Informality and Social Security*

Rasmus K. Berg† Martín Gonzalez-Eiras‡

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Abstract

We study the effects on the social security tax rate, labor supply and capital accumulation, of making pension benefits contingent on, or independent of, formal labor market participation. Due to the universal system's better insurance properties, society always prefers it over a contributive pension system. Nevertheless, higher tax rates might lead to lower steady state welfare under a universal system. We calibrate the model to Argentina, which between 2005 and 2011 switched from a contributive to a universal system. We show that the model accounts for the observed increase in pension spending of about 2.5% of GDP. It further predicts a reduction of one p.p. in private savings and an increase of almost 4 p.p. in informality.

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1 Introduction

Labor markets in Latin America are characterized by high levels of informality, ranging from around 25% in Chile and Uruguay to about 60% in Colombia and Mexico.[1] Workers in the informal sector typically earn 30% less than their formal sector counterparts. At the same time, to a large extent informality is voluntary, and unskilled young workers

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†University of Copenhagen. Oster Farimagsgade 5, 1353 Copenhagen K, Denmark. E-mail: rasmus.kehlet.berg@econ.ku.dk.

‡University of Copenhagen. Oster Farimagsgade 5, 1353 Copenhagen K, Denmark. E-mail: mge@alum.mit.edu.

[1]Estimates for the period 1989-2005 (Gasparini and Tornarolli, 2009).

accumulate human capital and business relations as wage earners and then start an informal business of their own (Maloney, 2004; Gasparini and Tornarolli, 2009). The fact that informality might be voluntary creates a challenge to policymakers trying to increase social welfare: should they attack informality or help workers transition to formal employment? Policy debate is further complicated by the fact that there are different views on what is meant by informality. Seen from the production technology, jobs are classified as informal if they have low productivity and take place in small firms, usually family-based. From a legal point of view, workers are considered to be informal if they have no access to social protection or labor benefits. From this perspective, formal workers receive several benefits alongside their wages, e.g. health and unemployment insurance and contributions to pensions.

Some workers remain formal or informal throughout their work life, while others alternate between the formal and informal sectors. Workers are also exposed to unemployment risk. Based on legal requirements to qualify for pension benefits, workers only learn close to retirement whether they will receive a benefit or not. To tackle the problem that a large share of informal workers are set to retire without any pension coverage, several countries in Latin America have started to implement a universal pension system.[2] In particular, Argentina recently implemented a large scale tax amnesty that allowed retirees to receive a benefit even if they did not qualify under the contributive system's rules. Thus, this policy can be seen as transforming the contributive system into a universal one, increasing the number of beneficiaries by 2.7 million between 2005 and 2011, see figure 1. This corresponds to an increase in coverage of the population aged above 65 years from 68% to 91% between these years.

In this paper we build on a standard overlapping generations setup with capital formation a tractable model to analyze the effects that switching from a contributive to a universal social security system has on the equilibrium tax rate, labor supply, and capital accumulation. In their role as economic agents, households in the model take prices, taxes, and retirement benefits as given when choosing consumption, savings, and labor supply. In their role as voters, households choose among office motivated parties that offer policy platforms comprising labor income taxes, and retirement benefits. Since policy choices are of different concern to young and old voters, we model the resolution of the ensuing conflict under the assumption of probabilistic voting. The political process lacks commitment, and elections take place every period.

Policy choices do not only affect economic outcomes. Absent commitment, they also

[2] Currently, one-third of pensions in the region are noncontributory, see (Cavallo and Serebrisky, 2016).

Figure 1: Pension coverage in Argentina

Increase in coverage from social security reforms 2005-2011. Source: The Economist.

affect, indirectly, future policy decisions. Voters internalize the indirect effects, reflected in the equilibrium relationship between future state variables and future policy choices. We focus on Markov perfect equilibria, and assume that only fundamental state variables enter this equilibrium relationship, excluding artificial state variables of the type sustaining trigger strategy equilibria.

We introduce endogenous labor supply by allowing households to supply labor to either the formal, or the informal labor sector.[3] To keep the model simple and tractable, we introduce a lottery mechanism to determine whether a retiree receives benefits or not under a contributive pension system. The probability of receiving benefits is positively correlated with labor market participation in the formal sector. On one hand, this assumption reflects the requirements on minimum number of years of contributions to receive benefits present in most real world social security systems. On the other hand it captures the stochastic nature of employment which makes some retirees unable to meet these requirements.

Note that our model assumptions capture, in a parsimonious way, both aspects of informality as described earlier. First, labor in the informal sector has a lower productivity and the productivity differential is increasing in the tax rate: as taxes increase, workers

[3]The informal sector comprises both home production and a black labor market for the underground economy.

spend more time in the informal sector driving down labor productivity there. Second, legal social protection, measured by the probability of getting access to retirement benefits, is increasing in the amount of time spent working in the formal sector.

We find that society would always prefer a universal social security system over a contributive one, if offered such a choice in the political process. The reason for this is that, conditional on expectations of future pensions, the only difference between having in the current period a contributive or a universal system is on the welfare of retirees. And these prefer the universal system as it insures them against the risk of not receiving pensions. In contrast, in a comparison of steady states, capital accumulation is depressed in a universal system. Thus, from a Ramsey perspective a contributive system might dominate.

The model predicts some adverse effects of implementing noncontributory, or universal, coverage systems to solve the problem of inadequate pension coverage in Latin America. For instance, while a large degree of informality might have caused the low degree of pension coverage to begin with, a model calibration to Argentina predicts that universal coverage would further increase the degree of informality and crowd out savings in general. Specifically we show that Argentina would experience a tax increase from 27.1% to around 32.6% when universal benefits are introduced. Informality would increase almost 4 p.p. and the saving rate would fal by one p.p.

We contribute to the existing literature on Markov Perfect voting equilibrium for social security (Forni, 2005; Gonzalez-Eiras and Niepelt, 2008; Gonzalez-Eiras and Niepelt, 2015; Song, 2011), in a number of ways. First, we develop a model that allows for different pension systems. Endogeneizing labor supply, by making workers allocate time in either the informal or formal sector as in (Song, Storesletten and Zilibotti, 2012), allows us to capture the problem of informality that persists in most Latin American countries. It turns out that when pension benefits are contributive, the standard numerical approach of iterating on the policy function over the relevant state space is inefficient when one is interested in the demographic transition. We circumvent this problem by developing a simple backwards recursive Endogenous Gridpoing Method (EGM) algorithm. Lastly, where a comparison is feasible, model predictions on the effects of ageing are in line with the above mentioned political models of intergenerational transfers.

Our work also relates to the literature estimating the effects of non-contributive pensions on saving and labor supply. Some negative effects have been identified in Argentina and Mexico, both for savings, (Gonzalez-Rozada and Ruffo, 2015; Amuendo-Dorantes, Juarez, and Alonso, 2012), as for labor supply in the formal sector, (Bosch and Guajardo, 2012; Galiani, Gertler and Bando, 2016). We provide a quantitative estimate for the Argentine context. The model can be calibrated to study the effect of non-contributive pensions in Mexico as well.

The rest of the paper is organized as follows: Section 2 presents the economic environment, and section 3 presents the economic and politico-economic equilibrium concepts. Section 4 analyzes politico-economic equilibria under the two pension systems, and shows that the universal system is always preferred to the contributive one if such a choice is offered on a popular vote. Section 5 outlines the numerical solution approach for the contributive equilibrium. In section 6 we calibrate the model to Argentina to get a quantitative comparison of the effects of both pension systems. Section 7 concludes, and an appendix collects proofs, auxiliary calculation, discussion of numerical methods and other ancillary discussions.

2 The Model

2.1 Demographics and Institutions

We consider an economy populated by overlapping generations of workers and retirees. Workers supply labor to the formal and informal sectors, pay taxes, consume and save. In the subsequent period they retire, consume the return on their savings and social security benefits, if they receive them, and die. The ratio of workers to retirees in period t follows a deterministic process, and is given by ν_t.

The government runs a pay-as-you-go (PAYG) pension system with a balanced budget. Every period the government taxes labor supplied in the formal sector and transfers the sum directly to retirees. When the system is universal, every retired household receives the same pension benefit, independent of her participation in the formal sector when young. When the system is contributive, some workers receive pensions while others do not. The probability of receiving pensions is proportional to labor market participation in the formal sector when young. This assumption is made to capture workers' uncertainty on whether they will meet legal requirements to receive benefits in contributive systems.[4]

Policy decisions are taken by a government that acts in the interest of voters, but lacks commitment.

[4]In most contributive social security systems workers need to have a minimum number of years of contributions to receive benefits. Thus, if affected by long or repeated unemployment spells, or if frequently switching between the formal and informal sectors, they might not qualify at the time of retirement.

2.1.1 Technology

In the formal sector, a continuum of competitive firms transforms capital and labor into output. Capital is owned by retirees and fully depreciates after a period. The economy-wide capital stock per worker, k_t, therefore corresponds to the economy-wide per-capita savings of workers in the previous period, s_{t-1}, normalized by ν_t. We assume that production technology is Cobb-Douglas with $\alpha \in (0,1)$ denoting the income share of capital. Furthermore, for tractability we assume productive externalities as in Romer (1986) such that firm i's output is given by

$$Y_t^i = A_t \left(K_t^i\right)^{\alpha} \left(H_t^i\right)^{1-\alpha}, \qquad A_t \equiv Ag\left(\frac{k_t}{h_t}\right) = A\left(\frac{k_t}{h_t}\right)^{1-\alpha}. \tag{1}$$

Here K_t^i and H_t^i denotes the individual firm's use of capital and labor, while k_t and h_t are economy per-capita aggregates that the representative firm takes as given. The function $g\left(\frac{k_t}{h_t}\right) \equiv \left(\frac{k_t}{h_t}\right)^{1-\alpha}$ measures the strength of productive externalities (note $g' > 0$, and $g'' < 0$). In equilibrium this entails the following factor prices

$$R_t = \alpha A, \qquad w_t = (1-\alpha)A\left(\frac{k_t}{h_t}\right). \tag{2}$$

Production in the informal sector is given by the technology

$$y_t = w_t^* F(h_t) = w_t^* \frac{\xi}{1+\xi} X\left(1 - h_t^{1+\frac{1}{\xi}}\right), \tag{3}$$

where w_t^* denotes the formal labor sector wage rate *if* labor taxes were zero, and note that $F(1) = 0$, $F' < 0$, $F'' \leq 0$, $F''' \leq 0$. Thus, informal production technology only uses labor $(1-h)$ and has weakly decreasing returns to scale.

2.2 Preferences and Household Choices

Workers and retirees in period t value consumption, $c_{1,t}^i$ and $c_{2,t}^i$ respectively. Workers discount the future at factor $\beta \in (0,1)$, and are endowed with a unit of time, supplying h_t in the formal sector and $1-h_t$ in the informal sector. For analytical tractability, we assume that period utility functions are logarithmic. Welfare of a worker who chooses savings, s_t^i, and labor supply in the formal market, h_t^i, is given by

$$\begin{aligned} & \ln(c_{1,t}^i) + \beta \ln(c_{2,t+1}^i) \\ \text{s.t.} \quad & c_{1,t}^i = w_t \left[(1-\tau_t)\, h_t^i + \frac{w_t^*}{w_t} F\left(h_t^i\right) \right] - s_t^i \equiv \mathcal{I}_t - s_t, \\ & c_{2,t+1}^i = s_t^i R_{t+1} + E_t[b_{t+1}^i]. \end{aligned} \tag{4}$$

Where τ_t is the social security tax rate, and the expectation for retirement benefits reflects that these are stochastic when the social security system is contributive. Total labor income is denoted by $\mathcal{I}_t$.

We assume optimal savings and labor supply decisions are characterized by

$$s_t^i = s(w_t, w_t^*, \tau_t, \tau_{t+1}), \tag{5}$$

$$h_t^i = h(w_t, w_t^*, \tau_t, \tau_{t+1}). \tag{6}$$

2.3 Social Security System

The government runs a pay-as-you-go pension systems with a balanced budget. Every period the government taxes labor supplied in the formal sector and transfers the sum directly to the current retired generation. We consider two polar types of systems. First, a contributive one such that workers need to satisfy a requirement on contributions to receive pension benefits. This makes pension benefits to be stochastic as a worker with long or repeated unemployment spells might fail to meet contribution requirements when they retire. To keep the model simple, we postulate that the probability that a worker receives pension benefits is given by the share of her time endowment supplied in the formal sector, h_t.[5] Since the budget is balanced, pension benefits are then given by

$$b_t^c = \frac{\nu_t h_t w_t \tau_t}{h_{t-1}}, \tag{7}$$

where the superscript c denotes that the system is contributive. Note that pension benefits are affected by past labor supply decisions.

The second pension system that we analyze is a universal one in which all retirees receive benefits, independently of contributions made. In this case we have that benefits are given by

$$b_t^u = \nu_t h_t w_t \tau_t, \tag{8}$$

[5] In this way we introduce the contributive principle of making pension benefits contingent on sufficient participation in the formal labor sector, while maintaining tractability by making all workers identical.

where the superscript u denotes that the system is universal. It is immediate that, for given tax rates, current labor supply, wage rate, and demographics, $b_t^u \leq b_t^c$.

2.4 Elections

Elections take place at the beginning of each period. We assume that preferences are aggregated through probabilistic voting.[6] Thus, policy maximizes a convex combination of the objective functions of all groups of voters, where the weights reflect the groups' sizes and their responsiveness to policy changes. We allow for age related variation in responsiveness, reflected in a per capita political influence weight of unity for young voters and a per capita weight of $\omega \geq 0$ for retired voters. Furthermore, we assume that the political weight of a retiree is independent of whether she receives benefits or not.

3 Equilibrium

3.1 Competitive Equilibrium

The state is given by z_t, which includes exogenous demographics as well as savings per capita, s_{t-1}, and past labor supply in the formal sector, h_{t-1}. Conditional on z_t, the production function as well as competition among firms determine factor prices, w_t and R_t. Policy, τ_t, and the type of pension system, then determines capital accumulation, s_t, labor supply, h_t, and thus z_{t+1}. Conditional on z_t, a policy sequence $\{\tau_s\}_{s\geq t}$ thus fully determines an allocation and price system.

Definition 1. A *competitive equilibrium* conditional on z_0 and a policy sequence $\{\tau_t\}_{t\geq 0}$ is given by an allocation and price system such that

i. households optimize: (5) and (6) hold for all i, t;

ii. capital evolves according to $k_t = s_{t-1}/\nu_t$, labor markets clear, and factor prices are determined according to (2) for all t; and

iii. the government budget constraints (7) or (8) are satisfied for all t;

Appendix A derives the competitive equilibria when pension benefits are universal or contributive. The following proposition summarizes the comparison of economic equilibria across regimes.

[6]See (Lindbeck and Weibull, 1987).

Proposition 1. Consider the economic equilibria.

i. For a given policy the propensity to save out of labor income (δ) and formal labor supply (h) are unambiguously higher when pension benefits are contributive.

ii. Propensity to save and formal labor supply are decreasing in the current tax rate under both type of pension benefits. When pension benefits are contributive propensity to save is decreasing in future taxes, while formal labor supply is increasing.

iii. When taking the path of future policies as exogenous, the elasticity of labor supply to the current labor tax is higher when pension benefits are universal.

Proposition 1 asserts that when we take policies as given, introducing universal benefits unambiguously leads to crowding out of private savings and lower formal labor supply. The economic mechanisms at play for both results in a contributive system can be explained as workers self-insuring against the risk of not receiving pension benefits when old. Workers achieve this self-insurance by (1) increasing formal labor supply, and (2) increasing private savings. This also explains why labor responses are smaller when benefits are contributive: A change in the current tax rate affects the profitability of supplying labor to the formal sector. However when benefits are contributive, optimal labor supply weights the profitability of the formal sector against the risk of losing out on pension benefits. This tempers the labor response compared to an economy with a universal system.

3.2 Politico-Economic Equilibrium

In politico-economic equilibrium political decision makers optimally choose tax rates, taking all implications of their actions into account and forming rational expectations about future policy choices. We assume that these choices are Markov, i.e. they are functions of the fundamental state variables. The decision maker at date t takes s_{t-1} and h_{t-1} as well as $\tau^{t+1}(\cdot)$ as given. Furthermore, given the continuation tax function the policymaker takes as given the following law of motion for the state variables as a function of current policy choices[7]

$$z_{t+1} = \zeta_t(z_t, \tau_t, \tau^{t+1}(\cdot)). \tag{9}$$

[7]These laws of motion formally follow from (5) and (6) when we replace future policy by the continuation policy function.

The policymaker chooses τ_t to maximize

$$\mathcal{W}_t(z_t, \tau_t; \tau^{t+1}(z_{t+1})) \equiv \omega \mathcal{O}(z_t, \tau_t) + \nu_t \mathcal{Y}(z_t, \tau_t; \tau^{t+1}(z_{t+1})) \quad (10)$$
$$\text{s.t.} \quad (2), (7) \text{ or } (8), (9),$$

where the objective function is the weighted sum of the indirect utility functions of workers, $\mathcal{Y}(z_t, \tau_t; \tau^{t+1}(z_{t+1}))$, and retirees, $\mathcal{O}(z_t, \tau_t)$.

Definition 2. A *politico-economic equilibrium* as of period t conditional on z_t consists of a sequence of tax functions, $\{\tau_\iota(\cdot)\}_{\iota \geq t}$; a sequence of continuation tax functions, $\{\tau^{\iota+1}(\cdot)\}_{\iota \geq t}$; a sequence of laws of motion for the state variables, $\{\zeta_\iota(\cdot)\}_{\iota \geq t}$; policy choices, $\{\tau_\iota^\star\}_{\iota \geq t}$; and a competitive equilibrium allocation such that

i. tax functions are optimal subject to continuation tax functions:

$$\tau_\iota(z_\iota) \in \arg\max_{\tau_\iota} \mathcal{W}_\iota(z_\iota; \tau^{\iota+1}(\cdot)) \text{ for all } z_\iota, \iota \geq t;$$

ii. continuation tax functions are consistent with tax functions:[8]

$$\tau^\iota(z_\iota) = \tau_\iota(z_\iota) \text{ for all } z_\iota, \iota \geq t;$$

iii. laws of motion are consistent with the policy and continuation policy functions according to (9);

iv. equilibrium tax choices are generated by the continuation tax function,

$$\{\tau_\iota^\star\}_{\iota \geq t} = \{\tau^\iota(z_\iota)\}_{\iota \geq t},$$

and $\{\tau_\iota^\star\}_{\iota \geq t}$ implements a competitive equilibrium allocation.

Note that for infinite horizon and a recursive time-autonomous structure, the policy and continuation policy functions are time-autonomous functions of the state as well, and conditions i. and ii. above are combined in a fixed point requirement.

[8]Note that we do not need to track policy choices in the future beyond one period since current voters, at most, live for two periods. Otherwise this consistency requirement should be written as $\tau^\iota(z_\iota) = (\tau_\iota(z_\iota), \tau^{\iota+1}(z_{\iota+1}(\cdot)))$.

4 Analysis

4.1 Universal pensions

When the system is universal every retired household receives the same pension rate, characterized by (8), independent of its participation in the formal labor sector when young. Households behavior is characterized by:

$$h_t^u = \left(\frac{1-\tau_t}{X}\frac{w_t}{w_t^*}\right)^{\xi}, \qquad s_t^u = \frac{\beta}{1+\beta}\mathcal{I}_t^u - \frac{b_{t+1}^u}{(1+\beta)R_{t+1}},$$

where the u superscript refers to social security being universal. Recall that w_t^* was defined as the equilibrium wage rate if taxes were zero. Using this in the households' first order condition, and the factor price equation for w_t, (2), we define $(w_t^*/w_t) = (h_t^u/h_t^*)$ where $h_t^* = (1/X)^{\xi}$. Combining households and firms' first order conditions and equilibrium conditions, we get:

$$\begin{aligned} h_t^u &= \left(\frac{1-\tau_t}{X^{1+\xi}}\right)^{\frac{\xi}{1+\xi}}, \\ s_t^u &= \delta^u\left(\tau_{t+1}\right)\mathcal{I}_t^u, \\ c_{1,t}^u &= \gamma^u\left(\tau_{t+1}\right)\mathcal{I}_t^u, \\ c_{2,t+1}^u &= \alpha A\eta^u\left(\tau_{t+1}\right)\delta^u\left(\tau_{t+1}\right)\mathcal{I}_t^u, \end{aligned} \tag{11}$$

where the functions $\delta^u, \gamma^u, \eta^u$ are defined by

$$\eta^u(\tau_{t+1}) \equiv \left(1+\frac{1-\alpha}{\alpha}\tau_{t+1}\right), \quad \delta^u(\tau_{t+1}) \equiv \frac{\beta}{\beta+\eta^u(\tau_{t+1})}, \quad \gamma^u(\tau_{t+1}) \equiv 1-\delta^u(\tau_{t+1}). \tag{EE_u2}$$

Note that the labor income function is given by

$$\mathcal{I}_t^u = (1-\alpha)A\frac{s_{t-1}^u}{\nu_t}\left[1-\tau_t+X^{\xi}F\left(h_t^u\right)\right],$$

which is independent of future policy choices and log-separable in state variables s_{t-1}^u, and ν_t.

When pensions are universal, the political aggregator function is given by

$$\mathcal{W}^u(z_t) = \omega u\left(c_{2,t}^u\right) + \nu_t\left[u\left(c_{1,t}^u\right) + \beta u\left(c_{2,t+1}^u\right)\right]. \tag{12}$$

The political process maximizes (12), subject to the constraints that the economy is in a competitive equilibrium, and taking the future policy function, $\tau_{t+1}(z_t)$, as given. Since the economic equilibrium —in particular the labor income function—, and the objective function do not depend on h_{t-1}, then only savings per capita might be a relevant endogenous state variable when pensions are universal.

The main results of the politico-economic equilibrium with universal pensions are summarized in the following proposition.

Proposition 2. Consider a universal pensions system. There is a unique Markov perfect equilibrium in the limit of the finite horizon. The equilibrium policy function is given by:

$$\tau^u(\nu_t) = \min\left\{1,\ \max\left\{0,\ \frac{1}{\omega + \nu_t(1+\beta)}\left[\omega\left(1+\xi X^{1+\xi}\right) - \frac{\alpha}{1-\alpha}\nu_t(1+\beta)\right]\right\}\right\}.$$

Proof. See appendix B.1. □

We note here that as the policy function does not depend on any endogenous states, future taxes τ_{t+1} become independent of τ_t as well. Notwithstanding this orthogonality, the trade-offs underlying the equilibrium tax rates are dynamic in nature as they relate contemporaneous tax revenue and benefits to future factor prices and tax revenue. The tractability of the model comes from specifying functional forms that render the factor price elasticities and the derivatives of the indirect utility functions orthogonal to the capital stock.[9] As we will see shortly, when pensions are contributive we lose the ability to generate closed form solutions since, although the capital stock remains orthogonal to the derivatives of the indirect utility functions, lagged labor supply affects political trade-offs.

Note that the equilibrium policy is increasing in ω and decreasing in ν_t. To gain intuition for this, consider the marginal effect of a tax increase on a worker and a retiree. For a retired household a marginal tax increase simply translates directly into higher benefits. Indirectly however, the higher tax on labor also crowds out the formal labor supply of workers, which lowers pension benefits for the retired. The direct effect always dominates, such that retirees prefer higher tax rates in equilibrium.[10] For workers, consumption is linear in labor income and thus political support can be summed up by how $\mathcal{I}_t^u$ is affected. The direct effect from a tax increase is a drop in labor income. The indirect effect

[9]As shown elsewhere, these functional form restrictions tend to be of minor importance for the quantitative predictions of the model. See Gonzalez-Eiras and Niepelt (2005) for an analysis in a related context.

[10]The assumption of productive externalities implies that the direct effect always dominates.However, even without this assumption the direct effect would at least always dominate locally around equilibrium.

is that formal labor supply in equilibrium drops, which drives up the wage rate, due to a standard capital deepening effect. The direct effect always dominates. Thus, workers will generally prefer lower taxes. In this respect the model with universal pension benefits confirms the finding in related literature that an ageing population (decrease in ν_t), and a larger political power of retirees, (higher ω), result in higher equilibrium taxes.

4.2 Contributive pensions

With contributive pensions some retirees do not receive pensions, as they presumably fail to meet eligibility criteria for receiving them due to long or repeated spells of unemployment. We assume that with probability $h_t^{c,j}$ retired household j receives pension benefits b_{t+1}^c given by (7), and with probability $(1 - h_t^{c,j})$ she receives nothing.[11] Note that the probability of receiving pension benefits depends on *individual* household's formal labor supply. In this way we introduce the contributive principle of making pension benefits contingent on sufficient participation in the formal labor sector, while keeping the model tractable by having workers identical in their youth. The economic equilibrium is given by:

$$\begin{aligned}
h_t^c &= \frac{1}{X^\xi}\left\{1-\tau_t+\beta ln\left[\eta^c\left(h_t^c,\tau_{t+1}\right)\right]\gamma^c\left(h_t^c,\tau_{t+1}\right)\frac{\mathcal{I}_t^c}{w_t}\right\}^{\frac{\xi}{1+\xi}}\\
s_t^c &= \delta^c\left(h_t^c,\tau_{t+1}\right)\mathcal{I}_t^c,\\
c_{1,t}^c &= \gamma^c\left(h_t^c,\tau_{t+1}\right)\mathcal{I}_t^c,\\
c_{2,t+1}^{c,0} &= \alpha A\delta^c\left(h_t^c,\tau_{t+1}\right)\mathcal{I}_t^c,\\
c_{2,t+1}^{c,1} &= \alpha A\eta^c\left(h_t^c,\tau_{t+1}\right)\delta^c\left(h_t^c,\tau_{t+1}\right)\mathcal{I}_t^c,
\end{aligned} \qquad (EE_c1)$$

where the superscript c refers to the contributive system, and for second period consumption superscript 1 denotes a retiree that receives benefits, while superscript 0 denotes a retiree that does not receive benefits. Functions $\delta^c, \gamma^c, \eta^c$ are defined by

$$\eta^c \equiv 1+\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c},\quad \delta^c\equiv\beta\frac{1+(\eta^c-1)(1-h_t^c)}{1+\beta+(\eta^c-1)\left(1+\beta(1-h_t^c)\right)},\quad \gamma^c\equiv 1-\delta^c. \qquad (EE_c2)$$

We note that while equilibrium consumption and savings are still linear in the labor-income function, and thus the capital stock is not a relevant state variable for the political process, two complications arise: (1) The equilibrium labor supply function is an implicit

[11] In reality, some social security systems still pay a pension to workers that do not meet eligibility criteria. This is usually a minimum benefit, and we abstract from it.

function of both current and future taxes. (2) Functions $\delta^c, \gamma^c, \eta^c$ are non-linear functions of state variables and future taxes.

When pension benefits are contributive, the political aggregator function is given by

$$\begin{aligned} \mathcal{W}^c(z_t) = \omega & \left[h^c_{t-1} \ln\left(c^{c,f}_{2,t}\right) + \left(1 - h^c_{t-1}\right) \ln\left(c^{c,if}_{2,t}\right)\right] \\ & + \nu_t \left[\ln\left(c^c_{1,t}\right) + \beta\left(h^c_t \ln\left(c^{c,f}_{2,t+1}\right) + (1 - h^c_t) \ln\left(c^{c,if}_{2,t+1}\right)\right)\right], \end{aligned} \tag{13}$$

and the political process seeks taxes to maximize it, subject to the constraints that the economy is in a competitive equilibrium, (EE_c1)-(EE_c2), and taking as given $\tilde{\tau}^c_{t+1}(\cdot)$, the policy function that determines future policy as a function of state variables. As mentioned above, the capital stock does not affect the marginal effects of taxes on the indirect utility functions. Compared to the case of universal benefits, h^c_{t-1} becomes a potentially relevant state variable. This makes the economic equilibrium labor supply function to depend explicitly on both τ_t and τ_{t+1}.

Given the policy function $\tilde{\tau}^c_{t+1}(\cdot)$, we implicitly define the *politico-economic equilibrium* labor function as:

$$\tilde{h}^c_t = \left\{\frac{1 - \tau_t}{X^{1+\xi}} + \beta ln\left(\eta^c\left[\tilde{h}^c_t, \tilde{\tau}^c_{t+1}\left(\tilde{h}^c_t, \nu_{t+1}\right)\right]\right)\gamma^c\left[\tilde{h}^c_t, \tilde{\tau}^c_{t+1}\left(\tilde{h}^c_t, \nu_{t+1}\right)\right]\frac{\mathcal{I}^c_t}{w_t}\right\}^{\frac{\xi}{1+\xi}}. \tag{14}$$

Before proceeding we impose some structure on this function.

Assumption 1. In the politico-economic equilibrium, labor supply and thus income are always strictly decreasing in the current tax rate τ_t.

The main analytical results of the contributive politico-economic equilibrium are summarized in the following proposition.

Proposition 3. Consider a contributive pensions system under assumption 1. In the finite horizon economy ending at time T, the terminal equilibrium policy function is unique and given by:

$$\tau^c_T\left(h^c_{T-1}, \nu_T\right) = \min\left\{1,\ \max\left\{0,\ \frac{1}{\omega + \nu_T/h^c_{T-1}}\left[\omega\left(1 + \xi X^{1+\xi}\right) - \frac{\alpha}{1-\alpha}\nu_T\right]\right\}\right\}. \tag{PEE_c}$$

For $t < T$ the politico-economic equilibrium has to be solved numerically. Some analytical results are: (i) The policy function is increasing in ω and decreasing in ν_t, (ii) the policy function is increasing in h^c_{t-1}, and (iii) retirees' support for taxes is unambiguously lower compared to the universal system.

Proof. See Appendix B.2. □

The basic intuition for why the equilibrium tax must be increasing in ω and decreasing in ν_t is the same as in the universal case: Retirees (workers) generally prefer higher (lower) taxes. Shifts in ω and ν_t move the relative weights the political process puts on the two groups of voters and thus the equilibrium tax. Compared to the universal case however, there are a number of additional effects.

When retirees evaluate an increase in the tax rate, only a share of them receive benefits. When this share (h^c_{t-1}) is large, more retired households benefits from higher taxes, thus τ^c_t is increasing in h^c_{t-1}. Since, for a given tax rate, households that receive benefits have lower marginal utility, there will be lower support for taxation from the retired, relative to the universal pension system.

Finally the effect of making pension benefits contributive on political support from workers is analytically ambiguous. When benefits are contributive, a rise in taxation has two effects on workers' welfare: A *labor income channel* and a *consumption smoothing channel*. In comparison, taxes only affect workers through a similar labor income channel, when benefits are universal.

As seen in (EE_c1), all economic equilibrium consumption functions are linear in labor income $\mathcal{I}^c_t$. As in the case of universal benefits, taxation produces a direct loss of income from working in the formal sector, and a positive effect on wages from capital deepening. In the universal case, the assumed constant labor supply elasticity is sufficient to ensure that the direct effect always dominates. In the contributive case we need to impose assumption 2 to ensure this. It is ambiguous whether the marginal effect of taxation due to changes in labor income is higher or lower than when pension benefits are universal.

When pension benefits are contributive, the pension system induces old-age consumption risk. The working households self-insures against this by increasing private savings and formal labor supply, see proposition 1. This increases the marginal utility of consumption (given labor income) and thus amplifies the marginal effect of a tax change on workers' indirect utility functions, thus reducing the political support for taxes.

4.3 Dominance of Universal System

We now show that if a vote were to take place on the type of pension system to use in the current period, a universal system would *always* be preferred. For this we start by conjecturing that the future choice of social security system is unaffected by the current choice, such that, for a given tax rate, workers' savings decisions would only be determined by the future type of social security system. If the pension system is contributive or universal today, the political objective functions, for given current and future taxes, are respectively given by:

$$\mathcal{W}_t^C = \omega h_{t-1} \ln\left(\eta^c(h_{t-1}, \tau_t)\right) + \nu_t \mathcal{Y}_t(\tau_t, \tau_{t+1}),$$
$$\mathcal{W}_t^U = \omega \ln\left(\eta^u(\tau_t)\right) + \nu_t \mathcal{Y}_t(\tau_t, \tau_{t+1}),$$

where $\mathcal{Y}_t$ is the indirect utility of workers. Crucially, for given tax rates, this is the *same* regardless of which pension system is in place today: When making their saving decisions, workers only care about what pension system will be in place when they retire.

It can be shown that $h_{t-1} \ln\left(\eta^c(h_{t-1}, \tau_t)\right) \leq \ln\left(\eta^u(\tau_t)\right)$.[12] Thus, retirees are always better off with universal pensions as they benefit from the risk sharing this provides. Therefore,

$$\mathcal{W}_t^U(\tau_t^{u,*}) \geq \mathcal{W}_t^U(\tau_t^{c,*}) \geq \mathcal{W}_t^C(\tau_t^{c,*}),$$

and a universal system would always be preferred. Note that this verifies our conjecture that the current choice of pension system would not affect future pension system choice.

If a universal pension would always defeat a contributive system in a vote, why do we not see more universal systems across the world? One possible explanation is that our modelling assumption that workers are homogenous and they are "surprised" by failure to meet retirement requirements is too blunt a description of behavior along the life cycle. But even if workers can foresee their retirement situation in advance and adjust accordingly, there will be a benefit from the universal system as this redistributes benefits towards the poor.

We can compare steady state welfare under the different social security systems. This can serve as an approximation of the preferences of a Ramsey planner that internalized political weights. To do this, we must assume that production has no externalities, as otherwise changes in savings would lead to divergence in GDP growth rates.[13] In this case the universal system will have the advantage of redistributing pension funds among all retirees. But, it will also have a cost as the tax rate will be higher and thus, capital accumulation will be depressed. Which of these two effects dominates depends on parameters. In appendix C we show that for our calibration it is the case that a contributive

[12]This follows since $\frac{dh_{t-1}\ln(\eta^c(h_{t-1},\tau_t))}{dh_{t-1}} \geq \frac{\ln(\eta^u(\tau_t))}{dh_{t-1}} = 0$, and $h_{t-1}\ln\left(\eta^c(h_{t-1},\tau_t)\right)|_{h_{t-1}=1} = \ln\left(\eta^u(\tau_t)\right)|_{h_{t-1}=1}$.

[13]Production externalities were assumed for tractability reasons, not because we cared about the effects of policies on output growth rate.

system has higher steady state welfare than a universal system.

5 Numerical Solution with Contributive Pensions

When pension benefits are contributive the politico-economic equilibrium has to be solved numerically. Nevertheless, our modelling assumptions allow us to numerically solve for each period's continuation policy function in an efficient way. More precisely, given that the labor supply function in (EE_c1) is a closed-form relation between τ_t, h_t, and τ_{t+1} that does not depend on h_{t-1}, we solve for the policy function using an EGM following (Carroll, 2006).

In the terminal period we construct a grid of the endogenous state h_{T-1}. The bounds are found by evaluating the labor supply function at $(\tau_{T-1}, \tau_T) = (0, 1)$ and $(\tau_{T-1}, \tau_T) = (1, 0)$ respectively. For each node on the grid we solve for the equilibrium terminal policy as given by (PEE_c). We approximate this policy function by interpolation over the solution grid. For each pair (h_{T-1}, τ_T) we use the labor supply function to define the implied tax rate τ_{T-1}. With the terminal policy given we solve for all policies $t < T$ recursively. For this we exploit the fact that we can express period t taxes as a close-form expression of current labor supply and period $t+1$ taxes.

Given parameter values and an initial value h_0 we can simulate a model realization using the identified policy functions. As with the EGM solution in general, this approach naturally takes corner solutions into account. Furthermore, the policy at time t may depend on the entire future path of the exogenous state $\{\nu_h\}_{h\geq t}$, in a way that is parsimoniously captured by the continuation policy function $\tau^{t+1}(z_{t+1})$.

As a robustness exercise we estimate the policy functions using traditional variations on the finite horizon EGM-type approach followed here. These include using an infinite horizon version of the model, using value function iteration instead of EGM, and using a steady state approximation. As appendix E shows, the resulting policy functions are essentially identical for all identification strategies while the finite horizon EGM approach is by far the fastest (3-46 times faster).

6 Quantitative Analysis

We now want to evaluate the model's quantitative performance. For this, we calibrate it to Argentina, a developing country that has experienced a high level of informality in the labor market for the last 30 years. In an attempt to solve persistent pension funding problems, Argentina introduced a fully-funded pillar to its social security system in 1994. High levels of unemployment and informality, coupled with strict requirements to receive benefits, led to close to one third of those of retirement age to be without benefits, even when the country had fully recovered from the deep crisis of 1999-2002 that reduced GDP by 18.4%. In December 2008 the fully-funded pension funds were nationalized, and its beneficiaries were transferred to the pay-as-you-go system.

Since 2005 a number of reforms were introduced to increase pension coverage. In particular, a tax amnesty allowed workers that had less than the required 30 years of contributions to receive a pension. This policy increased the number of beneficiaries by 2.7 million between 2005 and 2011. This increased coverage of the elderly from 68% in 2005 to 91% in 2011.[14] Furthermore, prior to the moratorium, minimum pensions had been raised significantly more than other pensions, such that by 2005 60% of beneficiaries were receiving minimum pensions. Thus, in less than a decade, Argentina experienced a significant increase in non-contributive pensions: While in 2005 pensions represented 4.3% of GDP, by 2011 they accounted for 6.8% in 2011.

To estimate the effects of switching from a contributive to a universal social security system on taxes, informality and savings—and how these are affected by ageing—we calibrate the model to Argentina in 2010. We assume pensions are contributive, and we take a period to be 30 years. The capital income share, $\alpha = 0.50$, comes from (Frankema, 2010) (see also (Restrepo-Echevarría, 2017)). In the baseline we take the elasticity of labor supply, $\xi = 0.35$, and explore values between 0.2 and 0.5 as robustness. The savings rate is used to calibrate β, and in 2010 is 20.6%, from World Bank national accounts data. We take the informality rate in 2010 to be 25.3%, since the reform increases coverage from 68% to 91%; this pins down parameter X.[15] To construct the time series for ν_t we follow (Gonzalez-Eiras and Niepelt, 2008) and use 30-year gross population growth rate and projections from census data. Finally, we use the social security tax rate in 2010 of $\tau_{2010} = 0.271$ to calibrate ω.[16]

As figure 2 illustrates, the model indicates that as ν_t decreases from 1.45 to around

[14]See (Bertranou, Cetrángolo, Grushka and Casanova, 2012) and (Rofman and Oliveri, 2012) for details on recent pension reforms in Argentina, and (Gonzalez-Rozada and Ruffo, 2015) for an estimate of the effect of the tax amnesty on savings.

[15]Thus, we take $(91 - 68)/91$ as our pre-reform estimate of informality. Note that International Labor Organizaton's ILOSTAT has a higher measure for labor informality for this time period at 48.1%, but using this would overestimate the likelihood of not receiving pensions, as workers that are informal in a given year might still be formal for enough years over their working life to claim a social security benefit when they retire.

[16]We do this by following the general outline of the nested fixed point algorithm, cf. (Rust, 1987): Given parameter values an inner loop solves the model and calculates the difference from target values τ_t. An outer hill-climbing algorithm searches the parameter space to minimize this difference.

Table 1: Calibration

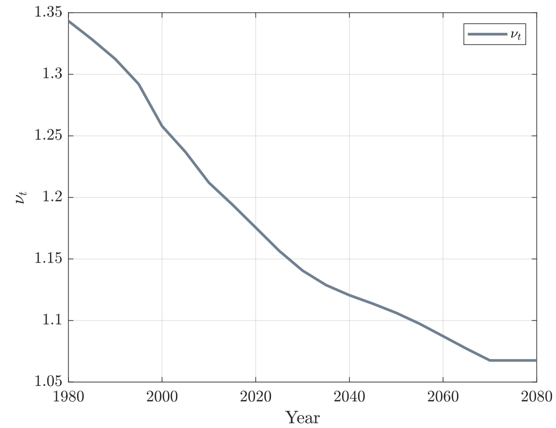
Parameter	Value	Calibration target
α	0.50	Factor income shares
ν_t	$[1.45, 1.06]$	30-year gross population growth rates
ξ	0.35	Elasticity of labor supply
β	0.32	Private savings rate of 20.6%
X	1.94	Informality rate of 25.3%
ω	1.39	Social security tax of 27.1%

1.06, the equilibrium tax rates and informality increase, whereas private savings decreases. What is more interesting, is that the model generally predicts significantly higher tax and informality rates under a universal system, and lower private savings and formal labor supply. Furthermore, the difference between pension systems is increasing with population ageing. In the calibration year the model predicts that a shift to a universal pension system increases tax rates by about 5.5 p.p. Given the labor income share, this translates into an increase of roughly 2.7% of GDP, a bit more than in the data. By the end of the demographic transition the increase would be of 7.2 p.p. with rates of respectively 34.6% and 41.8%. The same pattern is apparent for informality rate, private savings and formal labor supply: The savings rate drops from 20.5% to 19.5%, while formal labor supply drops from 0.753 to 0.715.

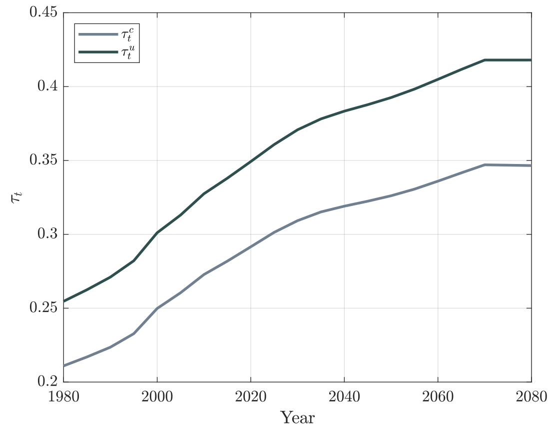
Finally, parts (c)-(d) further illustrates how much of the changes in savings rates and labor supply can be attributed to making pension benefits universal, and how much is due to changes in the tax rate. In particular, part (c) of figure 2 shows that savings drop primarily due to the increase in taxes, whereas the formal labor supply primarily responds to the changes in the pension system.

Taken together the figures in 2 tell an interesting story: Firstly, recall that the universal pension system is considered a potential solution to the problem of an increasingly ageing population set to retire with insufficient private savings and lack of coverage by the contributive pension system. However, while the implementation of a universal pension system reduces poverty among retirees, the adverse effect of such a treatment seems to be an exacerbation of the underlying high informality and a reduction in private savings. Secondly, figure 2 indicates that these adverse effects are both due the economic and political equilibrium responses, and that by ignoring the political process one neglects the fact that higher taxes are the outcome under a universal system.

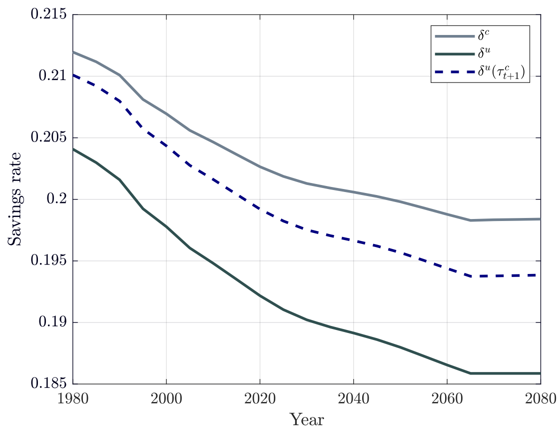
Figure 2: Comparing the universal- and contributive pension system in PEE



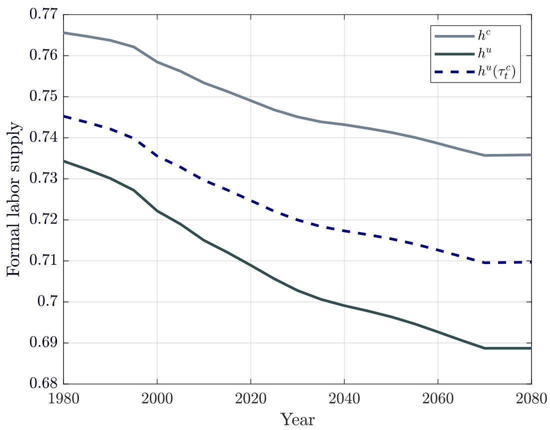
(a) Population growth rates



(b) PEE tax rates



(c) PEE private saving rates



(d) PEE formal labor supply

7 Conclusions

We have presented a politico-economic equilibrium model to assess the effects of universal social security benefits as opposed to a contributive system. Our modeling assumptions lead to analytical solutions under the universal system, and allowed us to find numerical solutions under the contributive pension system for the full demographic transition and not just for steady states. For this purpose we have developed a simple backwards induction algorithm that identifies how the politico-economic equilibrium evolves over time when the endogenous state-variable is forward-looking.

When the model is calibrated to Argentina it predicts that introducing universal pension benefits will increase taxes by 5.5 p.p. and informality by 3.8 p.p., and lower private savings rates by about one p.p. This suggests that the introduction of universal benefits might come at the cost of amplifying existing problems. Furthermore, we have shown that these costs can be attributed to both features of the economic and the political equilibrium features. Thus analyses neglecting to account for endogenous policy choice may severely underestimate the actual costs of introducing universal pension benefits.

The model can be used to analyze the effect of social security reforms in OECD countries in the aftermath of the Great Recession. Governments in these countries face future retirement income shortfall of a larger fraction of workers that are unable to meet the requirements to access social security benefits, or whose benefits are depressed due to low contributions or low real returns. For example, the model can be applied to Spain to rationalize the observed increase in minimum pension benefits in times of high unemployment since the restitution of democracy in the late 1970s. Increased unemployment raises the likelihood that workers do not meet the minimum number of years of contributions to qualify for pensions, or that their pensions fall short. Pension top-ups and non-contributory pensions (introduced in 1990) financed from general revenue are the political response. That universal pensions are politically preferred to a contibutive system explains why these changes have been persistent and have not been undone in periods of low unemployment.

Our results are subject to a number of caveats. Most importantly, we simplify the model by assuming that young households are all ex-ante identical and all employed. Thus we are not able to capture involuntary unemployment. Retirees that do not receive pension benefits because of unemployment would then not be able to engage in precautionary savings when young and generally not be ex-ante identical to retirees receiving those benefits. Following Song (2011) we could argue this distributional concern might make the universal pension system even more desirable.

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A The Economic Equilibria

A.1 Contributive equilibrium, new formulation

The representative young household solves (4). The first order conditions with respect to s_t and h_t for this problem are then given by,

$$\beta R_{t+1} c_{1,t} = c_{2,t+1}$$
$$\frac{w_t \left[1 - \tau_t + \frac{w_t^*}{w_t} F'(h_t^c)\right]}{c_{1,t}} = -\beta \frac{\nu_{t+1} w_{t+1} h_{t+1} \tau_{t+1}}{c_{2,t+1} h_t}.$$

Note that if $\tau_{t+1} = 0$ the right-hand side (RHS) of the first order condition wrt. h_t would simply be zero; in this case the first order condition is the same as under universal pension benefits. Using the budget constraints, as well as equilibrium relations for R_{t+1}, b_{t+1} and w_{t+1}, the savings rate is derived, as under the universal system:

$$s_t = \frac{\beta}{1 + \beta + \frac{1-\alpha}{\alpha}\tau_{t+1}} \mathcal{I}_t.$$

We note that the savings rate out of labor income is equivalent to that under universal pension benefits, δ^u. From the savings rate formulation and budgets this gives the consumption functions

$$c_{1,t} = (1 - \delta(\tau_{t+1}))\, \mathcal{I}_t$$
$$c_{2,t+1} = \alpha A \eta(\tau_{t+1}) \delta(\tau_{t+1}) \mathcal{I}_t$$
$$\delta(\tau_{t+1}) \equiv \frac{\beta}{\beta + \eta(\tau_{t+1})}$$
$$\eta(\tau_{t+1}) \equiv 1 + \frac{1-\alpha}{\alpha}\tau_{t+1}.$$

Rewriting the left-hand side (LHS) of the first order condition wrt. h_t, we use the following equilibrium results:

$$\frac{w_t^*}{w_t} = \frac{h_t}{h_t^*},$$
$$\nu_{t+1} h_{t+1} w_{t+1} = (1-\alpha) A s_t$$

where h_t^* is the labor supplied in equilibrium without taxation. We use this to write:

$$\frac{c_{1,t}}{w_t} = (1-\delta)\left[(1-\tau_t)h_t + \frac{h_t}{h_t^*}F(h_t)\right]$$
$$\frac{\nu_{t+1}h_{t+1}w_{t+1}\tau_{t+1}}{c_{2,t+1}} = \frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{\eta}$$

Using this we get (for $\tau_{t+1} > 0$)

$$\beta(1-\delta)h_t\frac{1-\tau_t+(1/h_t^*)F(h_t)}{1-\tau_t-(1/h_t^*)Xh_t^{\frac{\xi+1}{\xi}}} = -h_t\frac{\eta}{\tau_{t+1}}\frac{\alpha}{1-\alpha}.$$

Rearranging further we have the labor supply function:

$$h_t^c(\tau_t,\tau_{t+1}) = \left(\frac{(1-\tau_t)\left[\tau_{t+1}\beta(1-\delta)+\eta\frac{\alpha}{1-\alpha}\right]+\tau_{t+1}\beta(1-\delta)X^{1+\xi}\frac{\xi}{1+\xi}}{X^{1+\xi}\left[\eta\frac{\alpha}{1-\alpha}+\tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}\right]}\right)^{\frac{\xi}{1+\xi}}$$

Given h_t, τ_{t+1}, the implied tax rate τ_t can be backed out as

$$\tau_t = 1 + \frac{\tau_{t+1}\beta(1-\delta)X^{1+\xi}\frac{\xi}{1+\xi} - h_t^{\frac{1+\xi}{\xi}}X^{1+\xi}\left[\eta\frac{\alpha}{1-\alpha}+\tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}\right]}{\tau_{t+1}\beta(1-\delta)+\eta\frac{\alpha}{1-\alpha}}.$$

A.2 Proof of proposition 1, new formulation

(Some of this should be moved to online appendix.) Some properties for the labor supply:

- Labor is decreasing in current tax rate:

$$\frac{\partial h_t^c}{\partial \tau_t} = -\frac{\xi}{1+\xi}h_t^{-1/\xi}\frac{\eta\frac{\alpha}{1-\alpha}+\tau_{t+1}\beta(1-\delta)}{X^{1+\xi}\left[\eta\frac{\alpha}{1-\alpha}+\tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}\right]} < 0.$$

- Labor is increasing in future tax rate τ_{t+1}. To see this, we need a couple of derivations. Start with:

$$\frac{\partial(\tau_{t+1}\beta(1-\delta))}{\partial\tau_{t+1}} = \delta\Big(\eta+(\eta-1)\delta\Big).$$

Introduce a new short-hand (just temporarily):

$$
\begin{aligned}
h_t^c &= \left(\frac{f(\tau_{t+1})\,(\theta_1 + \theta_2\theta_3) + g(\tau_{t+1})\theta_1}{f(\tau_{t+1})\theta_2\theta_3 + g(\tau_{t+1})\theta_2} \right)^{\frac{\xi}{1+\xi}} \\
f(\tau_{t+1}) &\equiv \tau_{t+1}\beta(1-\delta) \\
g(\tau_{t+1}) &\equiv \frac{\alpha}{1-\alpha} + \tau_{t+1} \\
\theta_1 &\equiv 1 - \tau_t \\
\theta_2 &\equiv X^{1+\xi} \\
\theta_3 &\equiv \frac{\xi}{1+\xi}.
\end{aligned}
$$

The derivative wrt. τ_{t+1} is then given by

$$
\frac{\partial h_t^c}{\partial \tau_{t+1}} = \frac{\xi}{1+\xi} h_t^{-1/\xi} \frac{\theta_2\left[\theta_1 + \theta_3\left(\theta_2 - \theta_1\right)\right](f'g - g'f)}{\left(f(\tau_{t+1})\theta_2\theta_3 + g(\tau_{t+1})\theta_2\right)^2}
$$

To be sure of the sign of this derivative, we need to establish two things:

i. The terms regarding θ_i enters positively:

$$
\theta_1 + \theta_3(\theta_2 - \theta_1) = \frac{\xi}{1+\xi} X^{1+\xi} + \frac{1-\tau_t}{1+\xi} > 0.
$$

ii. The term $f'g - g'f$ is positive:

$$
f'g - g'f = \left(\delta\eta + \delta^2(\eta - 1)\right)\left(\frac{\alpha}{1-\alpha} + \tau_{t+1}\right) - \beta\tau_{t+1}\frac{\eta}{\beta+\eta}.
$$

Recall that

$$
\delta = \frac{\beta}{\beta+\eta}, \qquad \text{and that} \qquad \eta = 1 + \frac{1-\alpha}{\alpha}\tau_{t+1} \geq 1.
$$

Use this to simplify:

$$
f'g - g'f = \frac{\alpha}{1-\alpha}\left(\delta\eta + \delta^2(\eta-1)\right) + \tau_{t+1}\delta^2(\eta - 1) \geq 0.
$$

This establishes that $(\partial h_t^c / \partial \tau_{t+1}) > 0$.

With these things established, we can proceed to the proof of proposition 1. Note that for $\tau_{t+1} = 0$ the labor supply under universal and contributive pension coincide, $h_t^u = h_t^c$.

As we've established above, the labor supply function is strictly increasing in the future tax rate τ_{t+1} under contributive pension benefits. Thus $h_t^c \geq h_t^u$. Finally, we show the relative labor supply response (semi-elasticity) is largest under universal benefits:

$$\frac{\partial ln(h_t^c)}{\partial \tau_t} = -\frac{\xi}{1+\xi}(h_t^c)^{-\frac{1+\xi}{\xi}} \frac{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)}{X^{1+\xi}\left[\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}\right]}$$

$$\frac{\partial ln(h_t^u)}{\partial \tau_t} = -\frac{\xi}{1+\xi}(h_t^u)^{-\frac{1+\xi}{\xi}} \frac{1}{X^{1+\xi}}.$$

Thus the relative (relative) response is given by:

$$\frac{\partial ln(h_t^c)/\partial \tau_t}{\partial ln(h_t^u)/\partial \tau_t} = \left(\frac{h_t^c}{h_t^u}\right)^{-\frac{1+\xi}{\xi}} \frac{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)}{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}}$$

Using the formula for the labor supply in the two scenarios, we can express the relative labor supply as:

$$\frac{h_t^c}{h_t^u} = \left(\frac{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)}{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}} + \frac{\tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}}{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}}\frac{X^{1+\xi}}{1-\tau_t}\right)^{\frac{\xi}{1+\xi}}$$

Using this in the relative labor supply response yields

$$\frac{\partial ln(h_t^c)/\partial \tau_t}{\partial ln(h_t^u)/\partial \tau_t} = 1 + \frac{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)}{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}} \frac{\tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}}{\eta\frac{\alpha}{1-\alpha} + \tau_{t+1}\beta(1-\delta)\frac{\xi}{1+\xi}} \frac{X^{1+\xi}}{1-\tau_t} \geq 1.$$

A.3 Deriving the contributive equilibrium

The representative young household solves (4). The first order conditions with respect to s_t and h_t for this problem are then given by,

$$\beta\left(\frac{h_t^c R_{t+1}}{c_{2,t+1}^c} + \frac{(1-h_t^c)R_{t+1}}{c_{2,t+1}^{c,if}}\right) = \frac{1}{c_{1,t}}$$

$$\beta \ln\left(\frac{c_{2,t+1}^{c,f}}{c_{2,t+1}^{c,if}}\right) = -\frac{w_t\left[(1-\tau_t) + \frac{w_t^*}{w_t}F'(h_t^c)\right]}{c_{1,t}^c}.$$

Plugging in the budget constraints and rewriting the first order condition for savings,

$$w_t\left[(1-\tau_t)h_t^c + \frac{w_t^*}{w_t}F(h_t^c)\right] - s_t = \frac{s_t}{\beta\frac{s_tR_{t+1}+(1-h_t^c)b_{t+1}^c}{b_{t+1}^c+s_t^cR_{t+1}}}.$$

Note that using the equilibrium relations for R_{t+1} and pension benefits we can rewrite the denominator of the right hand side as

$$\frac{s_tR_{t+1}+(1-h_t^c)b_{t+1}^c}{b_{t+1}^c+s_t^cR_{t+1}} = \frac{1+\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c}(1-h_t^c)}{1+\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c}}.$$

Using this we can derive the savings function as

$$s_t^c = \delta^c(h_t^c, \tau_{t+1})w_th_t^c\left[(1-\tau_t)+X^\xi F(h_t^c)\right],$$

where the savings rate out of labor income is given by

$$\delta^c(h_t^c, \tau_{t+1}) = \beta\frac{1+\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c}(1-h_t^c)}{1+\beta+\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c}(1+\beta(1-h_t^c))}.$$

The consumption functions are then simply defined by plugging in the savings function into budgets.

From the first order condition with respect to labor start by plugging in budgets on the left-hand side (LHS) and use equilibrium conditions for b_{t+1}^c and w_t^*/w_t to get,

$$\ln\left(1+\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c}\right) = -\frac{w_t\left[(1-\tau_t)+h_t^cX^\xi F'(h_t^c)\right]}{c_{1,t}^c}.$$

Next we plug in budgets for $c_{1,t}^c$, use the functional form of $F(h)$ and rearrange as:

$$h_t^c = \left\{\frac{1-\tau_t}{X^{1+\xi}} + \beta\ln\left(1+\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c}\right)\gamma^c(h_t^c, \tau_{t+1})h_t^c\frac{1-\tau_t+X^\xi F(h_t^c)}{X^{1+\xi}}\right\}^{\frac{\xi}{1+\xi}}.$$

Given the value of h_t^c and τ_{t+1}, the tax rate τ_t can be found from

$${h_t^c}^{\frac{1+\xi}{\xi}} = \frac{1-\tau_t}{X^{1+\xi}} + \beta\ln(\eta^c)\gamma^ch_t^c\left[\frac{1-\tau_t}{X^{1+\xi}} + \frac{F(h_t^c)}{X}\right].$$

$$h_t^{c\,\frac{1+\xi}{\xi}} - \beta \ln(\eta^c)\gamma^c h_t^c \frac{F(h_t^c)}{X} = \frac{1-\tau_t}{X^{1+\xi}} \left[1 + \beta \ln(\eta^c)\gamma^c h_t^c\right]$$

$$\tau_t = 1 - \frac{X^{1+\xi}}{1 + \beta \ln(\eta^c)\gamma^c h_t^c} \left[h_t^{c\,\frac{1+\xi}{\xi}} - \beta \ln(\eta^c)\gamma^c h_t^c \frac{F(h_t^c)}{X}\right].$$

A.4 Proof of proposition 1

Comparing the private savings rates δ^c, δ^u is straightforward. First we note that $\partial\delta^c/\partial\tau_{t+1} \leq 0$, i.e a higher expected benefit reduces savings. One way to see that $\delta^c \geq \delta^u$ is to note that the economic equilibria in general coincide when $h_t^c = 1$ in the contributive solution. It is straightforward to show that $\partial\delta^c/\partial h_t^c \leq 0$ for all $h \in [0, 1]$, which proves that $\delta^c \geq \delta^u$.[17] A similar argument shows that $h_t^c \geq h_t^u$ (we show that $\partial h_t^c/\partial\tau_{t+1} \geq 0$ below).

The algebra from the labor response in equilibrium is tedious and thus relegated to an online appendix. It is however basically just an application of the implicit function theorem. It confirms that h_t^c is decreasing in τ_t, increasing in τ_{t+1} and more interestingly that the relative labor response of a tax-change (semi-elasticity) is larger under the universal system.

[17] Let X_1, X_2 denote numerator and denominator of δ^c.

$$\begin{aligned}\frac{\partial\delta^c}{\partial h_t^c} &= \frac{\beta}{X_2^2}\left[-\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{(h_t^c)^2}X_2 + X_1(1+\beta)\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{(h_t^c)^2}\right] \\ &= \frac{\beta\frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{(h_t^c)^2}}{X_2^2}\left[X_1(1+\beta) - X_2\right].\end{aligned}$$

Thus the sign of the derivative comes down to $X_2 \geq X_1(1+\beta)$:

$$1 + \beta + (\eta^c - 1)(1 + \beta(1 - h_t^c)) \geq (1+\beta)\left(1 + (\eta^c - 1)(1 - h_t^c)\right)$$

Reducing this expression term by term we have

$$h_t^c(\eta^c - 1) \geq 0, \qquad \text{where } \eta^c \equiv 1 + \frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c} \geq 1$$

A.5 A note the equilibrium labor income function

With our assumptions, the equilibrium labor income functions are given by:

$$\begin{aligned}\mathcal{I}_t^j &= (1-\alpha)A\frac{s_{t-1}^j}{\nu_t h_t^j}\left[(1-\tau_t)h_t^j + h_t^j X^\xi F(h_t^j)\right] \\ &= (1-\alpha)A\frac{s_{t-1}^j}{\nu_t}\left[1-\tau_t + X^\xi F(h_t^j)\right],\end{aligned}$$

with $j = \{u, c\}$. Note that in this case the effect of a tax change is given by

$$\frac{\partial \mathcal{I}_t^j}{\partial \tau_t} = (1-\alpha)A\frac{s_{t-1}^j}{\nu_t}\left[X^\xi F'(h_t^j)\frac{\partial h_t^j}{\partial \tau_t} - 1\right].$$

Note that there are opposing effects on $\mathcal{I}_t^j$ from a tax change. The reason is that in the economic equilibrium we have a capital thinning effect that lowers wages when h is raised. Thus suppressing the overall formal labor supply increases the per-unit wage rate. To ensure that the presence of our informal sector does not make it optimal for young households to impose a large labor tax on themselves we need to ensure that

$$1 > X^\xi F'(h_t^j)\frac{\partial h_t^j}{\partial \tau_t}. \qquad (15)$$

Furthermore one would expect that as taxes are increased, income losses should at least weakly increase. With the universal system condition (15) implies

$$1 > \frac{\xi}{1+\xi},$$

which is the case for $\xi \geq 0$. Note that while the marginal effect is constant, the level I_t^u decreases with τ_t. Consequently, the universal system can have corner solutions in the politico-economic equilibrium, but the policy-platform objective function will at least be concave in τ. For the contributive case we do not have the same guarantee automatically. This is addressed explicitly by assumption 1.

B The Politico-Economic Equilibrium

B.1 Proof of Proposition 2

We start by conjecturing that the policy function $\tau^u(\cdot)$ is independent of all endogenous states (in this case s_{t-1}^u). Under this conjecture, we can substitute the economic equilibrium consumption functions into the political problem and rewrite it as follows:

$$\tau^u(\nu_t) = \arg\max_{\tau_t \in [0,1]} \omega \ln\left(1 + \frac{1-\alpha}{\alpha}\tau_t\right) + \nu_t(1+\beta)\ln(I_t^u) + e_t,$$

where e_t contains all the terms that under our conjecture are independent of the choice of τ_t. This yields the FOC for equilibrium tax

$$\frac{\omega(1-\alpha)}{\alpha + (1-\alpha)\tau_t} + \nu_t(1+\beta)\frac{\partial I_t^u/\partial \tau_t}{I_t^u} = 0, \tag{16}$$

We note that

$$\frac{\partial \ln(I_t^u)}{\partial \tau_t} = -\frac{1}{1 - \tau_t + \xi X^{1+\xi}},$$

such that we can derive the equilibrium tax rate as:

$$\tau_t^* = \frac{1}{\omega + \nu_t(1+\beta)}\left[\omega\left(1 + \xi X^{1+\xi}\right) - \frac{\alpha}{1-\alpha}\nu_t(1+\beta)\right]. \tag{17}$$

Note that if $\tau_t^* > 1$ then the solution is in the corner of $\tau_t = 1$ and if $\tau_t^* < 0$ then we have the corner solution $\tau_t = 0$. Note furthermore that this confirms our conjecture that the policy function is indeed independent of endogenous states (s_{t-1}^u). It is straightforward to verify that this function is increasing in ω and decreasing in ν_t. If there is a terminal date, say T, the political objective at such date is derived in a similar manner, but with workers living only one period. This is given by:

$$\tau^u(\nu_T) = \max_{\tau_T \in [0,1]} \omega \ln\left(c_{2,T}^u\right) + \nu_T \ln\left(c_{1,T}^u\right),$$

where $c_{2,T}^u$ follows the usual household solution formula and $c_{1,T}^u = \mathcal{I}_T^u$ from imposing $s_T = 0$ in the budget. This yields the same problem as for $t < T$ where $\beta = 0$ is imposed. It is immediate that the policy function $\tau^u(\cdot)$ is the unique Markov perfect equilibrium in the limit of the finite horizon economy.

B.2 Proof of Proposition 3

Substituting the economic equilibrium constraints in the political program and omitting all terms independent of the choice of policy, we can write an equivalent objective function for the policy choice at time t as:

$$W_t^c = \omega\Big\{h_{t-1}^c \ln\left(\eta_{t-1}^c\right)\Big\} + \nu_t\Big\{\ln\left(I_t^c\right)(1+\beta) + \ln\left(\gamma_t^c\right) + \beta\ln\left(1-\gamma_t^c\right) + \beta h_t^c \ln\left(\eta_t\right)\Big\},$$

where the labor-income function can be written as

$$\mathcal{I}_t^c = (1-\alpha)A\frac{s_{t-1}^c}{\nu_t}\left[1-\tau_t + X^{\xi}F\left(h_t^c\right)\right],$$

and the maximization of W_t^c is subject to the constraint that future policies are defined by continuation policies consistent with the current maximization, i.e. $\tau_{t+1} = \tau_{t+1}^c\left(h_t^c, \nu_{t+1}\right)$. Before we proceed to the characterization of the equilibrium policy, we define the *politico-economic equilibrium labor function* as the labor function that internalizes the effect through the continuation tax function, i.e. the h_t^c implicitly determined from:

$$h_t^c \equiv h^c\left(\tau_t, \tau^{t+1}(z_{t+1})\right), \quad h_t^c \in z_{t+1}.$$

Assumption 1 in the main text defines the labor response of this function to be in the domain

$$\frac{-1}{X^{1+\xi}(h_t^c)^{1/\xi}} < \frac{dh_t^c}{d\tau_t} \equiv \frac{\partial h_t^c}{\partial\tau_t} + \frac{\partial h_t^c}{\partial\tau_{t+1}}\frac{d\tau^{t+1}}{d\tau_t} < 0, \qquad \frac{d\tau^{t+1}}{d\tau_t} \equiv \frac{\partial\tau^{t+1}}{\partial h_t^c}\frac{\partial h_t^c}{\partial\tau_t}. \tag{A2}$$

The marginal effect on political support from a marginal change in τ_t can be summed up in three terms.

First, the marginal effect on retirees' political support from a marginal tax increase ($\mathcal{E}_{2,t}^c$):

$$\mathcal{E}_{2,t}^c = \frac{\omega(1-\alpha)}{\alpha + (1-\alpha)\frac{\tau_t}{h_{t-1}^c}}.$$

This is always positive, but smaller than the corresponding term with universal pension benefits (they are equivalent when $h_{t-1}^c = 1$).

Second, the marginal effect on workers' political support from labor-income changes induced by a marginal tax increase ($\mathcal{E}_{1,t}^{c,I}$)

$$\mathcal{E}_{1,t}^{c,I} = \nu_t \left(1+\beta\right) \frac{X^{\xi} F'\left(h_t^c\right) \dfrac{dh_t^c}{d\tau_t} - 1}{1 - \tau_t + X^{\xi} F\left(h_t^c\right)}.$$

This marginal effect may be smaller/larger than the universal marginal effect depending on parameter values. For comparison we restate the effect in a universal system:

$$\mathcal{E}_{1,t}^{u,I} = \nu_t \left(1+\beta\right) \frac{X^{\xi} F'(h_t^c) \dfrac{dh_t^u}{d\tau_t} - 1}{1 - \tau_t + X^{\xi} F(h_t^u)}.$$

The problem with comparing the two analytically is that $\mathcal{E}_{1,t}^{c,I}$ depends on the politico-economic equilibrium function $d\tau_{t+1}^c/d\tau_t$, which has to be solved for numerically.

Finally, we collect the residual marginal effects on workers' political support from a marginal increase in τ_t as

$$\mathcal{E}_{1,t}^{c,r} = \nu_t \left[\frac{dh_t^c}{d\tau_t} \left(\ln(\eta_t^c) - \chi_h\right) + \left(\eta_t^c - 1\right) \frac{d\tau_{t+1}^c/d\tau_t}{\tau_{t+1}} \chi_\tau \right],$$

where coefficients $\chi_h, \chi_\tau \geq 0$ are relatively complicated functions defined as

$$\ln(\eta_t^c) \geq \chi_h \equiv \frac{\delta_t^c \left(1 + \beta h_t^c\right) + \beta^2 \gamma_t^c (1 - h_t^c) - (1+\beta)\gamma_t^c \delta_t^c \left(1 + \beta\left(1 - h_t^c\right)\right)}{\delta_t^c(\eta_t^c)}$$

$$\chi_\tau \equiv \frac{\gamma_t^c(\eta_t^c - 1)^2 h_t^c}{(\eta_t^c)^2 (1 + (\eta_t^c - 1)(1 - h_t^c))}.$$

Under assumption 1 we can further verify that $\mathcal{E}_{1,t}^{c,r} \leq 0$.

Consider first the terminal policy function in (PEE_c). To show this we proceed as for the universal system and set up the terminal period political objective function:

$$\tau_T^c\left(h_{T-1}^c, \nu_T\right) = \max_{\tau_T \in [0,1]} \omega \left\{ h_{T-1}^c \ln\left(c_{2,T}^{c,f}\right) + \left(1 - h_{T-1}^c\right) \ln\left(c_{2,T}^{c,if}\right) \right\} + \nu_T \ln\left(c_{1,T}^c\right),$$

where $c_{1,T}^c$ is defined from the budget of the young. Plugging in the economic equilibrium functions and omitting terms not relevant for the choice of τ_T we can equivalently present the political objective function as:

$$\tau_T^c\left(h_{T-1}^c, \nu_T\right) = \max_{\tau_T \in [0,1]} \omega \ln\left(\eta_{T-1}^c\right) h_{T-1}^c + \nu_T \ln\left(\mathcal{I}_T^c\right),$$

where the terminal period's choice of labor supply is equivalent to that of the universal pension system's such that $\mathcal{I}_T^c = \mathcal{I}_T^u$. Maximizing this yields (PEE_c). It is straightforward to verify that the terminal policy is increasing in h_{T-1}^c.

For $t < T$ we can characterize the equilibrium policy by use of the marginal effect terms $\mathcal{E}$. Firstly, note that if the equilibrium policy is not in a corner then it must be the case that

$$\mathcal{E}_{2,t}^c + \mathcal{E}_{1,t}^{c,I} + \mathcal{E}_{1,t}^{c,r} = 0.$$

First note that for us to have an interior solution at τ_t^*, we know that at least in a local region around τ^*, $\sum \mathcal{E}_t^c$ must be positive and decreasing towards the crossing of zero around τ_t^* and negative and decreasing in a region thereafter. Note furthermore that $\mathcal{E}_{2,t}^c$ is always positive, implying that in an interior equilibrium we conversely have that $\mathcal{E}_{1,t}^{c,I} + \mathcal{E}_{1,t}^{c,r} \leq 0$. Finally, note that a marginal change in h_{t-1}^c or ω simply shifts $\sum \mathcal{E}_t^c$ upwards implying an increase in equilibrium tax rate. Conversely, ν_t simply enters as a proportional factor in $\mathcal{E}_{1,t}^{c,I} + \mathcal{E}_{1,t}^{c,r} \leq 0$. Thus an increase in ν_t shifts $\sum \mathcal{E}_t^c$ downwards implying a lower equilibrium tax rate.

C Contributive Versus Universal System Steady States

As shown in 4.3, given a choice among social security systems, society would always choose the universal one. Since most countries have a contributive system, we conjecture that there are welfare loses from, presumably, higher taxes in the universal system that depress capital accumulation over longer horizons. To gauge this we compare steady state political welfare across systems. We note that with productive externalities the model features endogenous growth at rate

$$\left(\frac{k_{t+1}}{k_t}\right)_j^{BGP} = \frac{(1-\alpha)A}{\nu}\delta^j \left[1 - \tau^j + X^{\xi} F(h^j)\right], \qquad j \in \{u, c\}.$$

Given different levels of saving rates, tax levels and labor supply with the two pension systems the balanced growth paths diverge in the long run. To abstract from these effects, we now eliminate productive externalities.[18]

[18]Recall they were only introduced for analytical tractability.

C.1 Economic Equilibrium without productive externalities

The results are similar to those with externalities. In the universal case we have:

$$
\begin{aligned}
h_t^u =& \left(\frac{1-\tau_t}{X^{1+\xi}}\right)^{\frac{\xi}{1+\xi}}, \\
s_t^u =& \delta^u(\tau_{t+1})\mathcal{I}_t \\
c_{1,t}^u =& \gamma^u(\tau_{t+1})\mathcal{I}_t \\
c_{2,t+1}^u =& \alpha A \eta^u(\tau_{t+1}) \left[s_t\right]^{\alpha} \left(\nu_{t+1} h_{t+1}\right)^{1-\alpha} \\
\mathcal{I}_t =& (1-\alpha) A \left(\frac{s_{t-1}}{\nu_t}\right)^{\alpha} \left[(h_t)^{1-\alpha}(1-\tau_t) + X^{\alpha\xi} F(h_t)\right],
\end{aligned}
$$

where the functions $\delta^u, \gamma^u, \eta^u$ are the same as in the main part. In the contributive case the result is also similar: Formal labor supply is the same, savings rate and consumption when young is the same (except for $\mathcal{I}_t$ part which is identical to the part above) and $c_{2,t+1}^c$ is modified in the same manner as the universal part:

$$
\begin{aligned}
h_t^c &= \left\{\frac{1-\tau_t}{X^{1+\xi}} + \beta \ln\left(1 + \frac{1-\alpha}{\alpha}\frac{\tau_{t+1}}{h_t^c}\right) \gamma^c(h_t^c, \tau_{t+1}) h_t^c \frac{1-\tau_t + X^{\xi} F(h_t^c)}{X^{1+\xi}}\right\}^{\frac{\xi}{1+\xi}}, \\
s_t^c &= \delta^c(h_t^c, \tau_{t+1})\mathcal{I}_t \\
c_{1,t}^c &= \gamma^c(h_t^c, \tau_{t+1})\mathcal{I}_t \\
c_{2,t+1}^{c,0} &= \alpha A \left[\delta^c(h_t^c, \tau_{t+1})\mathcal{I}_t\right]^{\alpha} \left(\nu_{t+1} h_{t+1}^c\right)^{1-\alpha} \\
c_{2,t+1}^{c,1} &= \alpha A \eta^c(h_t^c, \tau_{t+1}) \left[\delta^c(h_t^c, \tau_{t+1})\mathcal{I}_t\right]^{\alpha} \left(\nu_{t+1} h_{t+1}^c\right)^{1-\alpha}.
\end{aligned}
$$

Imposing steady state implies the savings rates

$$
s^j = \left(\delta^j (1-\alpha) A \nu^{-\alpha} \left[(h^j)^{1-\alpha}(1-\tau^j) + X^{\alpha\xi} F(h^j)\right]\right)^{\frac{1}{1-\alpha}}, \qquad j \in \{u, c\}.
$$

C.2 Steady State welfare without productive externalities

The steady state political welfares are given by

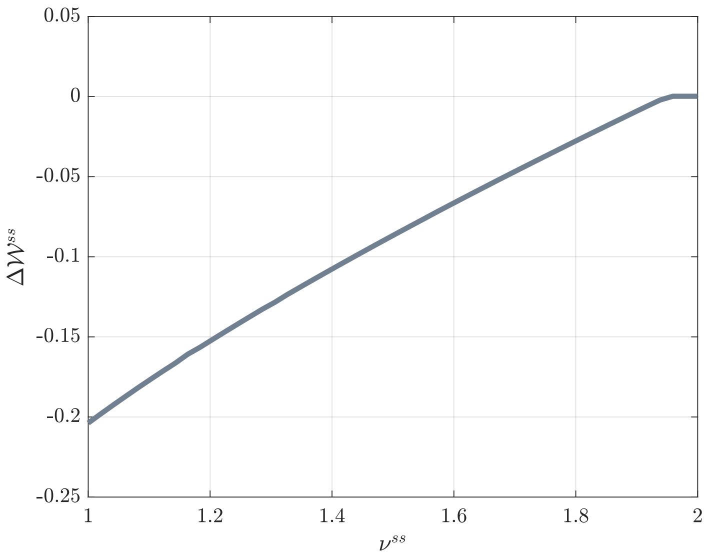
$$
\begin{aligned}
\mathcal{U}^u =& \omega \ln\left(c_2^u\right) + \nu\left[\ln\left(c_1^u\right) + \beta \ln\left(c_2^u\right)\right] \\
\mathcal{U}^c =& \omega\left[h^c \ln\left(c_2^{c,1}\right) + (1-h^c)\ln\left(c_2^{c,0}\right)\right] + \nu\left[\ln\left(c_1^c\right) + \beta\left[h^c \ln\left(c_2^{c,1}\right) + (1-h^c)\ln\left(c_2^{c,0}\right)\right]\right]
\end{aligned}
$$

The steady state difference can be reduced to

$$\Delta\mathcal{W} = \nu\left\{\Delta\ln(\gamma) + \frac{\alpha}{1-\alpha}\Delta\ln(\delta) + \frac{1}{1-\alpha}\Delta\ln\left[h^{1-\alpha}(1-\tau) + X^{\alpha\xi}F(h)\right]\right\}$$
$$+(\omega+\beta\nu)\left\{(1-\alpha)\Delta\ln(h) + \ln(\eta^u) - h^c\ln(\eta^c) + \frac{\alpha}{1-\alpha}\left[\Delta\ln(\delta) + \Delta\ln\left[h^{1-\alpha}(1-\tau) + X^{\alpha\xi}F(h)\right]\right]\right\},$$

where we have used the notation $\Delta f(x) = f(x^u) - f(x^c)$. For our calibration we show below that for all $\nu \in [1,2]$ the contributive system offers higher welfare in steady state. Figure 3 shows that $\Delta\mathcal{W} \leq 0$ and increasing in ν.

Figure 3: Difference in steady state (political) welfare



D Remarks on calibration

The calibration can either be *static* or *dynamic*. In general the calibration is based on the following variables and targets:

i. α is determined by the labor share of income that is analytically given by $1-\alpha$.

ii. Given α the interest rate is determined by $r_t = \alpha A$ (this A).

iii. The private savings rate δ_t^c is a function of parameters (β, α, A) and endogenous variables h_t^c, τ_{t+1}^c. With α and A as given and a target for the level of private savings in 2010, we can write β on the form

$$\beta \equiv f_{\beta}\left(h_{2010}, \tau^{c}_{2040}; \alpha, A, \delta_{2010}\right).$$

where τ_{2040} is the τ_{t+1} level from the generation that is young in 2010's perspective with 30 year cohorts.

iv. Since pension coverage was 68% and it increases to 91% after the amnesty we take the latter value as the maximum coverage in a universal system and normalize coverage under a contributive system accordingly

$$h^{c}_{2010} = \frac{68}{91}.$$

The equilibrium equation for labor supply involves parameters ξ, X (from the informal labor production function $F(h)$) as well as endogenous variable τ_t. With ξ calibrated to reflect an elasticity of labor supply of around 0.35 this condition implicitly gives X as a function of τ_t:

$$X = f_X\left(h_{2010}, \tau_{2010}; \xi\right).$$

v. Finally the pension tax level τ_t is targeted to reflect a rate of 27.1% in 2010. The equilibrium value of τ_t can only be found by solving the entire model, and simulating the equilibrium path forward. This equilibrium condition then involves all parameters and endogenous variables of the model.

In the *static* calibration we impose a steady state assumption for the year 2010. This simplifies the calibration quite a lot. With a steady state assumption we can proceed as follows:

i. Calibrate α, A from static requirements i.-ii. above.

ii. Given a steady state assumption solve simultaneously for the equilibrium level of β, X and h_{2010}. Note that the functions f_β and f_X both include the three variables in question here. Imposing $\tau_{2040} = \tau_{2010}$ the condition for equilibrium on the labor market further gives an equation in the same three variables.

iii. With $(\alpha, A, \beta, \xi, X)$ calibrated the only variable left to determine is ω. This is calibrated to ensure that the tax on pensions is on 27.1% in 2010. The PEE tax rate can only be solved for, by identifying the PEE tax functions and then simulate a path forward. Thus to calibrate the level of ω and target τ_{2010}, we follow the general outline of the nested fixed point algorithm (Rust, 1987). In an inner loop an algorithm solves for the PEE tax rate $\tau_{2010}^{simulated}$ given the level of ω, using the approach outlined in section 5. An outer loop use searches the parameter space for a value of ω that solves $\tau_{2010}^{simulated} = 27.1\%$.

In the *dynamic* calibration step ii. cannot be carried out separately from step iii.: Without the steady state assumption we cannot solve for (β, X, h_{2010}) independent of ω. Thus, in this case, parameters (β, X, ω) is solved for simultaneously in the way ω is solved for in the static case: Given a candidate of parameter values (β, X, ω) the model is solved and the PEE is simulated forward. An outer algorithm searches the parameter space for candidates where simulated values of $\tau_{2010}, h_{2010}, \delta_{2010}$ reach their respective targets.

The difference between the two calibration schemes are minor. There are however some computational differences. To give an overview:

- It takes roughly 25-30 seconds to solve for all policy functions and create interpolation functions that yield $(h_{t-1}, \tau_{t-1}, \tau_t)$ in PEE.
- It then takes 15 seconds to calibrate the model with the steady state assumption.
- The dynamic calibration is far slower with a run time of roughly 2-3 minutes.

Table 2: Calibrated parameters with static and dynamic method

Parameter	Static calibration	Dynamic calibration
β	0.3219	0.3315
X	2.4872	2.4841
ω	1.1731	1.1752

E Numerical Methods

This appendix deals with the numerical methods that can be applied when solving for the PEE with contributive pensions. The appendix discusses the difference between the *infeasible general problem*, the steady state infinite horizon assumption and the finite horizon version. Next, we discuss the merits of using either a standard Value Function Iteration (VFI) approach compared to an Endogenous Gridpoint Method (EGM) approach. Finally, the various solution methods are simulated to illustrate efficiency as well as differences in outcomes.

The conclusion from this appendix is that the four different solution methods listed above are very similar in terms of accuracy, but differ significantly when it comes to computational speed. In particular, this appendix shows that the finite horizon EGM approach is between 3 and 46 times faster than the competing approaches.

Table 3: Comparison of methods identifying the PEE

Solution method:	Time
Finite Horizon, EGM	26s
Infinite Horizon, Time-dependent, EGM	75s
Infinite Horizon, Time-dependent, VFI	119s
Infinite Horizon, SS-approximation, EGM	>600s
Infinite Horizon, SS-approximation, VFI	>1194s

Note: The infinite horizon solution with time-dependent policies that uses VFI, only uses the VFI to determine the steady state policy; then it iterates backwards through time using an EGM-type approach. The two infinite horizon steady state approximations are timed at '>', as the time only involves identifying the policy function, and not simulating the equilibrium path. As the effect of simulating forward is minuscule compared to the identification of the policy functions, it is also fair to replace '>' with '≈'.

E.1 The Infinite Horizon Version

E.1.1 The General Case vs. Steady State Assumption

Consider the infinite horizon version of the model. As outlined in section 3 the policymaker maximizes the *political aggregator* function, here repeated for convenience:

$$\mathcal{W}(z_t, \tau_t; \tau^{t+1}(z_{t+1})) = \omega \mathcal{O}(z_t, \tau_t) + \nu_t \mathcal{Y}\left(z_t, \tau_t; \tau^{t+1}(z_{t+1})\right). \tag{18}$$

Here z_t denotes the vector of relevant states for the political decision at time t. In the general infinite horizon case this consists of a single *endogenous* state variable (h_{t-1}), as well as the *entire* future path of population weights, that is:[19]

$$z_t = \Big(h_{t-1},\ \ \{\nu_\iota\}_{\iota \geq t}\Big).$$

With an infinite-dimensional state space this general infinite horizon problem is infeasible to solve without additional assumptions. The conventional way to get around this obstacle is to assume that the model converges on a steady state after T^{ss} periods. In this case the model has a recursive time-autonomous structure (for $t \geq T^{ss}$), in which case the identification of the policy and continuation policy functions is identified by a fixed-point requirement (see definition 2).

One of the traditional ways of solving for the politico-economic equilibrium in infinite horizon models is to identify the steady state time-autonomous policy function as a function of the simplified 2-dimensional steady state state space $z = (h_{-1},\ \nu)$, (Krusell, Quadrini and Ríos-Rull, 1997). We note that using the steady state policy function outside of steady state should however only be considered an approximation of the true *time-dependent* structure.

E.1.2 The Steady State Policy Function: Value Function Iteration

The standard VFI approach is well documented. However, a brief outline of the application in this setting facilitates a comparison with the EGM approach and finite horizon version discussed in the subsequent sections. Before we outline the solution algorithm, we consider the maximization problem in (18). The complication for the numerical solution is that there is no closed form representation of $\mathcal{W}$; thus the problem faced by the policymaker is for our purposes here more accurately presented by the constrained maximization problem

$$\tau = \arg\max_{\tau'} \left\{ W^c\left(h_{-1}, \nu, \tau', h, \tau_{+1}\right) \quad \text{s.t. } \tau_{+1} = \tau^{+1}(h, \nu) \quad \text{and } h = h^c(\tau', \tau_{+1}) \right\}. \quad (20)$$

[19] To see why this is the case, conjecture that the state space consists of (h_{t-1}, ν_t) and denote the corresponding continuation policy $\tilde{\tau}^t(\cdot)$. This entails that the relevant state space for the continuation policy is given by (h_t, ν_{t+1}). In this case ν_{t+1} becomes a relevant state for the political decision at time t **unless** the political decision defined by

$$\tau = \arg\max_{\tau'} \left(\omega \mathcal{O}(h_{t-1}, \nu_t, \tau') + \nu_t \mathcal{Y}\left(h_{t-1}, \tau'; \tilde{\tau}^{t+1}(h_t, \nu_{t+1})\right)\right) \quad (19)$$

is independent of ν_{t+1}. If ν_{t+1} is a relevant state at time t, then ν_{t+2} is a relevant state at time $t+1$. Following this argument all ν_ι, $\iota \geq t$ becomes a relevant state in the general case.

The mapping $W^c(\cdot)$ is given by equation (13) and the mapping $h^c(\cdot)$ is the economic equilibrium condition defined by equation (EE_c1). Importantly, the economic equilibrium condition only *implicitly* defines h as a function of τ, τ_{+1}, whereas the condition can be rewritten to yield an *analytical* solution for τ:

$$\tau = 1 - \frac{X^{1+\xi}}{1+\beta ln\left(\eta^c\left(h,\tau_{+1}\right)\right)h}\left[h^{\frac{1+\xi}{\xi}} - \beta ln\left(\eta^c\left(h,\tau_{+1}\right)\right)\gamma^c\left(h,\tau_{+1}\right)h\frac{F(h)}{X}\right]. \qquad (21)$$

With this in place define the ordered grids over the relevant state variables:

$$\mathcal{G}_h \equiv \left\{h_{-1}^1, h_{-1}^2, \ \dots\ , h_{-1}^{N_h}\right\}, \qquad h_{-1}^1 = h^c(1,0), \qquad h_{-1}^{N_h} = h^c(0,1).$$
$$\mathcal{G}_\nu \equiv \left\{\nu^1, \nu^2, \ \dots\ , \nu^{N_\nu}\right\}.$$

The VFI approach now proceeds as follows:

i. Let $n = 0$ and define an initial guess of a policy function $\tilde{\tau}^0(h_{-1}, \nu)$.

ii. Given $\tilde{\tau}^n$, for each pair of states (i,j) on the grids $(\mathcal{G}_h, \mathcal{G}_\nu)$, solve the maximization problem

$$\tau^{i,j} = \arg\max_{\tau'}\left\{W^c\left(h_{-1}^i, \nu^j, \tau', h, \tau_{+1}\right) \quad \text{s.t. } \tau_{+1} = \tilde{\tau}^n(h, \nu^j) \quad \text{and } h = h^c(\tau', \tau_{+1})\right\}.$$

Let $\mathcal{G}_\tau^n$ define the grid of solutions from the policy guess $\tilde{\tau}^n$ on the grids $(\mathcal{G}_h, \mathcal{G}_\nu)$.

iii. Update the policy guess $\tilde{\tau}^{n+1}(h_{-1}, \nu)$ using some functional approximation approach, e.g. interpolation.

iv. Define a tolerance level $\Delta > 0$. If $\sup_{i,j}\left|\tilde{\tau}^{n+1}(h_{-1}^i, \nu^j) - \tilde{\tau}^n(h_{-1}^i, \nu^j)\right| > \Delta$ then $n \rightsquigarrow n+1$ and repeat steps ii.-iv.

The computational cost of the VFI approach is the constrained maximization problem that is carried out in the innermost loop (in step ii.): Thus it is repeated for all (n, i, j).

E.1.3 The Steady State Policy Function: An EGM-type Approach

The idea of the EGM-type approach applied here, is to replace the constrained maximization problem with an unconstrained one that circumvents the root-finding operation of the labor equilibrium. To do this in the infinite horizon model, we have to perform two steps. First, we pre-approximate the labor equilibrium function $h^c(\tau, \tau_{+1})$ on a grid of

(τ, τ_{+1}). Secondly, we define the policy function on an endogenous grid of τ_{-1} instead of the exogenous grid of h_{-1}. This allows us to exploit the analytical mapping from h, τ_{t+1} to τ, (21), and thus circumvent a root-finding operation for each (n, i, j).

The EGM-type approach now proceeds as follows:

i. Approximate the labor equilibrium function $h = h^c(\tau, \tau_{+1})$ on grids of (τ, τ_{+1}). Denote the approximate function $h^a(\tau, \tau_{+1})$.

ii. Let $n = 0$ and define an initial policy guess $\tilde{\tau}^n(\tau_{-1}, \nu)$.

iii. Given $\tilde{\tau}^n$, for each pair of states (i, j) on the (exogenous) grids $(\mathcal{G}_h, \mathcal{G}_\nu)$, solve the *unconstrained* maximization problem

$$\tau^{i,j} = \arg\max_{\tau'} W^c \left(h^i_{-1}, \nu^j, \tau', \underbrace{h^a\left(\tau', \tilde{\tau}^n(\tau', \nu^j)\right)}_{\approx h}, \underbrace{\tilde{\tau}^n(\tau', \nu^j)}_{=\tau_{+1}} \right).$$

Note that in place of h we use the approximate function h^a that through $\tilde{\tau}^n$ only depends on the current tax rate (τ') and the exogenous state ν^j.

iv. From the exogenous grid of h_{-1} back out the corresponding value of τ_{-1} using (21) to obtain the *endogenous* state grid $\mathcal{G}^n_{\tau_{-1}} \equiv \left\{\tau^{1,n}_{-1}, \tau^{2,n}_{-1}, ..., \tau^{N^n_\tau,n}_{-1}\right\}$. Let $\mathcal{G}^n_\tau$ define the grid of solutions.

v. Update the policy function $\tilde{\tau}^{n+1}(\tau_{-1}, \nu)$ using some functional approximation approach, e.g. interpolation.

vi. Define a tolerance level $\Delta > 0$. If $\sup_{i,j} \left|\tilde{\tau}^{n+1}(h^i_{-1}, \nu^j) - \tilde{\tau}^n(h^i_{-1}, \nu^j)\right| > \Delta$ then $n \rightsquigarrow n + 1$ and repeat steps iii.-vi.

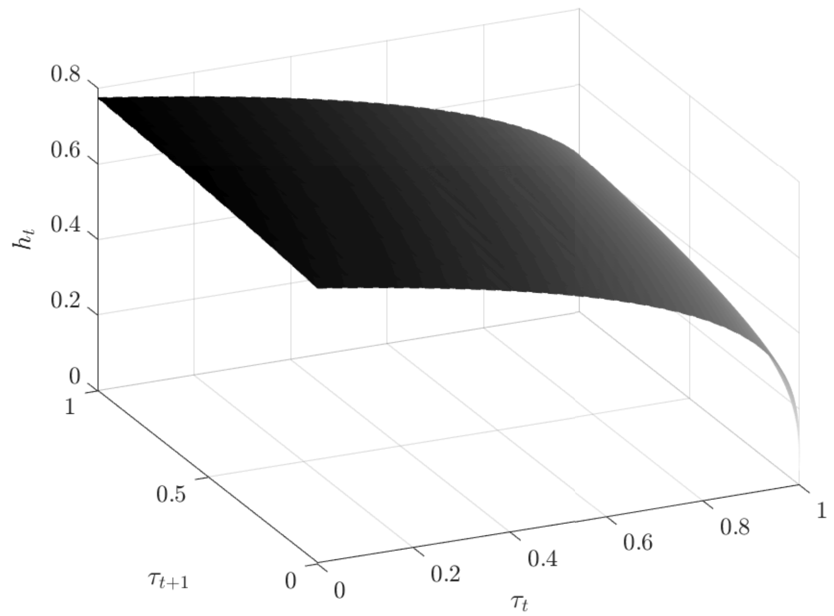
The EGM-type approach has the added computational cost of having to compute an approximation for the labor function in step i., whereas the gain comes from a simpler unconstrained maximization problem in step iii.. As discussed later in section E.1.4 the EGM approach with gradient-free solver is roughly three times faster than the standard VFI approach in the infinite horizon case.

Finally, we can set up a simple hybrid version of the VFI and the EGM approach to circumvent the computationally costly step *i.* of approximating the labor supply function. In the first round of the algorithm we use the VFI approach to compute the three grids $(\mathcal{G}^0_\tau, \mathcal{G}_h, \mathcal{G}_\nu)$. Using the grids $(\mathcal{G}^0_\tau, \mathcal{G}_h)$ and the analytical solution for τ_{-1} in equation (21) we establish the endogenous grid $\mathcal{G}^0_{\tau_{-1}}$. Using this we can establish an approximation of the policy function $\tau(\tau_{-1}, \nu)$, as well as the PEE labor equilibrium function using the grids of $(\mathcal{G}_h, \mathcal{G}_{\tau_{-1}})$.

E.1.4 Comparison of computational speed

To illustrate the difference between the two approaches we consider the following setup: For the EGM approach we define h^I as an interpolation over a relatively dense grid with 250^2 nodes of (τ, τ_{+1}). Figure 4 illustrates the resulting labor function.

Figure 4: The economic equilibrium labor supply



Parameter values are taken from the calibration to Argentina. For the VFI and PEE-EGM approaches we initialize both with a policy function on the form:

$$\tau^0_{VFI} = \tau^0_{PEE-EGM} = 0.5.$$

With the economic labor supply function in place, define the exogenous grids $\mathcal{G}_h \equiv \{h_1, ..., h_{N_h}\}$ and $\mathcal{G}_\nu \equiv \{\nu_1, ..., \nu_{N_\nu}\}$. We set $N_h = 250$ as with earlier grids and vary N_ν in the applications below. We compare four solution methods to the infinite horizon steady state policy function: VFI, EGM, EGM-GF, and VFI/EGM (hybrid). The EGM-GF method is a solution that utilizes a gradient-free optimization algorithm.[20]

[20]This is only implemented with the EGM approach, as it requires the object to be a scalar-function, which is only the case with EGM.

Table 4: Comparison of solution methods, infinite horizon

# Nodes	n	Solution time				$\min_n \sup_{i,j} \lvert\tilde{\tau}^{n+1}_{i,j} - \tilde{\tau}^{n}_{i,j}\rvert$			
		VFI	*EGM*	*EGM-GF*	*VFI/ EGM*	*VFI*	*EGM*	*EGM-GF*	*VFI/ EGM*
500	5	71s	52s	23s	27s	2.1e-6	1.7e-6	1.2e-6	1.0e-5
12500	5	19.9m	16.1m	9.3m	10.0m	2.1e-6	9.9e-6	1.2e-6	1.0e-5
25000	5	38.5m	31.1m	17.8m	20.0m	2.1e-6	8.3e-6	1.8e-6	1.1e-5

Note that for the EGM and EGM-GF the labor equilibrium function has been approximated outside the solution here. With 250^2 grid nodes this approximation takes roughly 300 seconds (5 minutes) to run.

As table 4 shows the fastest approach is *EGM-GF* followed closely by the hybrid approach *VFI/EGM*. While the *EGM-GF* approach uses a pre-computed labor equilibrium function, the hybrid approach does not. The right part of table 4 illustrates how far the algorithms are from convergence, by measuring the largest difference between two policy functions at n and $n+1$. Once again the *EGM-GF* approach performs best. We note furthermore that this measure favors the VFI method, as the other three methods iterate through n on *endogenous* grids. Thus when we measure the difference between n and $n+1$, we can use the exact gridpoints that the policy in $n+1$ is defined on, but the value at n will be an interpolated value adding an approximation error.[21]

E.1.5 Policy Functions out of Steady State

Assume that we have identified the steady state policy function $\tau^{ss}(h_{-1}, \nu)$, and let T^{ss} denote the time from which ν_t is in steady state. For $t < T^{ss}$ the structure of the policy maker's problem is no longer time-autonomous. At time $T^{ss} - 1$ however, the relevant continuation policy is exactly the identified steady state function. The implication is that for all $t < T^{ss}$ the relevant policy function is time-dependent and does not require solving a fixed-point problem; we can iterate backwards through time identifying policy functions $\tau^c_t(h_{t-1}, \nu_t)$ by simply maximizing the objective in (18).

As an alternative, one can approximate the out-of-steady-state policies ($t < T^{ss}$) using the steady state function. Figure 5 illustrates that in our setting the steady state

[21] In the computational literature a standard way of assessing the accuracy of approximation methods is to derive the so-called Euler-errors (Judd, 1992; Barillas and Fernández-Villaverde, 2007). As we do not have an analytical first order condition for the optimal choice of the tax rate, there is not a straightforward way of doing this in our setup. An alternative measure could be to compute deviations from the labor equilibrium condition; however as this constraint enters directly in the numerical problem, this level of error can be controlled by an option in the solver (tolerance level).

approximation performs very well: The largest deviation between the *true* infinite horizon solution and the steady state approximation is roughly 0.15%.

Figure 5: Comparison of time-dependent policies and steady state approximation

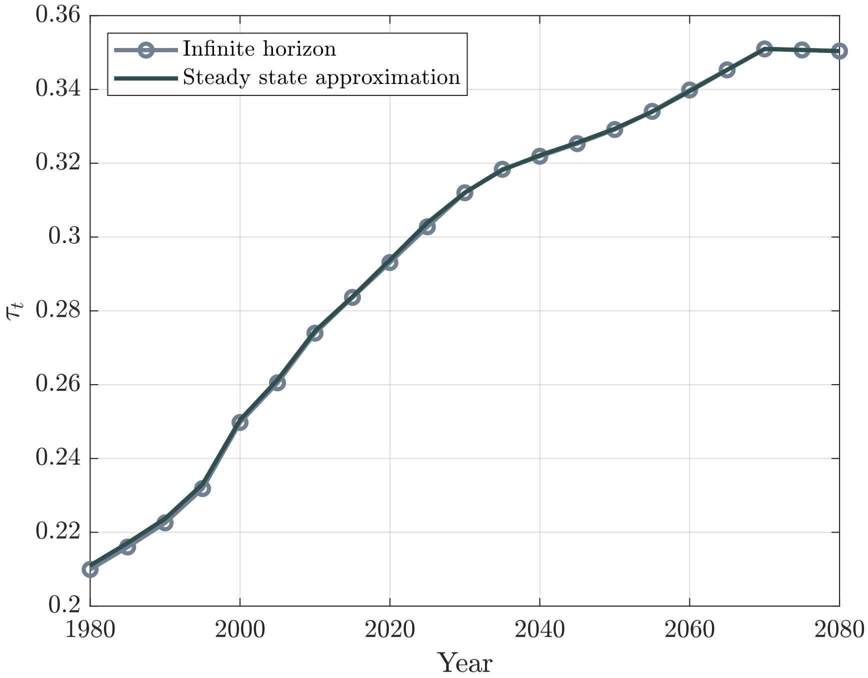


Figure 5 illustrates the validity of pursuing a steady state approximation. The approximation is however only desirable if the time-dimension is prohibitively large compared to the variation in ν_t. If the steady state value of ν is known, the time-dependent policy function solution involves: (1) Solving the fixed point problem of the steady state solution on a grid of h_{-1} (no ν-grid) and (2) maximizing the political aggregator function in (18) on a grid of h_{-1}, for each $t < T^{ss}$. In comparison the steady state approximation approach involves solving for the fixed point problem on a grid of (h_{-1}, ν). As the fixed-point problem is significantly slower to solve than a simple maximization problem, the relevant grid of ν has to be *much* smaller than the number of time-dependent policy function T^{ss} for the approximation to be faster. Letting the number of gridpoints for ν be the same as the number of policy functions outside steady state T^{ss} (25 in our case), the steady state approximation approach was roughly 8 times slower than the true time-dependent solution approach.

E.2 The Finite Horizon Version

Consider the finite horizon version of the model. In this case the terminal period policy (T) is defined analytically, here repeated for convenience:

$$\tau_T^c\left(h_{T-1}^c, \nu_T\right) = \min\left\{1,\ \max\left\{0,\ \frac{1}{\omega + \nu_T/h_{T-1}^c}\left[\omega\left(1+\xi X^{1+\xi}\right) - \frac{\alpha}{1-\alpha}\nu_T\right]\right\}\right\}.$$

In this case we can solve the model as follows:

i. Create an exogenous grid of the state $\mathcal{G}_h \equiv \{h_1, ..., h_{N_h}\}$. Given the exogenous value of ν_T this solves for the equilibrium τ_T on each node on the grid using (PEE_c). Denote the corresponding grid of solutions $\mathcal{G}_\tau^T$.

ii. Given the solutions for (τ_T, h_{T-1}, ν_T) back out an endogenous grid of τ_{T-1} using the economic labor equilibrium in equation (21). Denote the endogenous grid $\mathcal{G}_{\tau_{-1}}^T$.

iii. Define the terminal period policy function (τ_T^c) as the interpolation approximation over the grid of τ_{T-1}. Define the PEE labor equilibrium function ($\mathbf{h}_{T-1}^c$) as the interpolation approximation over the grid of τ_{T-1}.

   For $t < T$:

iv. On the exogenous grid of the state ($\mathcal{G}_h$) the political objective function is on node i given by

$$\tau_t^i = \underset{\tau_t \in [0,1]}{\arg\max}\, \mathcal{W}\left(h_{t-1}^i, \nu_t, \tau_t, \mathbf{h}^c(\tau_t), \tau^c(\tau_t)\right), \tag{22}$$

   where $h_t = \mathbf{h}^c(\tau_t)$ and $\tau_{t+1} = \tau^c(\tau_t)$ are the approximate solution functions from $t+1$.

v. Given the solution for (τ_t, h_{t-1}, ν_t) back out an endogenous grid of τ_{t-1} using the economic equilibrium in equation (21). Denote the endogenous grid $\mathcal{G}_{\tau_{-1}}^t$.

vi. Define the period t policy function (τ_t^c) as the interpolation approximation over the grids $(\mathcal{G}_\tau^t, \mathcal{G}_{\tau_{-1}}^t)$. Define the PEE labor equilibrium function ($\mathbf{h}_{t-1}^c$) as the interpolation approximation over over the grids $(\mathcal{G}_h, \mathcal{G}_{\tau_{-1}}^t)$.

Compared to the infinite horizon model, the finite horizon model solves steps *i.-iii.* instead of the functional fixed-point problem in E.1. Not only the steps *i.-iii.* in the finite horizon solution do not involve any fixed-point requirement, they are even solved analytically without any optimization or root-finding operations.

Furthermore, as discussed in the previous section, we could speed up the solution of the steady state infinite horizon by applying an EGM approach instead of the standard VFI. However, to use this EGM like approach in the infinite horizon, we either have to approximate the labor equilibrium function initially or run the standard VFI approach in the first iteration of the functional fixed-point problem. In the finite horizon version we circumvent these issues as well, as the terminal policy function and the terminal period PEE labor function are both solved analytically.

Finally, figure 6 illustrates that besides being significantly faster, the finite horizon version is virtually identical to the infinite horizon version (differing, as expected but only slightly, for t close to T).

Figure 6: Comparison of finite and infinite horizon models