An Optimal Voting Rule for Multilateral Financial Institutions

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Abstract

Governance in multilateral financial institutions is based on a quota system, and decisions take place by weighted voting. I show that, while weighted voting in general is not optimal, an optimal voting rule still assigns a quota to each country. When aggregating preferences, the vote of each country must be additionally weighted according to whether that country is a net creditor or net debtor. The model predicts that a country's quota be calculated from two components. First, a weighted sum of trade flows with other members, including domestic absorption i.e. trade with oneself, with weights proportional to the probabilities of each trade partner suffering a shock. Second, the ratio of the country's GDP PPP to GDP. The model shows how the total level of resources of multilateral institutions should evolve relative to world trade.

JEL Classification codes: D72, F32, F33, F41

1 Introduction

There are several multilateral financial institutions in the world, most of them development banks. The oldest and more widely known are the International Monetary Fund and the World Bank, set up in 1944. The latter has grown from a narrow focus on post-World War II reconstruction into a development bank focused on helping poor countries grow, and the Fund aims at preserving macroeconomic and financial stability. While capital flows have increased significantly in the last decades reducing the need for multilateral lending, as long as the effects of economic instability and underdevelopment spread

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through trade or immigration it is in the world's self-interest to coordinate actions and limit adverse spillovers.

Governance in multilateral financial institutions is based on a quota system, and usually modeled after the one used in the International Monetary Fund. This system assigns each country a weight derived from a quota formula whose inputs are indicators of countries' economic standing.[1] These weights then determines the amount of financial resources that members must contribute to the institution and their voting power in decision making. There is no explicit rationale for using this quota system instead of a more complex institution to aggregate preferences, as double majority voting.[2] Even before showing a strong performance in the last global crisis, emerging markets felt that quota formulas, and the composition of the institutions' governing bodies, have not been updated to reflect their rapid growth.[3]

The aim of this paper is to characterize an optimal voting rule for multilateral financial institutions, thus providing a normative benchmark that might help in framing the discussion on governance reform. To this effect I modify Barberà and Jackson's (2006) model on optimal voting rules for heterogeneous unions. Interaction through trade makes it possible to model how providing financial assistance to a given country (either to deal with a financial or balance of payments crisis, or to develop a productive project for which there is no private finance available) affects the welfare of all members. To quantify these welfare effects I use Waugh's (2010) variant of the Eaton and Kortum (2002) Ricardian model of trade, which by allowing for exporter fixed effects and introducing capital as a factor of production, explains income disparities across developed and developing countries. Providing assistance today requires resources from members. These trade off the positive impact of providing assistance against the cost of doing so when determining how to vote.

Numerical solutions show that weighted voting as determined by the quota system is *not optimal*, except for the Fund and only in the period before the breakdown of the Bretton Woods Agreement on exchange rates in 1973. The explanation for this result is that a country's own welfare effect from receiving assistance is much larger than the effect when other countries are the beneficiaries. Since developed countries are not recipients of assistance either from the Fund or from development banks, this asymmetry in preferences makes weighted voting not optimal.[4]

An alternative organizational structure, a "modified quota system" since it still is based on giving each member a weight, is suggested. The quota assigned to each country is used to weight its vote whenever a country is *opposing* the requested lending. When the country votes in favor of giving financial assistance this weight is multiplied by a factor that is different depending on whether the country is a net creditor or net debtor. Decisions are made according to the difference in weighted sum of votes in favor and

[1]In the Fund there are also basic votes common to each member with the objective of boosting the weights of small and poor countries. They represent 5.5% of total votes.

[2]See, for example, O'Neill and Peleg (2000) and Rapkin and Strand (2006).

[3]Eichengreen (2007) provides a comprehensive argument for IMF reform.

[4]Developed countries have recently received assistance from the Fund or the European Union. Except for the case of Iceland, these countries belong to the euro area.

against the proposal. A simpler way to implement this voting rule would have two votes taking place, one among net creditors and the other among net debtors. In these each countries' votes are weighted by their quota and different thresholds are required to pass the proposal. If both groups agree this is the collective decision. If they disagree a "grand assembly" is required in which the more complex mechanism is used.

Under a linear approximation of the equilibrium equations in Waugh's (2010) model, a formula to calculate members' optimal voting weights is derived. Quotas should be determined by two components. First, the weighted sum of a country's trade volume with the rest of the members of the organization, with weights related to the probabilities of each trade partner experiencing a need for financial assistance (i.e. suffering a financial or balance of payments crisis for the case of the Fund, or having an investment opportunity that requires outside finance for a development bank). Second, the ratio of the country's GDP converted at PPP exchange rates to GDP converted at market exchange rates. Since the model allows the determination of *total* quotas, it provides an objective measure of the volume of financial resources that multilateral financial institutions should manage, and how this should relate to the conditions of the world economy.

Two polar cases of interest are described. In the first every country has the same probability of experiencing a need for financial assistance, while in the second a subgroup of countries do not experience shocks, while the rest face the same risk. The first case serves as an approximation for the Fund between its creation and the breakdown of the Bretton-Woods fixed exchange rates regime in the seventies. The second case reflects the current situation with a distintion between net creditors and net debtors members, as well as the objective of development banks. The results show a significant change in the distribution of voting weights and help to explain the widespread criticism of the quota system in the Fund of recent decades.[5]

In recent years there have been many reform proposals that focused on different aspects of governance in multilateral financial institutions, mostly dealing with the IMF. Buira (2005), among others, calls for using PPP measures of GDP in the current quota formulas as a way to increase participation of developing countries and thus improve the Fund's "legitimacy". This suggestion was the subject of heated debate, and eventually incorporated in the reform proposal approved in 2008. Vaubel (2005) identifies as a problem the separation between the ultimate principals and the Fund executive directors, Woods (2005) calls for an increase in accountability, something that Frey and Stutzer (2006) suggest can be achieved through citizen participation in decision making. Eichengreen (2007) suggests adapting lending facilities to the growth of global financial markets and considers governance reform a prerequisite for the Fund to be able to pursue its goals. Rajan (2008) has a similar view and considers that for the World Bank and the Fund need substantial governance reform to be seen as "honest brokers". To the best of my knowledge, none of these, and other, reform proposals has considered an efficiency criterion to determine an optimal voting rule for multilateral financial institutions.

The remainder of the paper is structured as follows. Section 2 summarizes the history and salient features of governance in the Fund, including a reform of the quota system

[5]Bird and Rowlands (2005) argue that the post-Bretton Woods IMF makes the use of quotas for the simultaneous determination of contributions, access and voting rights untenable.

approved in April 2008. Section 3 describes the model, and compares quota distribution in the two polar cases described above. Section 4 discusses the effect of moral hazard. Section 5 concludes.

2 Governance in the Fund

Since most multilateral financial institutions structure their governance on the International Monetary Fund's quota system I briefly describe governance in this institution. The Fund was founded in 1944 with the main purpose of assisting members facing temporary balance of payments problems. From an initial membership of 44 states, today almost all the countries in the world participate in it. Members of the Fund do not have equal power. They contribute a quota subscription of financial resources, and this quota is the basis for determining voting power. Historically, quota allocations have been based mainly on economic size and external trade volume.

The role of the IMF has changed since the fall of the system which has brought the organization into life: the Bretton Woods Agreement. Before 1973, all members were more or less equally likely to request financial assistance. For example, from 1947 to 1967 industrial countries represented almost 70% of the total amount of resources withdrawn, close to the amount of resources they had contributed to the Fund. After the liberation of the world exchange rate regime in 1973, the main users of the IMF resources became emerging economies in Africa and Latin America with balance of payments crisis. This has widened the already divergent preferences between more or less developed countries on policy issues, and the disagreement over how this preferences are to be aggregated into collective decisions, i.e. about the methods to calculate quota allocations. In response to this concern, and also in the face of mounting criticism from academics and policymakers, the Fund embarked in September 2005 on a large-scale program of modernization. Salient among its objectives was governance reform, including adjusting quota shares to "reflect better the relative weight of members in the world economy". And in April 2008, a reform proposal representing a step in this direction was approved.

Decisions in the Fund are made by weighted majority of votes. The power structure is organized in the following way: the Board of Governors, which possesses all the powers of the Fund, is composed by representatives from all member countries. Each country initially received 250 basic votes plus one additional vote for each hundred-thousand Special Drawing Rights (SDRs) that it possesses. The basic votes were a compromise solution intended to reconcile the principle of sovereign equality with the fact of wide power asymmetries among members. The ratio of basic votes to total votes increased first, as new countries joined the IMF, reaching a historic high of 15.6% in 1958. Total quota increases thereof made this ratio decrease to roughly 2%, while the quota reform approved in April 2008 increased basic votes and introduced a mechanism to stabilize their number at 5.5% of total votes.[6] While initially there was a single formula for the calculation of quotas, in the early 1960s a complex multi-formula method was devised to determine quotas on the basis of GDP, exports and imports, variability of export

[6]In subsection 2.1 I will review in more detail the quota reform approved in April 2008.

receipts and reserves. While this allowed differing weights for the calculation of quotas for developed and developing countries, there was a significant loss of transparency. For a while ten formulas were used, later to be replaced by five, a system that was in place up to the reform of 2008 that returned to a single formula. There is no explicit rationale for using these and not other variables, nor for the weights attached to them in the mentioned formulas. In fact it appears that the formulas, as well as actual quotas, which in some cases diverge significantly from the calculated ones, are biased to produce a political outcome close to the one desired by the most powerful members of the IMF.[7] One of the objectives of the reform of 2008 is to increase the credibility of the Fund by increasing the transparency of the quota determination process, and by realigning actual quotas with calculated ones.

The Board of Governors can delegate certain decisions to the Board of Executive Directors, which is composed of one representative from each of the five members of the Fund having the largest quotas plus 19 other representatives, some of whom represent a certain subgroup of countries. Thus, each Executive Director has the number of votes equal to the sum of votes of the countries it represents. In this way, when the Board of Directors vote, there is at the beginning a first meeting in which each subgroup of countries determines how their representative will vote. Then, the Board of Executive Directors meets and cast their vote. There are different majority rules, including two supermajority provisions, and their use depends on the issue being discussed at the moment. A 70% majority, is required for issues of procedure (decisions involving matters of policy and operations) and an 85% majority, is required for issues of substance (for example, constitutional revisions or changes in quotas). An important observation of these majority rules and voting system is that the United States is the only country that retains a veto power since it possesses more than 15% of total quotas.

The quota system serves several functions, which creates the possibility of conflict, as pointed by Bird and Rowlands (2005). A member's quota defines four aspects of the relationship between the member country and the Fund: first, the amount of financial resources that members must contribute to the Fund; second, the amount of resources that they can draw from the IMF; third, their voting power in institutional decision making, and finally, the members' share of SDR allocations. After the breakdown of the Bretton Woods system of exchanges rates the IMF found it impossible to attain these multiple objectives with this single instrument. More developed capital markets made countries' request of financial assistance substantially larger than the quota structure had foreseen, as capital account imbalances grew significantly. And the Fund became bifurcated with the distinction of two types of members: rich country net lenders, and poor country net borrowers. This increased the conflict on using the quotas as a means of determining simultaneously contributions and access.

Related to the question of how are quotas determined, since its foundation, the Fund recognized that as it was going to make large disbursements of scarce financial resources, their decisions would have to be legally binding rather than merely advisory. More egalitarian decision methods, say a one-country, one-vote rule, would not be acceptable to the

[7] Mikesell (1944) acknowledges that the original Bretton Woods formula used to assign quotas among the first 44 members of the Fund were built with the objective to match a desired outcome.

major powers that contribute the bulk of the Fund's resources. Accordingly, a scheme was devised by which each national member of the IMF has a quota that roughly equates its voting power to its financial subscription to the organization.[8]

General Quota Reviews are typically undertaken at five-year intervals with the objective of adjusting to changes in members' relative position in the world economy, as well as to accommodate new members. Each quota increase is divided at the discretion of the Board into an equiproportional and selective components. The former is akin to an expansion of capital, simply extending proportionally the existing quotas, while the latter tends to shift the new quotas towards the calculated ones. Since historically the equiproportional component has averaged 70% of the quota increases, there has been a significant status quo in the distribution of power in the Fund.

2.1 Recent Governance Reforms

In March 2008 the Executive Board presented a reform proposal of its quota system that a month later was approved by the Board of Governors. The proposal was the final product of extensive discussions at the Executive Board along the guidelines set in the Fund's Annual Meeting in Singapore in September 2006, and aimed at realigning members quota shares with their relative economic weight. The participation and voice of low-income countries was enhanced through a substantial increase in the number of basic votes, and a mechanism that will maintain the ratio of basic votes to total votes constant in the future. Furthermore, the IMF will seek to make quotas and voting shares more responsive to changes in economic realities in future General Quota Reviews.

A salient characteristic of the reform is that quotas are once again calculated using a simple formula. This improves the transparency of governance at the Fund and helps in better reflecting the members' relative position in the global economy. In coming up with this formula the Board has taken into account a number of restrictions, which include the multiplicity of roles that quotas have, that they be based on available data, and that they be politically feasible. The new formula proposed includes four economic variables, GDP, openness, variability and reserves, expressed in shares of global totals. The weighted average is then compressed in order to reduce dispersion in calculated quota shares,

$$ICQ = (0.5Y + 0.3CC + 0.15V + 0.05R)^k$$

were ICQ is the intermediate calculated quota share, Y is a weighted average of GDP converted at market exchange rates and PPP exchange rates averaged over a three year period. The weight on market-based GDP is 60%. CC is the annual average of the sum of current payments and current receipts for a five year period. V is the variability of current receipts and net capital flows, measured as a standard deviation from the centered three-year trend over a thirteen year period. R is the average over a year of official reserves, and $k = 0.95$ is a compression factor. Calculated quotas are obtained after rescaling the sum of intermediate calculated quota shares to 100.

[8]The presence of basic votes introduces a wedge between financial subscription and voting power, which is significant mostly for less developed countries.

The computation of GDP both at market rates and at PPP rates reflects a compromise between the position of developing countries which supported the later as a better estimate of the relative volume of goods and services produce by their economies, and the position of developed countries which see market rate GDP as the relevant indicator, particularly as a measure of the contributive power of members. The compression factor is an artifact to moderate the dispersion of calculated quotas. Given the limitation of the new quota formula in enhancing the voice of emerging economies, a tripling of basic votes was decided, a measure that increases vote share beyond quota share for less developed nations. The inclusion of PPP GDP and the compression factor were the more controversial aspects of the reform proposal and the Executive Board has decided to include them in the formula for a period of 20 years. At the end of this period the argument for retaining these components would be reviewed. The quota formula only calculates the relative quota of IMF members. Total quota determination continues to be discretionally decided at General Quota Reviews.

3 Model

I assume that in a multilateral financial institution, "bank" for short, each member has a representative who votes on behalf of the citizens of her country.[9] I further assume that only decisions on providing financial assistance are made and thus votes take place over two alternatives: whether or not to provide financial assistance to a member country that requests it.[10] Formally there are n member countries, which are heterogeneous in terms of population, wealth, and openness. Country i has a population of N_i citizens, all of whom derive the same utility from per capita consumption of final goods, c_i. It is known that, with some probability a member will experience a need for financial resources (either because they suffer a financial or balance of payments shock in the case of the Fund, or they find a public investment opportunity that requires outside financing in the case of development banks) in which case a decision will have to be made on whether to financially assist the affected country, which will be denoted by j. A state of the world is then a description of members' preferences on whether or not to provide a loan to country j. Without loss of generality utilities can be normalized to zero if the status quo prevails and no assistance is provided, and preferences are then denoted by a vector $\vec{u}(j) \in \mathcal{R}^n$ with element $u(j)_i \equiv u_{ij}$ being the utility of a representative agent in country i if country j is given the requested loan (thus $u_{ij} \equiv c_i^{jb} - c_i^{jn}$ where superscripts differentiate consumption when country j is assisted or not).

After a shock takes place, each country's representative will decide to vote for granting the loan or not, based on whether the utility of doing so is positive or negative for that

[9]This is an approximation since most decisions are made by a smaller subset of representative, some of which themselves represent other members.

[10]The formalization generalizes in a straightforward way to cases in which a number of member countries simultaneously require a loan and the decision is whether to assist all of them, a subset of them, or none. Although multilateral financial institutions perform other roles besides lending, as long as they do not involve making large disbursements of scarce resources, they could be resolved by a different voting mechanism and are therefore irrelevant to the problem studied in this paper.

country's citizens. Thus the representative's voting behavior can be represented by a function $h_i : \mathcal{R} \longrightarrow \{b, n\}$, which maps the preferences of citizens into a vote. The notation $h_i(u_{ij}) = b$ shows that the representative of country i votes in favor of providing assistance. This means that $u_{ij} > 0$, and equivalently a vote against providing the loan, $h_i(u_{ij}) = n$, means that $u_{ij} < 0$.

In a second stage, the votes of the representatives are aggregated according to a voting rule. Let $v : \mathcal{R}^n \longrightarrow \{0, \frac{1}{2}, 1\}$ denote the outcome of this two-stage voting procedure as a function of the state of the world, $\vec{u}(j)$. Here $v(\vec{u}(j)) = 1$ corresponds to states for which a loan is approved, $v(\vec{u}(j)) = 0$ means that country j will not be assisted, and $v(\vec{u}(j)) = \frac{1}{2}$ denotes a tie that will be resolved by the toss of a coin.

Let an efficient voting rule be one that maximizes the expected social welfare function among the class of feasible voting rules.[11] The social welfare function is given by the expected total utility, giving equal weight to any citizen, independent of the country of residence, and given the independence of shocks between both periods we can restrict attention to the welfare function for the first period. Therefore I will consider voting rules that maximize the following welfare function:

$$E\left[\sum_i v(\vec{u}(j))N_i u_{ij}\right]$$

were the expectation is taken over the distribution of shocks, given by $\mu(\cdot)$. It will be assumed that the probability of experiencing a need for financial assistance in country j is independent of the policies it follows, i.e. that $v(\cdot)$ has no effect on $\mu(\cdot)$.[12]

We start by characterizing the first best outcome, i.e. the case in which the underlying preferences, u_{ij} are publicly observed (or perfectly inferred from the state of nature). In this case it is trivial to see that the decision rule should be,[13]

$$v^E(u) = \begin{cases} 1 & \text{if} \quad \sum_i N_i u_{ij} > 0, \\ 0 & \text{if} \quad \sum_i N_i u_{ij} < 0, \\ \frac{1}{2} & \text{if} \quad \sum_i N_i u_{ij} = 0. \end{cases} \tag{1}$$

Of more interest is the case in which the intensity of preferences for a given choice are private information of each country. For this case consider the following voting rule, proposed by Barberà and Jackson (2006) in a similar context where they study the optimal voting rule in a union where citizens of member countries differ on their preferences over policy. For each country, and for each possible state, two weights are assigned, one when

[11] These are voting rules that depend only on the information obtained from the votes of the representatives.

[12] I intend to later lift this assumption, and study how the optimal rule is determined when the voting rule affects domestic policies and thus the probability distribution of shocks.

[13] Since there is no private information in this case there is no need for a vote to take place. Note the implicit assumption, made here and henceforth, that a decision is made even if a subset of countries oppose it. This assumes either that there is an external enforcement mechanism or that decision-making in the bank satisfies a self-enforcement constraint. See Maggi and Morelli (2006) for an analysis of self-enforcing voting in international organizations.

the country votes in favor of giving a loan to country j, and another for votes against this. For the former we have,

$$\omega_{ij}^b = N_i E[u_{ij}|u_{ij} > 0, j].$$

Therefore the weight assigned to country i is proportional to the total expected welfare of its citizens when providing a loan to country j is indeed their preferred policy. Similarly, the weight assigned to country i when it votes against assisting country j is given by,

$$\omega_{ij}^n = -N_i E[u_{ij}|u_{ij} < 0, j].$$

The efficient voting rule $v^E(u)$ is then defined by,

$$v^E(\vec{u}(j)) = \begin{cases} 1 & \text{if} \quad \sum_{i:h_i(\vec{u}(j))=b} \omega_{ij}^b > \sum_{i:h_i(\vec{u}(j))=n} \omega_{ij}^n, \\ 0 & \text{if} \quad \sum_{i:h_i(\vec{u}(j))=b} \omega_{ij}^b < \sum_{i:h_i(\vec{u}(j))=n} \omega_{ij}^n, \\ \frac{1}{2} & \text{if} \quad \sum_{i:h_i(\vec{u}(j))=b} \omega_{ij}^b = \sum_{i:h_i(\vec{u}(j))=n} \omega_{ij}^n. \end{cases} \quad (2)$$

Proposition 1. If preferences are independent across countries (meaning that one country's utility for a given alternative does not depend on the full profile of votes of the rest of the countries), then a voting rule is efficient if and only if it is equivalent up to ties to v^E.

The proof of this proposition is in Barberà and Jackson (2006). It could be possible that for political reasons the voting rule can not be made contingent on the identity of the country that requires a loan. Ex-ante some potential members might feel unfairly treated by such a tailor-made governance structure and would decide not to join the organization. While modeling the determinants of multilateral financial institutions membership is beyond the scope of this paper, I will consider an additional constraint on the optimization problem, mainly that the voting rules can not be contingent on the state of the world. If we redefine the above voting rule correspondingly we get that each country is assigned the following weights when voting in favor or against providing a loan, irrespective of who is in need of financial assistance,

$$\omega_i^b = N_i E[u_{ij}|u_{ij} > 0], \quad (3)$$
$$\omega_i^n = N_i E[u_{ij}|u_{ij} < 0], \quad (4)$$

where now expectations are taken ex-ante over the joint probability distribution of the likelihood of country j requiring assistance, and over the effect this has on preferences of citizens in country i.

Now the efficient voting rule is given by (2) replacing the corresponding weights in the formulation of v^E. As we are now considering a non-contingent voting rule, chosen ex-ante under a veil of ignorance about the future state of nature, we can assume w.l.o.g. that ex-post the intensity of preferences, u_{ij}, are publicly observed. This simplifies the calculation of the weights ω_i^b and ω_i^n.

Weights are affected by the intensity of preferences inside a country for the alternatives, as captured by the values of u_{ij}. Thus countries that on average care more intensely about a decision on whether or not to provide a loan should be given more weight than countries that are less affected by the outcome. In their work, Barberà and Jackson (2006) consider an abstract decision and therefore have no reason for heterogeneity among members' *intensity* of preferences. They thus give every citizen of the union the same possible utilities, of $+1$ or -1.[14] Given the nature of the problem studied here I enrich the characterization of countries' preferences and the extent to which a shock in country j affects a representative citizen in country i.

I will assume all countries experience a positive effect from lending to country j, due to trade linkages, and a negative effect due to the need to raise funds to provide the bank with working capita when lending is approved. I model trade linkages using Waugh (2010) variation of the Eaton and Kortum (2002) Ricardian model of trade with a probabilistic representation of intermediate goods' production efficiencies.[15] Waugh (2010) has two types of goods produced in each country, with Cobb-Douglas production technologies. There is a single final good, c, which is not traded and is the only good valued by consumers. There are also a continuum of intermediate goods affecting production symmetrically through a Dixit-Stiglitz aggregate that are tradeable. Countries differ in the total factor productivity levels in the production of these intermediates, λ_i, and this is the driving force behind trade. These models are useful to estimate bilateral trade flows, showing that these take the form of a gravity equation with geographic barriers and importers' price levels creating trade frictions. Of particular interest to this application, Waugh (2010) by allowing for exporter fixed effects and introducing capital as a factor of production, explains income disparities across developed and developing countries.

Solving numerically for the equilibrium and for the different states of nature when each country is subject to a productivity shock gives an estimate to u^+_{ij}, the positive effects in country i when country j is given a loan. I model a negative shock (positive potential opportunity) in country j as a temporary decrease (potential increase) in its endowment of labor: $L_j(1-\epsilon_j)$ $(L_j(1+\epsilon_j))$ where ϵ_j measures the magnitude of the shock in relative terms.[16] It is assumed that lending takes the following form: each member country contributes a proportion of the decrease in the production factor in country j such that in total half the downfall (potential increase) is covered by the assistance.[17] At the end of the period once production and consumption take place, factor endowments in country j revert to its normal level and every country receives back the resources lent.

[14]The model of Barberà and Jackson (2006) has structure on how preferences for alternatives are distributed *inside* each country. I abstract from this complication by assuming that citizens of a given country have the same preferences with respect to the bailout decision.

[15]In Waugh (2010), which folows Alvarez and Lucas (2007), and in Eaton and Kortum (2002) with mobile labor, an increase in foreign technology always benefit every country in the world.

[16]To model a 5% change in GDP per capita, I used the following approximation:

$$\log(y_i) \approx \frac{\theta(1-\gamma)}{\beta} \log(\lambda_i)$$

I use this expression to translate changes in GDP to changes of the labor endowment. This ensures shocks to income per capita are of approximately the same magnitude in relative terms. See Appendix for more details on then numerical solution of Waugh (2010).

[17]Under this assumptions the model is static, with no real lending taking place.

The fraction of resources each country contributes are proportional to the voting weight to be derived. Since we will see that this weight is proportional to the exante welfare gain of being a member of the bank, it is reasonable that the capital contributions be proportional as well. Contrasting the current gain u_{ij}^{+} with the cost of resources that must be provided determines whether country i will vote in favor or against lending to country j.[18]

The following step is to recompute equilibrium after a shock is realized. The effect of lending to j is then found by comparing welfare levels c_i with and without the shock. With this information, coupled with information from labor endowments, we can construct matrices of the welfare effect in country i of lending to country j.

To gain insights into the determinants of voting power, an approximation of Waugh (2010) can be done to estimate the welfare gains to country i of having the bank give financial assistance to country j. Waugh (2010) considers equilibrium in a world of n countries with balanced trade. Besides total labor endownment, L_i, each country is characterized by its TFP and capital stock K_i. Total factor productivity for each of the intermediate goods is random, and it is assumed that its inverse, x, is distributed according to

$$x_i \sim exp(\lambda_i)$$

These draws are amplified in percentage terms by the parameter θ (assumed to be common across countries), such that $x^{-\theta}$ has a Frechet distribution. Equilibrium is obtained by imposing equality of demand for intermediate goods in country i with aggregate demand for intermediates produced in country i for all countries. For simplicity, I will work with the special case when only labor is a factor of production. For this case the equilibrium condition can be expressed as

$$L_i w_i = \sum_{k=1}^{n} L_k w_k D_{ki} \tag{5}$$

where w_i is the wage rate, and D_{ki} is the fraction of per capita spending in country k of intermediate goods produced in country i,

$$\begin{aligned} D_{ki} &= (AB)^{-1/\theta} \left(\frac{w_i^{\beta} p_{mi}(w)^{1-\beta}}{p_{mk}(w)\kappa_{ki}} \right)^{-1/\theta} \lambda_i \\ p_{mi}(w) &= AB \left(\sum_{k=1}^{n} \left(\frac{w_k^{\beta} p_{mk}(w)^{1-\beta}}{\kappa_{ik}} \right)^{-1/\theta} \lambda_k \right)^{-\theta} \end{aligned}$$

where p_{mi} is the price index of tradeables in country i, $\kappa_{ki} \leq 1$ measures transportation costs as units of goods shipped from i to k arriving in k per unit of goods shipped. Finally β is the share of labor in the production of intermediates, and A and B are functions of the parameters.[19]

[18] A country basically compares the gain today to the loss of not being able to assist another country with which it has closer ties, potentially itself, tomorrow.

[19] $A = A(\theta, \eta) = \left[\int_o^{\infty} e^{-z} z^{\theta(1-\eta)} dz\right]^{1/(1-\eta)}$, and $B = \beta^{-\beta}(1-\beta)^{-1+\beta}$, where η measures the elasticity of substitution in forming the tradeables aggregate and does not play a role in the equilibrium conditions.

I make an approximation which consists in estimating the effects that a shock has on wages and prices in country i keeping wages and prices in all other countries $k \neq i, j$ unaffected. Performing this approximation leads to the following expression for the positive effects of lending (see Appendix for details),

$$N_i u_{ij}^{+} = G \frac{N_i \frac{w_i}{p_i}}{L_i w_i} \left[(1-\beta)^2 L_j w_j D_{ji} + \beta L_i w_i D_{ij} \right] \epsilon_j \quad (6)$$

$$N_i u_{ij}^{+} = G \frac{N_i}{L_i} \frac{1}{p_i} \left[(1-\beta)^2 M_{ji} + \beta M_{ij} \right] \epsilon_j \quad (7)$$

where $G \equiv \frac{(\frac{1-\alpha}{\alpha})^{\alpha}}{\beta(1-\beta)(2-\beta)^2}$ and M_{ij} are total imports of country i from country j. For simplicity I assume that the ratio N_i/L_i is constant across countries. Approximating p_i by the ratio of GDP to GDP PPP, and assuming $\beta = (1-\beta)^2$ we have,

$$N_i u_{ij}^{+} = \beta G \frac{GDP\ PPP_i}{GDP_i} \left[M_{ji} + M_{ij} \right] \epsilon_j. \quad (8)$$

Thus positive effects of bailing out a country are proportional to bilateral trade with this country, and the constant of proportionality is country specific and depends on the ratio of GDP PPP to GDP.[20] The correction introduced by the ratio of GDP PPP to GDP is due to the presence of trade frictions that lead to a violation of the law of one price. Note that given that the positive effects are proportional to bilateral trade flows, in a world with trade frictions a loan request would have more support among neighbors of the country requiring assistance.

Giving a loan uses labor resources that reduce domestic production for each country except country j,

$$u_{ij} = N_i u_{ij}^{+} - \delta \epsilon_j GDP_j \omega_i^{*}, \quad (9)$$

$$u_{ij} = \beta G \frac{GDP\ PPP_i}{GDP_i} \left(\frac{M_{ji} + M_{ij}}{GDP_j} - \delta E\left[\left[M_{ki} + M_{ik} \right] \epsilon_k \right] \right) \epsilon_j GDP_j,$$

where δ is a factor of proportionality, and $\omega_i^{*} \equiv E[N_i u_{ij}^{+}] = Prob[u_{ij} > 0] E[N_i u_{ij}^{+} | u_{ij} > 0] + Prob[u_{ij} < 0] E[N_i u_{ij}^{+} | u_{ij} < 0]$ is the benefit to country i of being a member of the bank. The first thing to notice is that a country is biased towards voting in favor of lending to countries with which the ratio of bilateral trade to GDP, $\frac{M_{ij}+M_{ji}}{GDP_j}$, is high. Globally this favors assistance to countries that are relatively more open, and given the presence of trade frictions, country i would vote in favor of countries in its geographical neighborhood. Second, with this specifications for u_{ij}^{+}, preferences with respect to the bank's decision, given by (9), are indeed independent across countries and Proposition 1 holds. The calculation of weights ω_i^b, and ω_i^n fully characterizes the optimal voting rule for the bank,

$$\omega_i^b = N_i E[u_{ij}^{+} | u_{ij} > 0] - \delta \omega_i^{*} E[\epsilon_j GDP_j | u_{ij} > 0] = N_i E[u_{ij}^{+} | u_{ij} > 0] - \delta \omega_i^{*} \Phi$$

$$\omega_i^n = -N_i E[u_{ij}^{+} | u_{ij} < 0] + \delta \omega_i^{*} E[\epsilon_j GDP_j | u_{ij} < 0] = -N_i E[u_{ij}^{+} | u_{ij} < 0] + \delta \omega_i^{*} \Phi$$

[20]This result is consistent with the finding by XXXXX that welfare effects in a number of trade models are proportional to trade flows and the elasticity of XXXX.

where we assume $E[\epsilon_j GDP_j|u_{ij} > 0] = E[\epsilon_j GDP_j|u_{ij} < 0] = E[\epsilon_j GDP_j] \equiv \Phi$, i.e. the expected size of a loan arrangement is independent of whether country i is voting in favor or against this loan. This is a reasonable assumption given that we assume that the reduction in the probability that the bank will be able to provide lending in the future is linear in the size of the current loan affects, therefore the scale will affect the intensities of preferences in favor or against providing financial assistance but not the sign of u_{ij} (see (9)).[21] Recalling the definition of ω_i^* we get,

$$\begin{aligned}
\omega_i^b &= N_i E[u_{ij}^+|u_{ij} > 0](1 - \delta\Phi Prob[u_{ij} > 0]) - N_i E[u_{ij}^+|u_{ij} < 0]\delta\Phi Prob[u_{ij} < 0] \\
\omega_i^n &= -N_i E[u_{ij}^+|u_{ij} < 0](1 - \delta\Phi Prob[u_{ij} < 0]) + N_i E[u_{ij}^+|u_{ij} > 0]\delta\Phi Prob[u_{ij} > 0]
\end{aligned}$$

These formulas, together with (8), completely characterize the optimal voting rule. This will be a weighted voting rule when the ratio between weights ω_i^n and ω_i^b is the same across countries, i.e. all countries have a common bias given by $\frac{\omega_i^n}{\omega_i^b} \equiv \gamma$.[22] This requires

$$\frac{\omega_i^n}{\omega_i^b} = \frac{\delta\zeta\Phi Prob[u_{ij} > 0]\frac{E[u_{ij}^+|u_{ij}>0]}{E[u_{ij}^+|u_{ij}<0]} - (1 - \delta\zeta\Phi Prob[u_{ij} < 0])}{(1 - \delta\zeta\Phi Prob[u_{ij} > 0])\frac{E[u_{ij}^+|u_{ij}>0]}{E[u_{ij}^+|u_{ij}<0]} - \delta\zeta\Phi Prob[u_{ij} < 0]} = \gamma \quad \forall i.$$

Sufficient conditions for equality of these ratios across countries are that $Prob[u_{ij} > 0]$, and $\frac{E[u_{ij}^+|u_{ij}>0]}{E[u_{ij}^+|u_{ij}<0]}$, be independent of country i. The first condition is that every country in the bank be ex-ante equally likely to vote in favor of a loan. The second condition requires that the distribution of $\frac{M_{ij}+M_{ji}}{E[(M_{ik}+M_{ki})\epsilon_k]}$ be the same across countries. Under these conditions, the ratio $\frac{\omega_i^*}{\omega_i^b}$ will also be constant across countries. This allows to characterize the optimal voting rule as assigning weights

$$\omega_i^* = \beta G \frac{GDP\ PPP_i}{GDP_i} \sum_j \mu(j)(M_{ji} + M_{ij}),$$

to each country, and a threshold of $\frac{\gamma}{1+\gamma}\sum_{i=1}^n \omega_i^*$ of votes to approve a loan.

If $\frac{E[u_{ij}^+|u_{ij}>0]}{E[u_{ij}^+|u_{ij}<0]}$ varies across countries the weighted voting rule is not optimal. From the numerical simulations the ratio $\frac{\omega_i^n}{\omega_i^b}$ is approximately constant only for the case in which all countries have the same probability of being affected by a shock. When there are net creditors and net debtors (and under the assumption that the second have the same probability of suffering a shock), the ratio of weights is approximately distributed around two values, one characterizing net creditors, the other net debtors. For this case, and calling γ^A the value of $\frac{\omega_i^n}{\omega_i^b}$ for net creditors, and γ^B for debtors, the optimal voting rule

[21] Alternatively if we assume that the size of a loan is not related to the size of the shock experienced by country j then countries experiencing larger shocks are more likely to be assisted, and we no longer can assume $E[\epsilon_j GDP_j|u_{ij} > 0] = E[\epsilon_j GDP_j|u_{ij} < 0]$. This only complicates the expressions derived, but does not affect the results.

[22] See Corollary 1 in Barberà and Jackson (2006).

still gives weights ω_i^* to each country but now a decision to grant a loan to country j requires that

$$\sum_{u_{ij}>0} \gamma^i \omega_i^* > \sum_{u_{ij}<0} \omega_i^* \tag{10}$$

An alternative institutional arrangement that employs a simpler decision rules, at least for some states of nature, would be to have two separate votes, one for countries characterized by γ^A and another for the rest. Countries of group A would favor giving country j a loan if among them a weighted vote has a support above threshold $\frac{\gamma^A}{1+\gamma^A}\sum_{\gamma^i=\gamma^A}\omega_i^*$. A similar rule holds for countries in group B. If both votes agree then their common decision is the one taken by the bank. If they disagree then the above rule (10) is used.

Given that weights are derived from the expected welfare gain of lending to member countries, they also relate to the absolute level of capital contributions that each country should make to the bank (i.e. each member's contributions to the bank should be proportional to the benefits from belonging to it). Therefore the model also helps to determine the total level of working capital that multilateral financial institutions should have.

To use these formulae to calculate quotas we need to estimate the likelihood that country j suffers a financial or balance of payments crisis for the IMF, or that might experience a positive investment opportunity requiring external finance for development banks. For the Fund, an alternative is to use sovereign bond spreads, or credit ratings, for those countries that have issued public debt, assigning a common (high) value to those poor countries that have not been able to issue debt, and a value of zero to rich countries having no outstanding debt issues. To get an idea of what drives the allocation of voting rights we compare now the optimum voting weights in a symmetric world where every country has the same probability of suffering a shock, with a "North-South" world where a subgroup of countries no longer is affected by shocks (or if affected, does not need external finance).

In the symmetric case $\mu(\cdot) = \mu$ for all countries, i.e. the product of the probability of a shock times its relative size is constant, and independent of country j. This makes the optimal voting weights to be,

$$\omega_i^* = 2\mu\beta G \frac{GDP\ PPP_i}{GDP_i} GDP_i$$

Thus we see that quotas are proportional to GDP PPP. This expression would change to the following in a North-South world,

$$\omega_i^* = \mu\beta G \frac{GDP\ PPP_i}{GDP_i} (M_{iS} + X_{iS})$$

where M_{iS} and X_{iS} refer to trade volume of country i with all South countries, i.e. countries that are subject to shocks (and still assuming $\mu(\cdot) = \mu$ for all South countries). Note that in this calculation we have to include domestic absorption of those developing countries that are exposed to shocks, while for developed countries only bilateral trade flows are considered.

Table 1 has actual and calculated quotas for the Fund for the 19 countries in the G20 (i.e. excluding the EU). For comparison I include the results that would obtain in the symmetric case where every country has the same probability of facing a balance of payments shock. As can be seen in Table 1, an application of the derived quota formula leads to a reduction in the influence of the richest industrial countries. But this effect is not driven by the pattern of trade between North and South countries. It is produced by the fact that developed countries do not suffer shocks while developing ones do.

From the formula for optimal voting weights we see that the total weights, and correspondingly the working capital of multilateral financial institutions should respond to developments in the volume and distribution of trade, changes in the relative wealth of nations, and likelihood of balance shocks, $\mu(\cdot)$. With respect to the possibility of a multi-country crisis, the model assumes that the negative effects of lending are linear in the characteristics of the country receiving the loan. Thus the trade off between positive and negative costs is independent for each country and thus the situation is equivalent to having separate votes on each request.

4 Moral Hazard

Suppose now that the probability that a country experiences a shock depends on its domestic policies. Furthermore the ruler of country i receives a private benefit of $B > 0$ if she does not exert effort into pursuing "good" policies that reduce the likelihood of a financial shock (increase the likelihood of a productive investment opportunity). Suppose further that when a country experiences a shock this has a cost $C_i > 0$ for the ruler if there is no loan from the bank. Then a ruler will decide whether to exert effort or not based on a private cost benefit analysis, taking into consideration the likelihood of receiving a loan as implied from the voting rule. Denoting $\Delta\mu$ the change in the likelihood of experiencing a shock if effort is not exerted, the ruler chooses to adopt "good" policies if

$$B < \Delta\mu\left[(1 - s_i)C_i\right] \tag{11}$$

where s_i is the probability of receiving outside finance. This probability is given by

$$s_i = Prob[\sum_{u_{ki}>0} \gamma^k \omega_k^* > \sum_{u_{ki}<0} \omega_k^*] \tag{12}$$

If this probability is high enough that a large group of countries decides not to adopt "good" policies, then the bank might find optimal to increase the threshold of votes required to approve a loan. From (12) this reduces every country's probability of being rescued, and as can be seen in (11), this has a disciplining effect, as the cost of shirking effort increases. The threshold should also be raised if there is an increases in moral hazard problems, as measured by B.

Note that members' weights need not be adjusted, as they do not affect their incentives to exert effort. Heterogeneity in members' preference with respect to domestic policy, reflected in differences in C_i, will likely result in a threshold that, while disciplining some countries, will still not be high enough to rule out moral hazard throughout the world.

5 Conclusions

I have derived a theoretical model for the optimal voting rule in multilateral financial institutions. In general the quota system currently used in these organizations is not optimal. A better arrangement would still assign a quota to each country. This quota determines capital contributions and voting power, but the voting rule weights each country vote according to whether that country is a net creditor or net debtor and whether it is voting in favor or against providing a requested loan. Implementation can be done through separate simple weighted voting among net creditors and net debtors. If both groups agree on a proposal this is the collective decision. If they disagree, then a "grand assembly" uses the more complex aggregation of preferences.

Optimal weights are proportional to a weighted sum of a country's trade flows with the rest of the members with weights related to the probabilities of each trade partner suffering a crisis, including domestic absorption if the country can suffer a shock. Weights are also proportional to the ratio of GDP PPP to GDP reflecting trade frictions that affect domestic consumption price levels. Introducing moral hazard in the model, the optimal response is to leave weights unchanged, but to increase the threshold of votes required to approve a bailout.

Given that weights are derived from the expected welfare impact of lending on members, they also relate to the absolute level of capital contributions that each country should make to the institution. Therefore the model also helps to determine the total level of resources that different multilateral financial institutions should have. The model predicts that changes in the international financial system, such as the breakdown of Bretton Woods fixed exchange rates arrangement, should be reflected in the distribution of voting shares, and in the total level of quotas.

In future research I intend to extend this model to a dynamic setting where every period a shock might take place. Furthermore in this context a question of interest is how the optimal voting rule is affected when the same rule must be used to determine whether or not to adjust members' voting power to developments in their weight in the global economy. Finally it would be desirable to extend the intensity of preferences to include linkages due to flows of capital and labor between countries.

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6 Appendix

6.1 Using Waugh (2010) to estimate welfare gains

To test the model, I used data from Waugh (2010) for 77 countries. The data contains information on labor and capital endowments, as well as all the information needed to run a gravity equation regression. Following Waugh (2010), exporter fixed effects were specified in order to generate predictions on prices consistent with the data.

In order to compute countries' optimal weights in the bank, I need to specify the distribution and size of the shocks. To explore the workings of the model and the intuitions it holds within, two polar cases are analyzed. In the first case, there are two sets of countries. On the one hand, the first set, which we will call "North"[23] , is formed by countries assumed to be immune to shocks. This means they never face a balance-of-payments crisis. On the other hand, the remaining countries, which we will denominate "South", are equally prone to suffer an approximately 5% change in its GDP (a decrease for the case of the Fund providing assistance for a financial crisis, or an increase in the case of development banks). This case is meant to reflect roughly the current configuration in all multilateral financial institutions, with rich countries net lenders and poor countries net borrowers.

The second polar case features a simmetric world, where all countries face the same probability of suffering a 5% change in their GDP. This setup stands for the Fund under the Bretton Woods system where, arguably, all countries were seen as being equally likely to require temporary financial assistance.

To model a 5% change in GDP per capita, I used the following approximation. We know that the following relationship holds in equilibrium:

$$
\begin{aligned}
y_i &= X_{ii}^{\frac{-\theta(1-\gamma)}{\beta}} \lambda_i^{\frac{\theta(1-\gamma)}{\beta}} k_i^{\alpha} \\
\log(y_i) &= \frac{-\theta(1-\gamma)}{\beta} \log(X_{ii}) + \frac{\theta(1-\gamma)}{\beta} \log(\lambda_i) + \alpha k_i
\end{aligned}
$$

Since k_i is fixed in the short run, if we assume the share of domestic absorption does not change much, then:

$$\log(y_i) \approx \frac{\theta(1-\gamma)}{\beta} \log(\lambda_i)$$

I use this expression to translate changes in GDP to changes of the productivity parameter. This ensures shocks to income per capita are of approximately the same magnitude in relative terms.

The following step is to recompute equilibrium after a shock is realized. This is performed using Waugh (2010)'s algorithm. In particular, a good fit for GDP per capita

[23]The full list of "Northern" countries is Australia, Austria, Bahrein, Belgium, Canada, Cyprus, Denmark, Finland, France, Germany, Greece, Iceland, Israel, Italy, Japan, Kuwait, Luxemburg, Malta, New Zealand, Norway, Oman, Portugal, Saudi Arabia, South Korea, Spain, the Netherlands, United Arab Emirates, the United Kingdom, and the United States

is desired, since this is the measure of welfare of the model. The algorithm calculates how much GDP per capita is affected in country "i" after country "j" suffers a shocks. Therefore, we have a 77x77 matrix with a typical element ΔGDP_{ij}. In the North-South case, all columns that pertain to North countries will be zeros. As expected, diagonal elements (own effects) are the greatest by far.

With this information, coupled with information from labor endowments, we can construct matrices of the welfare effect in country i of lending to country j. $L_i \Delta GDP_{ij}$ is the benefit of country i of country j receiving a loan (u_{ij}^{+}). On the other hand, the costs of saving this country are measured by $\delta E(L_i u_{ij}^{+})\zeta L_j \Delta GDP_{jj}$. Since it is assumed that all countries have the same probability of suffering a crisis:

$$E(L_i u_{ij}^{+}) = \frac{L_i}{77} \sum_j \Delta GDP_{ij}$$

We set $\zeta = 1$ and calibrate δ in the following way:

$$\delta = \frac{1}{\sum_i \Delta GDP_{ii}}$$

Comparing benefits and costs, it can determined whether country i votes in favor or against bailing out country j. With this I then compute $P(u_{ij} > 0)$ and $P(u_{ij} < 0)$ to compute optimal weights when voting for and against bail out:

$$\begin{aligned} w_i^b &= L_i E(u_{ij}^{+}/u_{ij} > 0) - \delta E(L_i u_{ij}^{+})\zeta E(L_j \Delta GDP_{jj}/u_{ij} > 0) \\ w_i^n &= -L_i E(u_{ij}^{+}/u_{ij} > 0) + \delta E(L_i u_{ij}^{+})\zeta E(L_j \Delta GDP_{jj}/u_{ij} > 0) \end{aligned}$$

With this information, we compute relative weights for each country:

$$\frac{w_i^b}{w_i^n} = \frac{L_i E(u_{ij}^{+}/u_{ij} > 0) - \delta E(L_i u_{ij}^{+})\zeta E(L_j \Delta GDP_{jj}/u_{ij} > 0)}{-L_i E(u_{ij}^{+}/u_{ij} > 0) + \delta E(L_i u_{ij}^{+})\zeta E(L_j \Delta GDP_{jj}/u_{ij} > 0)}$$

For the quota system to be plausible, this ratio must be independent of country i. Although this is not far from reality in the simmetric case, we find that this ratio diverges in the North-South scenario. In that case, I suggest defining two different ratios. This would represent two different chambers, one for each set of countries. Then, a decision would be reached when both cameras agreed. In the last program, we explored this issue, computing the odds that both cameras will agree on the decision making, thus, this system admissible.

6.2 Derivation of positive effects of a bailout

In Alvarez and Lucas (2007) equilibrium is obtained by imposing trade balance for the n countries in the world.

$$L_i w_i = \sum_{k=1}^{n} L_k w_k D_{ki}(w) \tag{13}$$

where L_i is the total units of labor in country i measured in efficiency units, w_i is the wage rate, and D_{ki} is the fraction of per capita spending in country k of intermediate goods produced in country i,

$$\begin{aligned} D_{ki}(w) &= (AB)^{-1/\theta}\left(\frac{w_i^{\beta}p_{mi}(w)^{1-\beta}}{p_{mk}(w)\kappa_{ki}}\right)^{-1/\theta}\lambda_i, \\ p_{mi}(w)^{-1/\theta} &= (AB)^{-1/\theta}\sum_{k=1}^{n}\left(\frac{w_k^{\beta}p_{mk}(w)^{1-\beta}}{\kappa_{ik}}\right)^{-1/\theta}\lambda_k, \end{aligned} \tag{14}$$

where θ is a parameter that measures the variability in productivity draws, λ_k is a technology parameter associated with the level of TFP in country k, p_{mi} is the price index of tradeables in country i, $\kappa_{ki} \leq 1$ measures transportation costs as units of goods shipped from i to k arriving in k per unit of goods shipped. Finally β is the share of labor in the production of intermediates, and A and B are functions of the parameters.[24] For convenience I use the following notation,

$$\begin{aligned} \psi_{ik} &= \left(\frac{w_k^{\beta}p_{mk}^{1-\beta}}{\kappa_{ik}}\right)^{-1/\theta}\lambda_k, \\ \psi_{ij} &= \left(\frac{w_j^{\beta}p_{mj}^{1-\beta}}{\kappa_{ij}}\right)^{-1/\theta}\lambda_j(1-\epsilon_j). \end{aligned}$$

Equations (13) and (14) can be seen as a system of $2n$ equations in $2n$ unknowns, w_i and p_{mi}. Once these are found, the price level of final goods, p_i can be calculated using the following relation,

$$p_i = \alpha^{-\alpha}(1-\alpha)^{-1+\alpha}w^{\alpha}p_{mi}^{1-\alpha}. \tag{15}$$

I model a crisis in country j as a decrease in its TFP parameter: $\lambda_j(1-\epsilon_j)$ where ϵ_j measures the magnitude of the crisis in relative terms. The effect of bailing out j is then found by solving the above system of equations and comparing welfare levels $c_i = \frac{w_i}{p_i}$ with and without the shock. To this effect I make an approximation which consists in estimating the effects that this productivity shock has on wages and prices in country i keeping wages and prices in all other countries $k \neq i, j$ unaffected. The first step is to estimate the effect of a TFP shock on wages and prices in country j. Taking derivatives with respect to ϵ_j in (13) and (14) for the case $i = j$ we find,

$$\begin{aligned} \left[1 - D_{jj} - \sum_k \frac{L_k w_k}{L_j}\frac{\mathrm{d}D_{kj}}{\mathrm{d}w_j}\right]\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} + \left[-\sum_k \frac{L_k w_k}{L_j}\frac{\mathrm{d}D_{kj}}{\mathrm{d}p_{mj}}\right]\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} &= 0 \\ \left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\psi_{jk}}{\mathrm{d}w_j}\right]\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} + \left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\psi_{jk}}{\mathrm{d}p_{mj}} - \frac{1}{\theta}p_{mj}^{-(1+\theta)/\theta}\right]\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} & \\ +(AB)^{-1/\theta}\psi_{jj}|_{\epsilon_j=0} &= 0 \end{aligned}$$

[24] $A = A(\theta,\eta) = \left[\int_o^{\infty} e^{-z}z^{\theta(1-\eta)}dz\right]^{1/(1-\eta)}$, and $B = \beta^{-\beta}(1-\beta)^{-1+\beta}$, where η measures the elasticity of substitution in forming the tradeables aggregate and does not play a role in the equilibrium conditions.

Now to simplify, note that for the first equation, $\frac{\mathrm{d}D_{kj}}{\mathrm{d}w_j} = -\frac{\beta}{\theta}\frac{D_{kj}}{w_j}$, $\frac{\mathrm{d}D_{kj}}{\mathrm{d}p_{mj}} = -\frac{1-\beta}{\theta}\frac{D_{kj}}{p_{mj}}$ when $k \neq j$ and $\frac{\mathrm{d}D_{jj}}{\mathrm{d}p_{mj}} = -\frac{1-\beta}{\theta}\frac{D_{jj}}{p_{mj}} + \frac{1}{\theta}\frac{D_{jj}}{p_{mj}}$, and $\sum_k \frac{L_k w_k}{L_j w_j} D_{kj} = 1$. Dividing the second equation by $p_{mj}^{-1/\theta} = (AB)^{-1/\theta}\sum_k \psi_{jk}$, and using $\frac{\mathrm{d}\psi_{jk}}{\mathrm{d}w_j} = \frac{\mathrm{d}\psi_{jk}}{\mathrm{d}p_{mj}} = 0$ if $j \neq k$, $\frac{\mathrm{d}\psi_{jj}}{\mathrm{d}w_j} = -\frac{\beta}{\theta}\frac{\psi_{jj}}{w_j}$, $\frac{\mathrm{d}\psi_{jj}}{\mathrm{d}p_{mj}} = -\frac{1-\beta}{\theta}\frac{\psi_{jj}}{p_{mj}}$, and $\frac{\psi_{jj}}{\sum_k \psi_{jk}} = D_{jj}$,[25]

$$
\begin{aligned}
\left[1 - D_{jj} + \frac{\beta}{\theta}\right]\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} + \left[\frac{1-\beta-D_{jj}}{\theta}\frac{w_j}{p_{mj}}\right]\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} &= 0 \\
\left[\frac{\beta}{\theta}\frac{D_{jj}}{w_j}\right]\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} + \left[\frac{1-\beta}{\theta}\frac{D_{jj}}{p_{mj}} - \frac{1}{\theta}\frac{1}{p_{mj}}\right]\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} + D_{jj} &= 0
\end{aligned}
$$

Multiplying the first equation by θ, and the second by θw_j we get

$$
\begin{aligned}
\left[\beta + \theta(1 - D_{jj})\right]\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} + (1-\beta-D_{jj})\frac{w_j}{p_{mj}}\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} &= 0 \\
\beta D_{jj}\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} - (1-(1-\beta)D_{jj})\frac{w_j}{p_{mj}}\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} &= -\theta w_j D_{jj}
\end{aligned}
$$

Solving for the effects of ϵ_j on wages and prices in country j we find

$$
\begin{aligned}
\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} &= = \frac{1}{\Delta_j}\theta\frac{w_j^2}{p_{mj}}(1 - D_{jj}^2) \\
\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} &= -\frac{1}{\Delta_j}\theta^2 w_j D_{jj}(1 - D_{jj}) \\
\Delta_j &= -\frac{w_j}{p_{mj}}\left[\beta + \theta - \theta(2-\beta)D_{jj} + (\theta(1-\beta) - \beta)D_{jj}^2\right]
\end{aligned}
$$

In order to get closed form solutions for the weights ω_i^b, and ω_i^n, we need to make the following assumption

$$\theta = \frac{\beta}{1-\beta}$$

This assumption means, for example, that if $\beta = 0.38$[26] then $\theta = 0.61$, a larger value than the ones used by Alvarez and Lucas in their calibrations (they considered the range $[0.1, 0.25]$), or the benchmark estimate in Waugh (2009), $\theta = 0.18$. This implies that

$$
\begin{aligned}
\Delta_j &= -\frac{w_j}{p_{mj}}\beta\frac{2-\beta}{1-\beta}(1 - D_{jj}) && (16) \\
\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} &= = -\frac{1}{2-\beta}w_j(1 + D_{jj}) && (17) \\
\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} &= \frac{\beta}{(1-\beta)(2-\beta)}p_{mj}D_{jj} && (18)
\end{aligned}
$$

[25] In what follows I simplify notation such that $\psi_{jj}|_{\epsilon_j=0}$, and the same holds for all expression that feature a term λ_j (as D_{jj}).

[26] This is the ratio of value added in manufacturing to total value of production in the US for 1996-1999 according to BEA input-output data. See Alvarez and Lucas (2007).

We can now approximate the effect of ϵ_j on wages and prices in country $i \neq j$ by considering both the direct effect and the indirect effect through w_j and p_{mj}. To do this we take derivatives with respect to ϵ_j in (13) and (14) we find,

$$\begin{aligned}\left[1 - D_{ii} - \sum_k \frac{L_k w_k}{L_i}\frac{\mathrm{d}D_{ki}}{\mathrm{d}w_i}\right]\frac{\mathrm{d}w_i}{\mathrm{d}\epsilon_j} + \left[-\sum_k \frac{L_k w_k}{L_i}\frac{\mathrm{d}D_{ki}}{\mathrm{d}p_{mi}}\right]\frac{\mathrm{d}p_{mi}}{\mathrm{d}\epsilon_j} \\ -\frac{L_j}{L_i}D_{ji}\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} - \frac{L_j w_j}{L_i}\frac{\mathrm{d}D_{ji}}{\mathrm{d}p_{mj}}\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} &= 0 \\ \left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\psi_{ik}}{\mathrm{d}w_i}\right]\frac{\mathrm{d}w_i}{\mathrm{d}\epsilon_j} + \left[-(AB)^{-1/\theta}\sum_k \frac{\mathrm{d}\psi_{ik}}{\mathrm{d}p_{mi}} - \frac{1}{\theta}p_{mi}^{-(1+\theta)/\theta}\right]\frac{\mathrm{d}p_{mi}}{\mathrm{d}\epsilon_j} \\ -(AB)^{-1/\theta}\frac{\mathrm{d}\psi_{ij}}{\mathrm{d}w_j}\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} - (AB)^{-1/\theta}\frac{\mathrm{d}\psi_{ij}}{\mathrm{d}p_{mj}}\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j} + (AB)^{-1/\theta}\psi_{ij}|_{\epsilon_j=0} &= 0\end{aligned}$$

Simplifying these equations as done before and multiplying by θ and θw_i we get

$$\begin{aligned}[\beta + \theta(1 - D_{ii})]\frac{\mathrm{d}w_i}{\mathrm{d}\epsilon_j} + (1 - \beta - D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}p_{mi}}{\mathrm{d}\epsilon_j} &= \left[\frac{L_j}{L_i}\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} + \frac{L_j}{L_i}\frac{1}{\theta}\frac{w_j}{p_{mj}}\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j}\right]D_{ji} \\ \beta D_{ii}\frac{\mathrm{d}w_i}{\mathrm{d}\epsilon_j} - (1 - (1 - \beta)D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}p_{mi}}{\mathrm{d}\epsilon_j} &= -\left[\theta w_i + \beta\frac{w_i}{w_j}\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} + (1 - \beta)\frac{w_i}{p_{mj}}\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j}\right]D_{ij}\end{aligned}$$

Notice the similar structure between this system of equations and the one solved for country j. Using the expressions (17) and (18) these equations simplify to

$$\begin{aligned}[\beta + \theta(1 - D_{ii})]\frac{\mathrm{d}w_i}{\mathrm{d}\epsilon_j} + (1 - \beta - D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}p_{mi}}{\mathrm{d}\epsilon_j} &= -\frac{L_j}{L_i}\frac{1}{2 - \beta}w_j D_{ji} \\ \beta D_{ii}\frac{\mathrm{d}w_i}{\mathrm{d}\epsilon_j} - (1 - (1 - \beta)D_{ii})\frac{w_i}{p_{mi}}\frac{\mathrm{d}p_{mi}}{\mathrm{d}\epsilon_j} &= -\frac{\beta}{(2 - \beta)(1 - \beta)}w_i D_{ij}\end{aligned}$$

Solving for the effects of ϵ_j on wages and prices in country i we find

$$\begin{aligned}\frac{\mathrm{d}w_i}{\mathrm{d}\epsilon_j} &= \frac{1}{\Delta_i}\left[\frac{w_i}{p_{mi}}(1 - (1 - \beta)D_{ii})\frac{L_j}{L_i}\frac{1}{2 - \beta}w_j D_{ji} + (1 - \beta - D_{ii})\frac{w_i^2}{p_{mi}}\frac{\beta}{(2 - \beta)(1 - \beta)}D_{ij}\right] \\ \frac{\mathrm{d}p_{mi}}{\mathrm{d}\epsilon_j} &= \frac{1}{\Delta_i}\left[\beta D_{ii}\frac{L_j}{L_i}\frac{1}{2 - \beta}w_j D_{ji} - (\beta + \theta(1 - D_{ii}))\frac{\beta}{(2 - \beta)(1 - \beta)}w_i D_{ij}\right]\end{aligned}$$

We are now ready to estimate the approximate positive effect that a bailout of country j has on country i: $u_{ij}^+ \equiv -\frac{\mathrm{d}\frac{w_i}{p_i}}{\mathrm{d}\epsilon_j}$, using (15) and the assumption $\theta = \frac{\beta}{1-\beta}$ that lead to Δ_i given by (16) with j replaced by i. Again start with country j.

$$u_{jj}^+ = -(1 - \alpha)(\frac{w_j}{p_{mj}})^{-\alpha}\frac{\mathrm{d}\frac{w_j}{p_{mj}}}{\mathrm{d}\epsilon_j}\epsilon_j = -(1 - \alpha)(\frac{w_j}{p_{mj}})^{-\alpha}\left[\frac{1}{p_{mj}}\frac{\mathrm{d}w_j}{\mathrm{d}\epsilon_j} - \frac{w_j}{p_{mj}^2}\frac{\mathrm{d}p_{mj}}{\mathrm{d}\epsilon_j}\right]\epsilon_j.$$

Using (17) and (18)

$$u_{jj}^+ = (1 - \alpha)(\frac{w_j}{p_{mj}})^{1-\alpha}\frac{(1 - \beta + D_{jj})}{(2 - \beta)(1 - \beta)}\epsilon_j = \frac{(\frac{1-\alpha}{\alpha})^\alpha}{(2 - \beta)(1 - \beta)}(1 - \beta + D_{jj})\frac{w_j}{p_j}\epsilon_j,$$

where in the last equality we used (15). Note that u^+_{jj} is proportional to welfare. Using the same logic we find welfare effects on country i

$$u^+_{ij} = (1-\alpha)(\frac{w_i}{p_{mi}})^{1-\alpha}\frac{1}{\beta(1-\beta)(2-\beta)^2}\frac{1}{L_i w_i}\left[(1-\beta)^2 L_j w_j D_{ji} + \beta L_i w_i D_{ij}\right]\epsilon_j.$$

Multiplying by population N_i and defining $G \equiv \frac{(\frac{1-\alpha}{\alpha})^\alpha}{\beta(1-\beta)(2-\beta)^2}$ we get the expression used for the calculation of quotas:

$$N_i u^+_{ij} = G\frac{N_i\frac{w_i}{p_i}}{L_i w_i}\left[(1-\beta)^2 L_j w_j D_{ji} + \beta L_i w_i D_{ij}\right]\epsilon_j.$$

6.3 Alternative characterization of voting weights

Since giving financial assistance eventually requires that members make capital injections to the IMF, some countries might oppose to a bailout if they are experiencing a tight fiscal position, or if voting for a bailout creates a conflict with its domestic political agenda.[27] I will model this as a random cost independently distributed across countries, and independent of the balance of payments shocks that affects country j. The negative effects of a bailout then take the form

$$N_i u^-_{ij} = -\delta(j)\phi_i\omega^b_i$$

where ω^b_i is the weight that rule v^E assigns to a country when it votes in favor of a bailout, and here I make the conjecture that a country's quota, which also determines capital contributions, is proportional to this weight. ϕ_i is the random cost associated to country i's fiscal position or domestic politics, and $\delta(j)$ is a discount factor that captures the possibility that the capital injection takes place in the future.[28] $\delta(j)$ may be an increasing function of the size of the loan required by country j, reflecting that approving a larger rescue package increases the probability of future capital injections more than approving a smaller one. For simplicity I will assume that: a) ϕ_i can only take two values,

$$\phi_i = \begin{cases} \bar{\phi} > \frac{1}{E[\delta(j)]} & \text{with probability } \pi_i, \\ 0 & \text{with probability } 1-\pi_i \end{cases}$$

and, b) when $\phi_i = \bar{\phi}$ the negative effect always dominates the positive one.[29] Note that with this specifications for u^+_{ij} and u^-_{ij}, preferences with respect to the bailout decision, given by $u_{ij} = u^+_{ij} - u^-_{ij}$, are indeed independent across countries and Proposition 1 holds. The calculation of weights ω^b_i, and ω^n_i fully characterizes the optimal voting rule for the

[27]It is likely that the first concern is more relevant for voting decisions among poor and middle-income countries, while the latter might affect decision-making in high income countries.

[28]Since this is a static model this parameter only captures the expected lag between a bailout and a capital injection into the IMF. Thus a high $E[\delta(j)]$ reflects times of significant lending activity by the IMF, while a low $E[\delta(j)]$ signals periods of relative tranquility.

[29]Thus, $\delta(j)$ must satisfy: $\inf_j \delta(j)\bar{\phi}w^b_i > \sup_j N_i u^+_{ij}\ \forall i$. This is a simplification that allows a simple calculation of weights w^b_i, and w^n_i.

IMF. We will start with ω_i^b and consider the case where country j is not allowed to vote (see (3)),

$$\begin{aligned}
\omega_i^b &= N_i E[u_{ij}^+ + u_{ij}^- | u_{ij}^+ > u_{ij}^-, j \neq i] = N_i E[u_{ij}^+ | \phi_i = 0, j \neq i] = \\
&= \beta G \frac{GDP\ PPP_i}{GDP_i} e^{-gH_i} \sum_{j \neq i} \mu(j) \left(M_{ji} + M_{ij}\right)
\end{aligned}$$

And ω_i^n is given by (see (2)),

$$\begin{aligned}
\omega_i^n &= -N_i E[u_{ij}^+ + u_{ij}^- | u_{ij}^+ < u_{ij}^-] = -N_i E[u_{ij}^+ + u_{ij}^- | \phi_i = \bar{\phi}, j \neq i] = \left(E[\delta(j) | j \neq i]\bar{\phi} - 1\right) w_i^b \\
&= \left(E[\delta(j) | j \neq i]\bar{\phi} - 1\right) \beta G \frac{GDP\ PPP_i}{GDP_i} e^{-gH_i} \sum_{j \neq i} \mu(j) \left(M_{ji} + M_{ij}\right)
\end{aligned}$$

Under the assumption that $E[\delta(j)|j \neq i] = E[\delta(j)]$, the ratio $\gamma \equiv \frac{\omega_i^n}{\omega_i^b} = E[\delta(j)]\bar{\phi} - 1$ is constant and independent of country i. γ measures a country's bias against a bailout, and the fact that in this case all countries have a common bias means that the efficient voting rule can be written as a weighted voting rule.[30] This rule assigns weights,

$$\omega_i^* = \omega_i^b$$

to each country, and a threshold of $\frac{E[\delta(j)]\bar{\phi}-1}{E[\delta(j)]\bar{\phi}} \sum_{i=1}^n \omega_i^*$ of votes to approve a bailout. Given that weights are derived from the expected welfare impact of a bailout on member countries, they also relate to the absolute level of capital contributions that each country should make to the IMF (i.e. each member's contributions should be proportional to the benefits from belonging to the IMF). Therefore the model also helps to determine the total level of working capital that the IMF should have, a decision that has so far been discretionally determined at General Quota Reviews.

[30]See Corollary 1 in Barberà and Jackson (2006).

Table 1: Quota Proposal - G20 countries

Country	Current	Symmetric	North-South
Argentina	0.888	0.576	0.886
Australia	1.358	0.610	0.523
Brazil	1.783	1.455	1.764
Canada	2.672	2.034	0.835
China	3.996	11.227	7.689
France	4.505	3.768	4.101
Germany	6.110	7.251	8.307
India	2.442	2.239	2.173
Indonesia	0.872	1.107	0.783
Italy	3.307	3.264	3.809
Japan	6.556	3.355	3.248
Korea	1.412	2.197	2.192
Mexico	1.521	2.319	0.905
Russia	2.494	2.447	3.224
Saudi Arabia	2.930	1.244	1.005
South Africa	0.784	0.583	0.584
Turkey	0.611	1.218	1.467
United Kingdom	4.505	2.690	2.696
United States	17.670	9.764	10.730