were promised a high level of current consumption. This helps to explain recent survey data that shows that the old have stronger preferences for repayment than the young, even when controlling for other determinants of repayment preferences.

The model has implications for empirical research suggesting that regressions of sovereign risk, or actual default episodes, should control for past shocks (not relying on outstanding debt as summarizing these), and the share of the elderly in the population. In future research I intend to extend the model to a production economy and to study the effect of finite lives in a quantitative model with incomplete markets.

References

Abreu, D. (1988), 'On the Theory of Infinitely Repeated Games with Discounting', *Econometrica* **56**(2), 383-396.

Aguiar M. and Amador, M. (2014), 'Sovereign Debt', *Handbook of International Economics*, **4**, 647-687.

Angrist, J. D. and Pischke, J-S. (2006), Mostly Harmless Econometrics, *Princeton Univesity Press*.

Belloni, A., Chernozhukov, V. and Hansen, C. (2014), 'Inference on treatment effects after selection amongst high-dimensional controls', *Review of Economic Studies* **81**(2) 608-650.

Broner, F. A., Martin, A. and Ventura, J. (2010), 'Sovereign Risk and Secondary Markets', *American Economic Review* **100**(4), 1523-1555.

Bulow, J. I. and Rogoff, K. (1989), 'Sovereign Debt: Is To Forgive To Forget?', *American Economic Review* **79**(1), 43-50.

Calahorrano, L. (2011), 'Population Aging and Individual Attitudes toward Immigration: Disentangling Age, Cohort and Time Effects', *Review of International Economics* **21**(2), 342-353.

Chari, V. V. and Kehoe P. J. (1990), 'Sustainable Plans', *Journal of Political Economy* **98**(4), 783-802.

den Haan, W. J. and Marcet, A. (1990), 'Solving the Stochastic Growth Model by Parameterizing Expectations', *Journal of Business and Economic Statistics* **8**(1), 31-34.

Dovis, A., Golosov, M. and Shourideh, A. (2015), 'Sovereign Debt vs Redistributive Taxes: Financing Recoveries in Unequal and Uncommitted Economies', working paper, Princeton University.

Fahri, E., Sleet, C., Werning, I. and Yeltekin S. (2012), 'Non-linear Capital Taxation Without Commitment', *Review of Economic Studies* **79** 1469-1493.

Eaton, J. and Gersovitz, M. (1981), 'Debt with Potential Repudiation: Theoretical and Empirical Analysis', *Review of Economic Studies* **48**(2) 289-309.

Giuliano, P. and Spilimbergo, A. (2014), 'Growing up in a Recession', *Review of Economic Studies* **81**(2) 787-817.

Gonzalez-Eiras, M. (2000), 'Essays on Incomplete Markets, Liquidity and Risk Sharing', PhD thesis, MIT.

Guembel, A. and Sussman, O. (2009), 'Sovereign Debt without Default Penalties', *Review of Economic Studies* **76**(2) 1297-1320.

Kletzer, K. M. and Wright, M. (2000), 'Sovereign Debt As Intertemporal Barter', *American Economic Review* **90**(3), 621-639.

Marcet, A. and Marimon, R. (1992), 'Recursive Formulation of Dynamic Contracts with Non Recursive Constraints', working paper, Universitat Pompeu Fabra.

Persson, T. and Tabellini, G. (2000), Political Economics, *MIT Press.*

Tomz, M. and Wright, M. (2007), 'Do Countries Default in "Bad Times"?', *Journal of the European Economic Association* **5**(2-3), 352–360.

Tomz, M. (2004), 'Interest, Information, and the Domestic Politics of International Agreements', working paper, Stanford University.

Worrall, M. (1995), 'Debt with potential repudiation' *European Economic Review* **34**(5), 1099–1109.

7 Appendix

7.1 Probabilistic voting

The proposed utilitarian functions postulated for period governments, (3), can be rationalized if policy preferences are aggregated via probabilistic voting. Assume that the young and old alive in period t vote on candidates representing competing platforms. Voters support a candidate not only for the private utility derived from her policy platform, but also for other characteristics like "ideology" that are orthogonal to the economic policy dimensions of interest. These characteristics are permanent and cannot be credibly altered in the course of electoral competition; their valuation differs across voters (even if voters agree about the preferred policy platform) and is subject to random aggregate shocks, realized after candidates have chosen their platforms. This setup thus renders the probability of winning a voter's support a continuous function of the competing policy platforms.

In a Nash equilibrium with two candidates maximizing their expected vote share, both candidates propose the same policy platform.[32] This platform maximizes a convex combination of the objective functions of young and old voters, where the weights reflect the groups' sensitivity of voting behavior to policy changes. Groups that care a lot about policy platforms relative to the candidates' other characteristics have more political influence as they are more likely to shift their support from one candidate to the other in

---

[32]See Persson and Tabellini (2000) for a textbook discussion on probabilistic voting.

response to small changes in the proposed platform. In equilibrium, these groups of "swing voters" thus tilt policy in their own favor. Letting ω and $1 - \omega$ denote the per-capita political influence of old and young, respectively, the objective function of candidates for period t government is precisely (3).

7.2 Preferences for Default

When the incentive constraint (5) is binding we have that

$$
\omega \left(u(c^o(h^t)) - u(c^{oD}(y_t)) \right) \quad + \quad (1 - \omega) \left(u(c^y(h^t)) + \beta E_t[u(c^o(h^{t+1}))] \right.
$$
$$
\left. - (u(c^{yD}(y_t)) + \beta E[u(c^{oD}(y_{t+1}))]) \right) = 0.
$$

Thus the weighted average of the surplus form the contract for y9 oung and old is zero. This implies that in general one group will be in favor of default while the other will be against. Whenever $c^o(h^t) > c^{oD}(y_t)$ the old will be against a default.

7.3 Proof of proposition 1

We need to relate the change in current and expected future consumption for the current young, when the credibility constaint is binding. We start by differentiating (9) for period $t + 1$[33]

$$
u'(c^o(h^{t+1}))\frac{dc^o(h^{t+1})}{d\lambda_{t-1}} = u'(c^y(h^t))\frac{dc^y(h^t)}{d\lambda_{t-1}}\frac{\frac{-u''(c^y(h^t))}{u'(c^y(h^t))}}{\frac{-u''(c^o(h^{t+1}))}{u'(c^o(h^{t+1}))}}\frac{u'(c^o(h^{t+1}))}{u'(c^y(h^t))}
$$
$$
+ \frac{-u'(c^o(h^{t+1}))}{u''(c^o(h^{t+1}))}u'(c^y(h^t))\left[\frac{\omega(1-\omega)\lambda_{t+1}}{(1 + \omega\lambda_{t+1} + (1-\omega)\lambda_t)^2}\right]\frac{d\lambda_t}{d\lambda_{t-1}}
$$

The last term is positive while the other two are negative. Thus we can write $\beta E_t[u'(c^o(h^{t+1}))\frac{dc^o(h^{t+1})}{d\lambda_{t-1}}] \equiv \psi u'(c^y(h^t))\frac{dc^y(h^t)}{d\lambda_{t-1}}$, with $\psi > 0$ and bounded (e.g. if preferences have constant absolute risk aversion then, since $c^o(h^{t+1}) \geq c^y(h^t)$, $\psi < \beta$). Then from (12) we have that $\omega u'(c^o(h^t))\frac{dc^o(h^t)}{d\lambda_{t-1}} = -(1-\omega)(1+\psi)u'(c^y(h^t))\frac{dc^y(h^t)}{d\lambda_{t-1}}$. Using this relation to see the marginal effect of λ_{t-1} on total consumption we get

$$
\frac{d\left(c^o(h^t) + c^y(h^t)\right)}{d\lambda_{t-1}} \quad = \quad \frac{dc^o(h^t)}{d\lambda_{t-1}}\left[1 - \frac{\omega u'(c^o(h^t))}{(1+\psi)(1-\omega)u'(c^y(h^t))}\right]
$$
$$
\frac{dc^o(h^t)}{d\lambda_{t-1}}\left[1 - \frac{\omega(1 + (1-\omega)\lambda_t)}{(1+\psi)(1-\omega)(1 + \omega\lambda_t + (1-\omega)\lambda_{t-1})}\right].
$$

Thus, at least for some values of λ_{t-1} and λ_t, for ω high enough the term in brackets will be negative and total consumption falls with an increase in λ_{t-1}.

From this last equation we can see that if $\omega \leq \omega^S$, with $\omega^S \geq \frac{1}{2}$ ($\omega^S = \frac{1}{2}$ i.f.f. $\psi = 0$), then $\frac{d\left(c^o(h^t)+c^y(h^t)\right)}{d\lambda_{t-1}} > 0$.

---

[33]In what follows it is understood that $\frac{dc^o(h^{t+1})}{d\lambda_{t-1}} = \frac{dc^o(h^{t+1})}{d\lambda_t}\frac{d\lambda_t}{d\lambda_{t-1}}$.

7.4 Proof of proposition 2

To show that $\hat{y}_t(\lambda_{t-1})$ is unique, we start noting that capital flows cannot be positive (or negative) for all states. The country can get positive flows in some states at the expense of being willing to make net payments in other states. Thus at least one threshold $\hat{y}_t(\lambda_{t-1})$ exists. Then we note that $E_t[u(c^o(h^{t+1})] - E_t[u(c^{oD}(y_{t+1})]$ is weakly increasing in the current output realization.[34] Finally, for states with higher current output in (12), the ratio of the marginal utilities of the consumption pair $c^o(h^t)$ and $c^y(h^t)$ moves closer to its value under autarky (see (4), (8) and (9)). And in the limit as output, and thus λ_t, goes to infinity, the ratio of marginal utilities converges to

$$\lim_{\lambda_t \to \infty} \frac{u'(c^o(h^t))}{u'(c^y(h^t))} = \frac{1-\omega}{\omega} = \frac{u'(c^{oD}(y_t))}{u'(c^{yD}(y_t))}.$$

The country has to be a recipient of funds in the worst state for which consumption is stabilized as seen in (10) and (11). Since $E_t[u(c^o(h^{t+1})] - E_t[u(c^{oD}(y_{t+1})]$ is weakly increasing in output, and the ratio of marginal utilities converges to its autarky value, $\frac{1-\omega}{\omega}$, as current output increases, for the highest realization of output, from (12) it must be the case that $c^y(s^t) + c^o(s^t) < y_t$, i.e. there are capital outflows.

If capital flows are positive in the worst state, and negative in the best state, then by continuity there must be an odd number of thresholds, $\hat{y}_t(\lambda_{t-1})$. Since consumption is stabilized when $y_t \leq \bar{y}_t(\lambda_{t-1})$, then at most one of these thresholds might lie at the left of $\bar{y}_t(\lambda_{t-1})$. I will now show that a contradiction arises if two thresholds exist to the right of $\bar{y}_t(\lambda_{t-1})$. Let's call them $\hat{y}_t^1(\lambda_{t-1})$ and $\hat{y}_t^2(\lambda_{t-1}) > \hat{y}_t^1(\lambda_{t-1}) > \bar{y}_t(\lambda_{t-1})$. At both thresholds (12) holds and $c^{o1}(s^t) + c^{y1}(s^t) = \hat{y}_t^1(\lambda_{t-1})$, $c^{o2}(s^t) + c^{y2}(s^t) = \hat{y}_t^2(\lambda_{t-1})$. And since $E_t[u(c^o(h^{t+1})] - E_t[u(c^{oD}(y_{t+1})]$ is weakly increasing in the current output it must be the case that

$$\omega u(c^{o2}(h^t)) + (1-\omega)u(c^{y2}(h^t)) - \big(\omega u(c^{oD}(\hat{y}_t^2(\lambda_{t-1}))) + (1-\omega)u(c^{yD}(\hat{y}_t^2(\lambda_{t-1})))\big) <$$
$$\omega u(c^{o1}(h^t)) + (1-\omega)u(c^{y1}(h^t)) - \big(\omega u(c^{oD}(\hat{y}_t^1(\lambda_{t-1}))) + (1-\omega)u(c^{yD}(\hat{y}_t^1(\lambda_{t-1})))\big) < 0$$

But for higher income the optimal contract's ratio of marginal utilities moves towards its autarky value. And since the latter, by definition, maximizes $\omega u(c^o) + (1-\omega)u(c^y)$, then it cannot be the case that the weighted average of utilities moves away from its maximum value as income increases. That establishes the desired contradiction and proves that $\hat{y}_t(\lambda_{t-1})$ is unique.

To show that it might be the case that $\hat{y}_t(\lambda_{t-1}) > \bar{y}_t(\lambda_{t-1})$, assume that it is always the case that $\hat{y}_t(\lambda_{t-1}) \leq \bar{y}_t(\lambda_{t-1})$, and suppose $y_t = \hat{y}_t(\lambda_{t-1})$. Since at this output level the

---

[34]Note that this is straightforward since shocks are i.i.d. The second term is constant and the former increases with y_t as this increases $c^y(h^t)$ and therefore the consumption floor in old age. Additional assumptions would be required for a general Markov process.

credibility constraint is not binding, $c^o(h^t) = c^y(h^{t-1})$ and $c^y(h^t) = \underline{c}$. Since $y_t = \hat{y}_t(\lambda_{t-1})$, $c^y(h^t) + c^y(h^{t-1}) = y_t$. It must be the case that

$$\omega\left(u(c^y(h^{t-1})) - u(c^{oD}(y_t))\right) + (1-\omega)\left(u(c^y(h^t)) - u(c^{yD}(y_t)) + B\right) > 0,$$

where $B = \beta\left(E_t[u(c^o(h^{t+1}))] - E[u(c^{oD}(y_{t+1}))]\right) > 0$. Since $c^y(h^t) = \underline{c} \le c^y(h^{t-1})$, and $c^y(h^{t-1})$ are fixed, and $\omega\left(u(c^y(h^{t-1})) - u(c^{oD}(y_t))\right) + (1-\omega)\left(u(c^y(h^t)) - u(c^{yD}(y_t))\right) \le 0$ (with equality only when ω is such that the autarky allocation coincides with the optimal contract), then for high values of ω the weighted average of current utility surplus becomes larger in absolute value than $(1-\omega)B$. This is due to high values of ω leading to increasing divergence between the autarky allocation and the optimal contract, $c^{oD}(y_t) > c^y(h^{t-1})$, as well as the reduction in the term $(1-\omega)B$. Thus, the credibility constraint becomes binding and $\hat{y}_t(\lambda_{t-1}) > \bar{y}_t(\lambda_{t-1})$, contrary to out initial assumption.

7.5   Proof of proposition 3

It is illustrative to consider the cases of extreme political preferences. We first consider the case that only the welfare of the old enters the utilitarian welfare function. In this case, $\omega = 1$ and the credibility constraint when binding, (12), reduces to

$$u(c^o(h^t)) = u(y_t),$$

since in this case the optimal consumption under a deviation gives all the endowment to the old. Thus when the constraint is binding net exports are zero. Therefore the economy is unable to receive resources from the rest of the world in bad states and the optimal allocation corresponds to $c^o(h^t) = y_t$ in every period and state.

The opposite polar case has the political process only caring about the welfare of the young, i.e. $\omega = 0$. In this case consumption for the old will not be driven to zero since future old age consumption enters the utilitarian welfare function through the preferences of the current young. From (9) it can be seen that $c^o(h^t) = c^y(h^{t-1})$ regardless of $\lambda(h^t)$. This makes sense since the current old do not need to be given incentives in the political process. Thus promises of future expected utility made to the young are best delivered through a constant old age consumption. In states that the credibility constraint is binding, (12) reduces to

$$u(c^y(h^t))(1 + \beta) = u(y_t) + \beta u(0).$$

If $\lim_{c \to 0} u(c) = -\infty$, then the credibility constraint would not bind in any state and the first best allocation would be achieved. By continuity, this result would hold in a neighborhood of $\omega = 0$. With a finite $u(0)$, when output is high there might be a need to provide incentives, as in the general case.

To see how incentives to deviate change with an increase in the political power of the old, rewrite (12) as

$$\omega u(c^o(h^t)) + (1-\omega)u(c^y(h^t)) = \left[\omega u(c^{oD}(y_t)) + (1-\omega)u(c^{yD}(y_t))\right] - (1-\omega)B(\omega)$$

with $B(\omega) \equiv \beta\left(E_t[u(c^o(h^{t+1}))] - E[u(c^{oD}(y_{t+1}))]\right)$. Taking derivative with respect to ω, keeping initially the optimal contract unchanged, the derivative of the term in brackets in the RHS is positive when $\omega > \frac{1}{2}$ and negative when $\omega < \frac{1}{2}$ since

$$\frac{d\left[\omega u(c^{oD}(y_t)) + (1-\omega)u(c^{yD}(y_t))\right]}{d\omega} = u(c^{oD}(y_t)) - u(c^{yD}(y_t)),$$

where the autarky FOC (4) was used. This is $\leq 0 (> 0)$ when $c^{oD}(y_t) \leq (>)c^{yD}(y_t)$, which happens when $\omega \leq (>)\frac{1}{2}$. Next, $\frac{-d(1-\omega)B(\omega)}{d\omega} > 0$, since $B(\omega) \geq 0$ and $c^{oD}(y_{t+1})$ is increasing in ω. Then there exists $\omega_2^P < \frac{1}{2}$ such that for $\omega \geq \omega_2^P$, $\underline{V}(y_t)$ is increasing in ω.

To show that $\underline{V}(y_t)$ is increasing in ω for $\omega \leq \omega_1^P$ we consider first the case that $\lim_{c\to 0}u(c) = -\infty$. Then as seen above, the first best allocation follows and $\lim_{\omega\to 0}\underline{V}(y_t) = -\infty$. By continuity for ω close to zero we also get the first best. Thus locally around $\omega = 0$ an increase in ω leads to an increase in $\underline{V}(y_t)$. If $\lim_{c\to 0}u(c) > -\infty$, then to establish the result we must look at the limit behavior of $\frac{-d(1-\omega)B(\omega)}{d\omega}$. In particular $\frac{d\underline{V}(y_t)}{d\omega}|_{\omega=0}$ is given by[35]

$$u(0) - u(y_t) + \beta\left(E_t[u(c^o(h^{t+1}))] - u(0)\right) + \beta u'(0)\frac{dc^{oD}}{d\omega} > 0.$$

Although the first term is negative it is bounded. And we are assuming that $\lim_{c\to 0}u'(c) = \infty$. Thus $\underline{V}(y_t)$ is increasing in ω when $\omega = 0$, and by continuity also on a neighborhood of $\omega = 0$.

This implies that $\omega u(c^o(h^t)) + (1-\omega)u(c^y(h^t))$, and therefore both $c^o(h^t)$ and $c^y(h^t)$ (and with it $c^o(h^{t+1})$), is increasing in ω when $\omega \geq \omega_2^P$ or when $\omega \leq \omega_1^P$. Applying this reasoning when $y_t = \hat{y}_t(\lambda_{t-1})$, an increase in ω leads to an increase in $\hat{y}_t(\lambda_{t-1})$ when $\omega \geq \omega_2^P$ or when $\omega \leq \omega_1^P$. Similarly, when $y_t = \bar{y}_t(\lambda_{t-1})$, an increase in ω leads to an decrease in $\bar{y}_t(\lambda_{t-1})$ when $\omega \geq \omega_2^P$ or when $\omega \leq \omega_1^P$.

It is illustrative to write $V(y_t)$ as a function of $\Gamma \equiv \frac{\omega}{1-\omega}$, since the sign of $\frac{d\underline{V}(y_t)}{d\omega}$ is the same as the sign of $\frac{d\underline{V}(y_t)}{d\Gamma}$. The latter is given by

$$\frac{d\underline{V}(y_t)}{d\Gamma} = u(c^{oD}) + \beta E_t\left[u'(c^{oD}(y_{t+1}))\frac{dc^{oD}(y_{t+1})}{d\Gamma}\right]$$

Since $\frac{dc^{oD}(y_{t+1})}{d\Gamma} > 0$, when preferences are CRRA with a coefficient of risk aversion below unity the above derivate is always positive (because then $u(c^{oD}) \leq 0$), and an increase in the political power of the old unambiguously reduces a country's risk sharing possibilities.

---

[35]It should be worth reminding that this comparative exercise keeps the contact, and therefore $c^o(h^{t+1})$, constant.

7.6   Proof of proposition 4

When $\lambda_{t-1} = 0$ and the credibility constraint starts to bind, i.e. $y_t = \bar{y}_t(0)$, we have $c^o(h^t) = \underline{c}$. At the same time, the autarky consumption of the old is an increasing function of ω as seen in (4). Thus there exists ω^D such that $c^{oD}(\bar{y}_t(0)) = \underline{c}$, and when $\omega > \omega^D$ the old favor a default. And since we showed that the old favor a default when $\lambda_{t-1} = 0$ and $\omega = \frac{1}{2}$, this implies that $\omega^D \leq \frac{1}{2}$.

This reasoning can be replicated when $\lambda_{t-1} > 0$. There exists $\omega^D(\lambda_{t-1})$ increasing in λ_{t-1} such that $c^{oD}(\bar{y}_t(0)) = c^o(h^t) \geq \underline{c}$, and when $\omega > \omega^D(\lambda_{t-1})$ the old favor a default.