Resolution of Financial Crises*

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Abstract

A financial crisis creates substantial wealth losses. How these losses are allocated determines the magnitude of the crisis and the path to recovery. We study how institutions and technological factors that shape default and debt restructuring decisions affect the amplification and persistence of aggregate shocks. For sufficiently large shocks, agents renegotiate. This limits the losses borne by borrowers, dampening the initial output loss. The range of shocks that trigger renegotiation is decreasing in repossession costs and increasing in default costs, if the latter are public information. Private information about default costs leads to "V-shaped" recoveries: default depresses output on impact, but by shielding borrowers' net worth facilitates the recovery. The model is consistent with evidence from real estate markets in the U.S. during the Great Recession, and with features of the crises of Japan and South East Asia in the 1990s.

JEL codes: E32, E44, G01.

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1 Introduction

Financial crises often are preceded by asset price booms and increased borrowing, typically against appreciating assets. Once the boom reverses and prices collapse, the allocation of the ensuing financial losses between debtors and creditors determines of the magnitude of the crisis and its path to recovery. In some economies, such as Japan in the early 1990s, the adjustment entails a long-drawn process of corporate debt repayments with few breakdowns or restructurings of large business firms. In others, such as the South East Asian countries in the late 1990s, the economy suffers an initial period of significant output losses, characterized by debt restructurings and transfers of property and control, followed by swift growth − a "V-shaped" recovery.

In this paper, we build a tractable framework to study the resolution of financial crises. Our analysis emphasizes how institutions and technological factors determine the extent of socially wasteful default costs and the distribution of financial losses among agents. Factors such as bankruptcy law, the ease of shutting down and starting businesses, and the opacity of lending relationships (i.e. what lenders know about borrowers in distress), play a fundamental role.[1]

The baseline model builds on the seminal contribution by Kiyotaki and Moore (1997) − henceforth KM. Like KM, we study an economy where shocks depress the price of assets, leaving highly levered entrepreneurs underwater. Unlike KM, who assume that entrepreneurs honor their debts ex-post, we allow for default and bargaining between creditors and debtors. More precisely, we assume that entrepreneurs can default, keeping their output but paying a cost to do so. Financiers, i.e. their lenders, can repossess the collateralized asset at a cost. To avoid these social losses, entrepreneurs and financiers may bargain, with financiers offering a haircut on the entrepreneurs' outstanding debt.

Adding this salient real world phenomenon to KM has important implications. Indeed, our results imply that the institutional environment and technological considerations that shape default and debt restructuring decisions are critical for the amplification and persistence of macroeconomic shocks. When shocks are small, the threat of default is not credible. Entrepreneurs honor existing debts, which depresses the demand for capital and leads to a collapse in asset prices. The response of the economy thus is the same as in the original KM model. By contrast, when shocks are large, the threat of default is credible, which triggers a renegotiation. Thus, entrepreneurs manage to extract haircuts from financiers, cutting their financial losses. This cushions the reduction in capital demand and dampens the decrease

[1] In the United States there is considerable variation across states in the homestead exemption in the personal bankruptcy procedures under Chapter 7. States also differ in whether foreclosed sales should take place through courts or not. These sources of variation in the data are useful to test our model.

in asset prices.

How large do shocks have to be to trigger debt renegotiations? We show this depends on who bears the financial losses. When financiers' repossession costs are low, default costs are high, or entrepreneurs have little bargaining power, entrepreneurs bear the lion's share of financial losses. As a result, only large shocks trigger renegotiation. In addition, when agents renegotiate, bargained haircuts are smaller, leading to a larger drop in capital and asset prices. Importantly, however, while the allocation of financial losses determines the size of the initial output drop, it is irrelevant for the speed of the ensuing recovery. In this sense, our model shares with the original KM analysis the ability to rationalize the sluggish recovery experienced by some countries, e.g. Japan in 1990s.

In the original KM analysis, only productivity shocks are considered. Yet, financial crises often resemble the consequences of preference shocks that affect the supply of credit and have a direct impact on asset prices. We consider a temporary shock that reduces lenders' discount factor, which in equilibrium increases the interest rate. This shock has two effects on entrepreneurs. By reducing their required downpayment, it allows them to increase leverage. But, by lowering asset prices, it also generates capital losses. We show that the net effect on entrepreneurs' asset demand depends on their incentive to default. When default is credible, renegotiation dampens their capital losses, and their asset demand increases. Otherwise, their demand falls.

In our baseline model default never occurs in equilibrium: financiers and entrepreneurs avoid socially wasteful outcomes. In reality, debtors have private information about their businesses, and it is challenging for creditors to assess default costs. This observation motivates an extension of our model, where entrepreneurs' default costs are private information. Financiers only know the distribution of these costs in the population. Thus, in equilibrium, they propose a haircut accepted only by entrepreneurs with relatively high default costs. The remaining entrepreneurs default, which creates substantial output losses.

Our main result in the private information extension of the model is that asymmetric information leads to V-shaped recoveries. On the one hand, default creates inefficient output losses in the period of the shock, i.e. the crisis is deeper than in the economy with observable default costs. On the other hand, asymmetric information prevents financiers from extracting rents from high default costs entrepreneurs. These entrepreneurs thus suffer from a smaller drop in their net worth. As a result, their capital stock falls less under asymmetric information, accelerating the recovery after the shock.

We then study how financiers' repossession costs and entrepreneurs' default costs interact with asymmetric information. When repossession costs are high, financiers offer substantial haircuts, few entrepreneurs default, and the economy recovers fast. By contrast, the effect of (potential) default costs is more nuanced. The key factors play at the extensive margin, i.e. they affect who defaults. If the set of defaulting agents is fixed, and default costs go up, then output falls more on impact due to higher effective default costs. It also recovers more slowly, as entrepreneurs obtain smaller haircuts. However, financiers may find it optimal to affect entrepreneurs' incentive to default. Perhaps surprisingly, we construct examples where higher *potential* default costs may translate into lower *effective* default costs and more sizeable haircuts.

An important assumption of the model is that the ex-post resolution of debt crises does not affect the ex-ante behavior of agents. We believe this is a reasonable approximation of behavior in credit markets for rare events such as financial crises. For example, in the credit boom before the Great Recession, lenders paid little attention to borrowers' repayment capacity. Mian et al. (2015) show that in the late 1990s and early 2000s lenders did not differentiate lending based on states' foreclosure requirements.[2] In commercial real estate markets debt was often issued with minimum covenants, and commercial real estate had low risk premia relative to other assets. These facts point to lenders assigning a very low probability to states of the world in which foreclosure requirements and covenants would be important.[3]

Our model can rationalize developments in real estate markets in the United States during the Great Recession, and their implications for the rest of the economy. In particular, we can explain the findings of Mian et al. (2015) and Agarwal et al. (2017), namely the causal effect of foreclosures on economic outcomes and the impact of the 2009 Home Affordable Modification Program. It can also shed light on the contrasting experiences of countries after financial crises. Japan in the early 1990s is a well-known example of a long-drawn balance sheet recession. The South East Asian economies in the late 1990s are examples of sharp, but temporary, contractions. In the aftermath of the Great Recession, the experiences of Iceland and Ireland, on the one hand, and Spain and Portugal, on the other, offer a similar contrast.

Our work contributes to the theoretical literature on the balance sheet channel, going back to the seminal work of Bernanke et al. (1999), Carlstrom and Fuerst (1997), and Kiyotaki and Moore (1997) and, more recently, the work of Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013), among others. This strand of work stresses how the concentration of aggregate risk in one sector of the economy leads to significant amplification of shocks

[2]They show such a differential lending behavior was seen in the early 1990s, consistent with findings by Pence (2006), who documents 3% to 7% smaller mortgages in states with a judicial foreclosure requirement in the mid 1990s, as expected given the higher foreclosure costs in these states.

[3]Similarly, all eurozone countries paid roughly the same interest rate on their public debts before the Great Recession.

via their effect on balance sheets. A critique of this channel is that it would disappear if agents were allowed to write contracts contingent on the aggregate state of the economy (Krishnamurthy, 2003 and Di Tella, 2017). This motivated papers to explain why insurance contracts may not be available (e.g. Cooley et al., 2004, Krishnamurthy, 2003) or why agents may optimally decide to become exposed to aggregate risk (e.g. Asriyan, 2015, Di Tella, 2017). By contrast, our paper does not seek to explain balance sheets from an ex-ante perspective. Rather, we ask how the possibility of default and bargaining ex-post affect the depth and posterior recovery of a financial crisis. We derive new results characterizing the evolution of the macroeconomy ex-post as a function of the size of the shock, the institutional and technological background, and the observability of default costs.

Our model is also related to the literature on the limited enforceability of debt contracts, allowing for strategic default. Cooley et al. (2004) assume lending can take the form of long term state-contingent debt contracts, borrowers can divert capital, and default is costly. They solve for the optimal dynamic contract that is self-enforceable and find that the equilibrium features amplification. Jermann and Quadrini (2012) also allow borrowers to default and derive borrowing constraints by assuming that lenders can recover the collateral with an exogenous probability (otherwise, recovery is zero). They interpret this time-varying probability as "financial shocks" and find that they can explain a large share of observed dynamics of real and financial variables. These two papers abstract from the effect of (endogenous) asset prices on borrowing constraints, while in our model, as in KM, it is precisely this variable that drives results. Furthermore, we also allow for financial shocks as we consider a temporary increase in the discount factor of lenders (and thus in the equilibrium interest rate).[4]

The paper is organized as follows. Section 2 presents the basic framework, which introduces default costs and renegotiation into KM's model. Section 3 develops an extension with asymmetric information about default costs that rationalizes V-shaped recoveries. Section 4 discusses how our model can be used to interpret existing empirical findings in the context of real estate markets in the United States during the Great Recession, and the contrasting experiences of Japan and South East Asia in the 1990s. Section 5 concludes. Appendix A contains all proofs and detailed derivations. Appendix B describes the parametrization and calibration used to create the figures.

[4]Other recent contributions of the effect of financial shocks are Christiano et al. (2010), Del Negro et al. (2017), and Liu et al. (2013).

2 Baseline Model

We are interested in studying how the possibility of renegotiation shapes the aftermath of a financial crisis. To highlight the novel features of our analysis, we build on the work of KM, a renowned model of a financial crisis. To ensure clarity, we purposefully deviate from this framework as little as possible.

2.1 Setup

There are two sets of agents: entrepreneurs and financiers, each with measure 1.[5] We use plain notation for entrepreneurs and $'$ for financiers. Both are risk neutral and maximize their utility, given respectively by

$$\sum_{t=0}^{\infty} \beta^t x_t \qquad \text{and} \qquad x_0 + (1-\epsilon) \sum_{t=1}^{\infty} \beta'^t x'_t,$$

where x_t and x'_t denote their respective consumptions, $0 < \beta < \beta' < 1$ are their respective discount factors and $\epsilon \in [0, 1 - \beta/\beta')$ is a discount factor "shock" in the first period.[6] These assumptions imply that entrepreneurs and financiers are, respectively, borrowers and lenders in equilibrium.

There is a fixed aggregate endowment of a productive asset, or capital, $\bar{K}$. Capital is the only factor of production and creates output with a one-period lag. Agents have access to different technologies. Entrepreneurs are endowed with a linear production technology,

$$y_{t+1} = (a_t + c)k_t,$$

where a_t is the "tradable" share of output, i.e. it can be used for market transactions, while c is the "nontradable" share, i.e. it can only be consumed by the entrepreneur. We will consider cases where entrepreneurs' productivity falls,

$$a_0 = a(1 - \Delta) \qquad \text{and} \qquad a_t = a \quad \forall t \geq 1$$

[5] In KM, entrepreneurs are "farmers" while financiers are "gatherers".

[6] The discount factor shock is absent in the original KM formulation, which focused on a technology shock (see below). We include it to capture, in reduced form, a shock to risk-aversion that induces a sharp drop in asset prices unrelated to the underlying productivity of the asset. This shock is not to be confused with the kind of preference shock used in the New Keynesian literature to model liquidity traps, which would have the opposite sign (e.g. Fernández-Villaverde et al., 2015). The latter is intended to capture the drop in the riskless rate experienced during the Great Recession, while ours is intended to capture the large drop in the price of risky assets (see Caballero and Farhi (2017) for a model where both phenomena are tightly linked).

with $a > 0$, $\Delta \in [0, \tilde{\Delta}]$ and $\tilde{\Delta} < 1$. By contrast, lenders are endowed with a standard production technology with decreasing returns:

$$y'_{t+1} = G(k'_t)$$

with $G' > 0$, $G'' < 0$, and $\beta' G'(0) > a > \beta' G'\left(\overline{K}\right)$.

Agents also differ in their access to credit. Whereas lenders are unconstrained, entrepreneurs must satisfy a collateral constraint,

$$R_t b_t \leq q_{t+1} k_t,$$

where q_t is the price of capital, b_t is one-period debt contracted at t, and R_t is the gross interest rate. This constraint is widely used in the literature, and it is typically microfounded through the impossibility of the borrower to pre-commit to making use of the firm's assets (see, e.g. KM). The constraint determines how much debt the entrepreneur can take ex-ante at t depending on what agents think the future price of capital q_{t+1} will be. However, this constraint is silent on how conflict is resolved if an unexpected event lowers the price of capital below the value of outstanding debt. In this paper, we analyze how different institutional arrangements ex-post affect the propagation of economic shocks. To do so, we enrich the original KM model to contemplate the possibility of default and renegotiation.[7] More precisely, we assume entrepreneurs always have the option to renege on their debts ex-post. However, if they do so, they lose D units of tradable output.[8] Entrepreneurs also have the possibility of renegotiating their debts with their creditors to avoid the default cost. We assume the surplus is split according to Nash bargaining and let φ denote the haircut on the outstanding value of debt.

Since agents have perfect foresight, these considerations will be of no consequence for the equilibrium at dates $t \geq 1$. However, in period 0, entrepreneurs have some legacy debt b_{-1} and capital k_{-1} and, depending on their levels and the state of the economy, renegotiation may be optimal. Henceforth, we assume that the level of legacy debt and capital are exactly their respective "steady state" levels where the economy would stabilize if $a_t = a \; \forall t$.[9] This is the outcome that would arise if agents in this economy were expecting $\Delta = \epsilon = 0$. By

[7] KM also consider the threat of default and renegotiation, but only at an *interim* stage, i.e. after the financial contract is written but *before* agents commit their labor. That is, once the productivity shock is realized, the borrower has to repay her debt in full. Here, by contrast, we consider renegotiation *ex-post*, i.e. after shocks and output are realized.

[8] To ensure everything is well-defined, we choose $\tilde{\Delta}$ to satisfy $(1 - \tilde{\Delta})a = D$, i.e. default is always feasible. Given our normalization below of $D = \alpha q^* K^*$, this requires $\tilde{\Delta} = 1 - \frac{R^*}{R^* - 1} \alpha K^*$.

[9] Convergence to this steady-state requires $c > (\beta^{-1} - 1)a$, which is "assumption 2" in the original KM paper (see KM for a proof). We assume this condition also holds in our environment.

contrast, we analyze cases where $\Delta \neq 0$ and $\epsilon \neq 0$, i.e. we study the response to "one-time" unexpected shocks. Henceforth, we use $*$ to denote quantities that correspond to the steady state. We also parametrize the default cost as a share of the steady-state level of debt to make the comparison across economies more transparent, i.e. we set $D = \alpha q^* K^*$.[10]

2.2 Solving the model

We solve the model backwards. We first solve for the equilibrium at dates $t \geq 1$. Then, we use these results to determine the bargained haircut. We complete this subsection with a system of two equations that characterize the equilibrium at date $t = 0$.

Continuation equilibrium $t \geq 1$ We start by characterizing financiers' demand for capital. Since they are unconstrained, they must be indifferent between lending and investing, i.e.

$$q_t = \frac{G'(\bar{K} - K_t) + q_{t+1}}{R_t} \quad \forall t, \tag{1}$$

where $K_t \equiv \int_0^1 k_t(i)di$ denotes the aggregate amount of capital in the hands of entrepreneurs. This must hold at all dates t. Furthemore, since they have linear utility

$$R_t^{-1} = \beta' \quad \forall t \geq 1.$$

This completes the characterization of financiers' decisions.

Next, we solve for entrepreneur demand. Given our assumptions, entrepreneurs will borrow as much as they can and invest the proceeds in capital.[11] Since there is perfect foresight, there will be no default or renegotiation and the borrowing constraint will bind at every date $t \geq 1$. Letting hats denote proportional deviations from the steady state (e.g. $\hat{k} = \frac{k_t - K^*}{K^*}$) we obtain

$$1 + \hat{k}_t = \frac{a}{u(\hat{K}_t)}(1 + \hat{k}_{t-1}) \quad \forall t \geq 1 \tag{2}$$

where $u(\hat{K}_t) \equiv \beta' G'(\bar{K} - K^* - K^* \hat{K}_t)$, following KM's notation. Note that equation (2) already solves for equilibrium in the continuation dates $t \geq 1$, since financiers' demand is

[10] If the default cost were kept constant across economies, then economies with higher K^* would have lower default costs. Similarly, if it were proportional to output (instead of the value of capital), then economies with lower interest rates and hit by larger shocks would have lower default costs. The current modeling choice helps isolate the direct effect of default costs.

[11] The assumption $c > (1/\beta - 1)a$ guarantees that investing to the maximum dominates consumption around the steady state (see footnote 9). In the solution, $\{\hat{q}_t\}$ is an increasing sequence that converges to q^* (provided $\hat{K}_t < 0$, which is guaranteed by lemma 1), which increases the attractiveness of investing even further. To see this, note that (2) implies $\{\hat{K}_t\}$ is an increasing sequence, and $\hat{q}_t$ is monotonic in $\{\hat{K}_{t+s}\}$ for $t \geq 1$ and even lower at $t = 0$ if $\epsilon > 0$. Finally, lending is always dominated given $\beta \leq R_t^{-1}$ $\forall t$.

encoded in $u(\hat{K}_t)$. The only remaining step is to aggregate entrepreneurs' decision, which is straightforward since (2) is linear in k_t and k_{t-1}. Iterating backwards, we may summarize the date $t \geq 1$ equilibrium via an increasing relationship $\hat{K}_t = f_t(\hat{K}_0)$.

Solving for the haircut At $t = 0$, the entrepreneur has two options: to renegotiate or to default. The amount of capital the entrepreneur can buy will be impacted by this decision,

$$\begin{aligned} 1 + \hat{k}_0^R &= \frac{a}{u(\hat{K}_0)(1-\epsilon)} \left(1 - \Delta + \frac{R^*}{R^*-1}(\hat{q}_0 + \varphi)\right) \\ 1 + \hat{k}_0^D &= \frac{a}{u(\hat{K}_0)(1-\epsilon)} \left(1 - \Delta - \frac{R^*}{R^*-1}\alpha\right), \end{aligned} \tag{3}$$

where $\hat{k}_0^R$ and $\hat{k}_0^D$ denote, respectively, the amount of capital that can be purchased in the case of renegotiation and default, respectively. Note that we used that $R_0^{-1} = (1-\epsilon)\beta' = (1-\epsilon)R^{*-1}$, since financiers are indifferent between consuming and lending.

Next, we compute the implied entrepreneurs' utilities of default and renegotiation given the shocks, aggregate capital K_0, and the proposed haircut φ,

$$U^i = cK^* + \beta c k_0^i + \beta^2 c k_1^i + ... + \lim_{t\to\infty} \beta^t c k_{t-1}^i$$

with $i = R, D$. Using our previous results, we obtain

$$U^R - U^D = \frac{ac^*K^*}{u(\hat{K}_0)(1-\epsilon)} \frac{\beta R^*}{R^*-1} (\hat{q}_0 + \varphi + \alpha) \left(\sum_{t=0}^{\infty} \beta^t \left(\prod_{s=0}^{t} \frac{a}{u(f_s(\hat{K}_0))}\right)\right). \tag{4}$$

By renegotiating, a borrower saves on the default costs, $\alpha q^* K^*$ and, in exchange, accepts to keep a share of the (negative) capital gains, $(\hat{q}_0 + \varphi) q^* K^* \leq 0$, which translates in a uniformly lower level of capital, both initially and in subsequent periods.

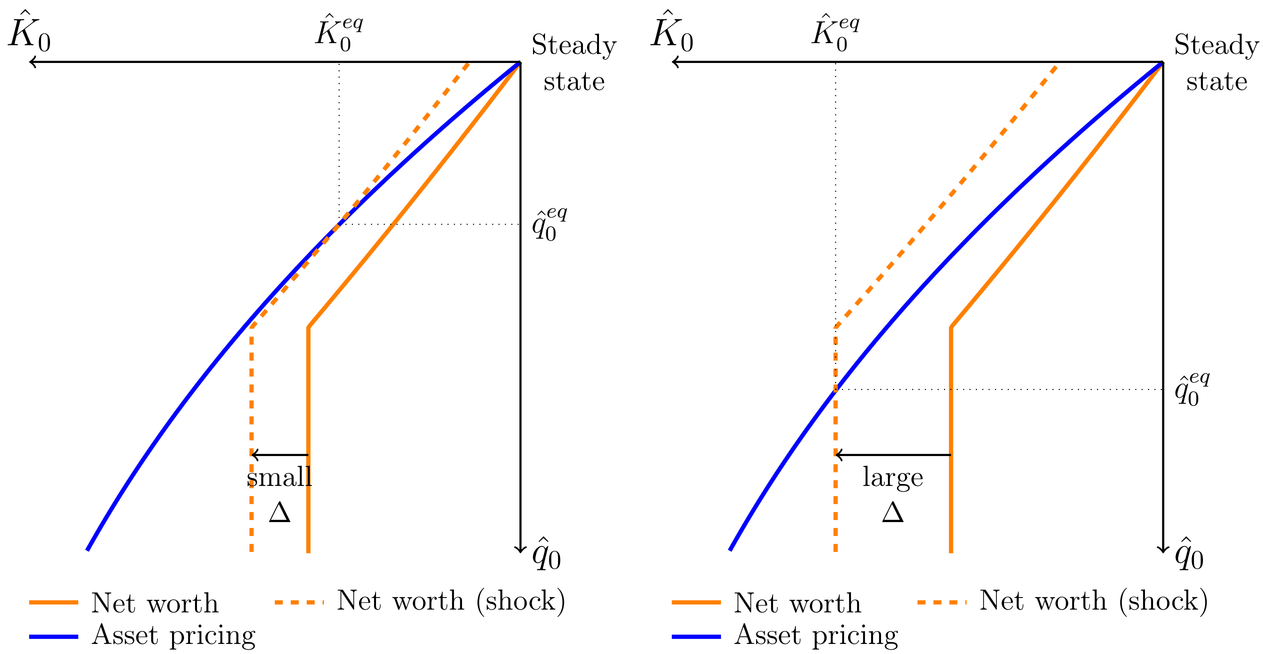
Renegotiation gives an entrepreneur surplus $U^R - U^D$ while a lender gets surplus $(1-\varphi)q^*K^* - (1 + \hat{q}_0 - \mu)q^*K^* = -(\hat{q}_0 + \varphi - \mu)q^*K^*$, where $\mu q^* K^*$ is a repossession cost (e.g. the cost of foreclosing in the case of real estate). We assume these surpluses are split according to Nash-bargaining. Letting θ denote financiers' bargaining power, the equilibrium haircut φ is given by[12]

$$\varphi = \max\{-\hat{q}_0 - \theta\alpha + (1-\theta)\mu, 0\}. \tag{5}$$

The equilibrium haircut depends on the effect that φ has on entrepreneurs' surplus. Let $\bar{q} \equiv -\theta\alpha + (1-\theta)\mu$. When $\hat{q}_0 \geq \bar{q}$, the price of capital is sufficiently high that the threat

[12] See appendix A.1.1 for details.

Figure 1: Technology shock.



Note. This figure illustrates the date-0 equilibrium for technology shocks Δ of different sizes. On the left, the shock is small and entrepreneurs get no haircut ($\varphi = 0$). On the right, the shock is large and entrepreneurs get a positive haircut ($\varphi > 0$). See appendix B for details.

of default is not credible even if $\varphi = 0$. Hence, entrepreneurs bear all the capital losses in this region and do not default. By contrast, when $\hat{q}_0 < \bar{q}$, entrepreneurs can bargain a positive haircut. Everything else equal, entrepreneurs extract a larger haircut when their default costs are lower (low α), their bargaining power is higher (low θ) and when financiers' repossession costs are higher (high μ).

Equilibrium at date 0 Equilibrium at date 0 is fully characterized by

$$\frac{u(\hat{K}_0)}{a}(1+\hat{K}_0) = \frac{1}{1-\epsilon}\left(1-\Delta+\frac{R^*}{R^*-1}\max\{\hat{q}_0,\bar{q}\}\right) \tag{6}$$

$$1+\hat{q}_0 = \frac{R^*-1}{R^*}\frac{1-\epsilon}{a}\left(u(\hat{K}_0)+\sum_{t=1}^{\infty}\frac{1}{R^{*t}}u(f(\hat{K}_0))\right). \tag{7}$$

The first equation is the "net worth" relation, which links the size of the capital losses faced by the entrepreneur with the amount of capital she can retain the first period. Noting that renegotiation always dominates default, using equation (3) and noting every entrepreneur is identical yields equation (6). The orange solid line in both panels of figure 1 plots this relationship in the $(\hat{K}_0, \hat{q}_0)$ space when $\Delta = 0$ and $\epsilon = 0$. Since this is consistent with the steady state, this curve passes through $(0,0)$. Around this point, this curve describes an increasing relationship: Since entrepreneurs are heavily levered, a lower price of capital damages their net worth more than one-to-one, decreasing their purchasing power. However, when capital becomes low enough, renegotiation is triggered putting a lower bound on how much capital can fall in equilibrium. This level of capital solves

$$\frac{u(\underline{\hat{K}})}{a}(1+\underline{\hat{K}}) = \frac{1}{1-\epsilon}\left(1 - \Delta + \frac{R^*}{R^*-1}\bar{q}\right). \tag{8}$$

Thus, when $\hat{q}_0 \leq \bar{q}$, this curve becomes a vertical line at $\underline{\hat{K}}$. Note that if the repossession cost and/or the preference shock are large, entrepreneurs may find it optimal to default and take advantage of the depressed asset prices to buy even more capital than they had originally. Assumption 1 rules out this unrealistic case.

Assumption 1. *The following holds*

$$\epsilon \leq \frac{R^*}{R^*-1}\left(\theta\alpha - (1-\theta)\mu\right).$$

The second equilibrium equation (7) is a standard "asset-pricing" relation, which states that the price of capital is the present sum of future dividends. It comes from iterating forward on (1) and imposing a standard no-bubbles condition. The blue solid line in both panels of figure 1 plots this relationship in the $(\hat{K}_0, \hat{q}_0)$ space when $\Delta = 0$ and $\epsilon = 0$. Since both u and f are increasing in $\hat{K}_0$, this curve also describes an increasing relationship between $\hat{q}_0$ and $\hat{K}_0$. The next lemma shows an equilibrium always exists.

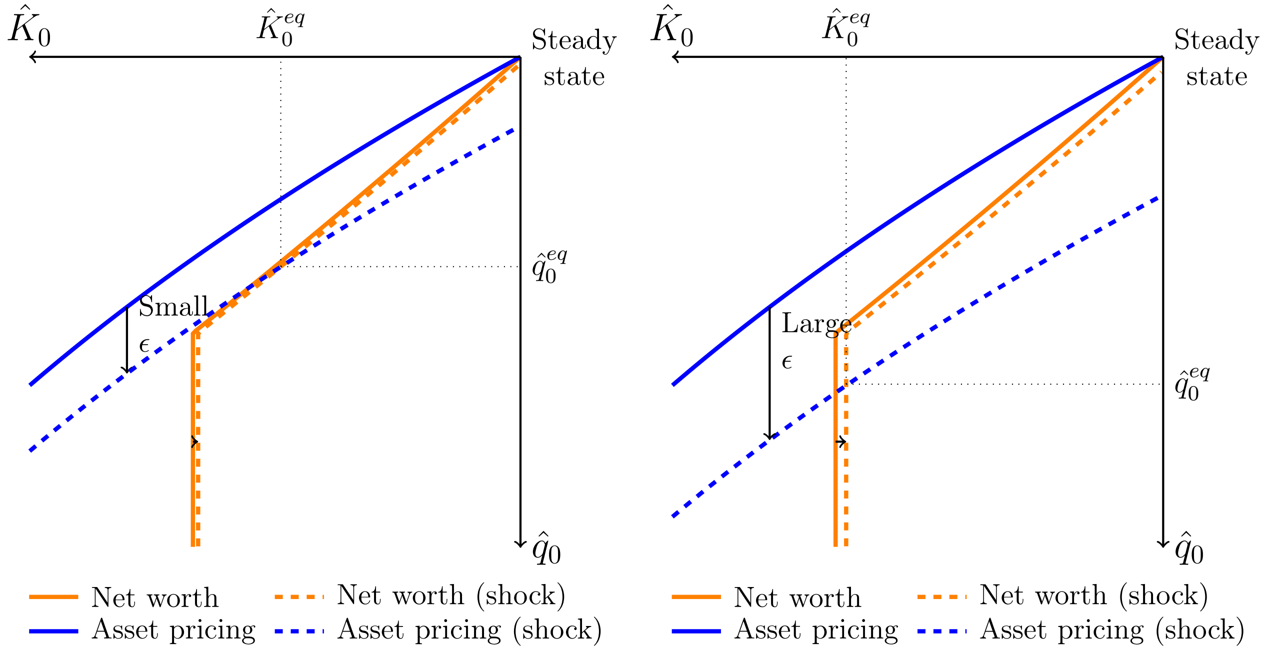
Lemma 1. *An equilibrium exists. The equilibrium features $\hat{q}_0 \leq 0$ and $\hat{K}_0 \leq 0$.*

2.3 Shocks

Technology shocks The left panel in figure 1 illustrates the effect of a small negative technology shock. Given capital prices, entrepreneurs can now buy less capital, shifting the net worth curve to the left. Since the shock is small, $\hat{q}_0$ remains above $\bar{q}$. Thus, entrepreneurs bear all the losses and capital demand increases with its price. On the other hand, the asset-pricing relationship is independent of the shock. The interaction of both upward sloping curves leads to significant amplification of the original shock and a substantial drop in entrepreneur capital and asset prices - exactly as in the original KM analysis.

The right panel in figure 1 illustrates the effect of a large negative shock. In this case, the drop in asset prices is so large that renegotiation is triggered. This puts a lower bound on

Figure 2: Preference shock.



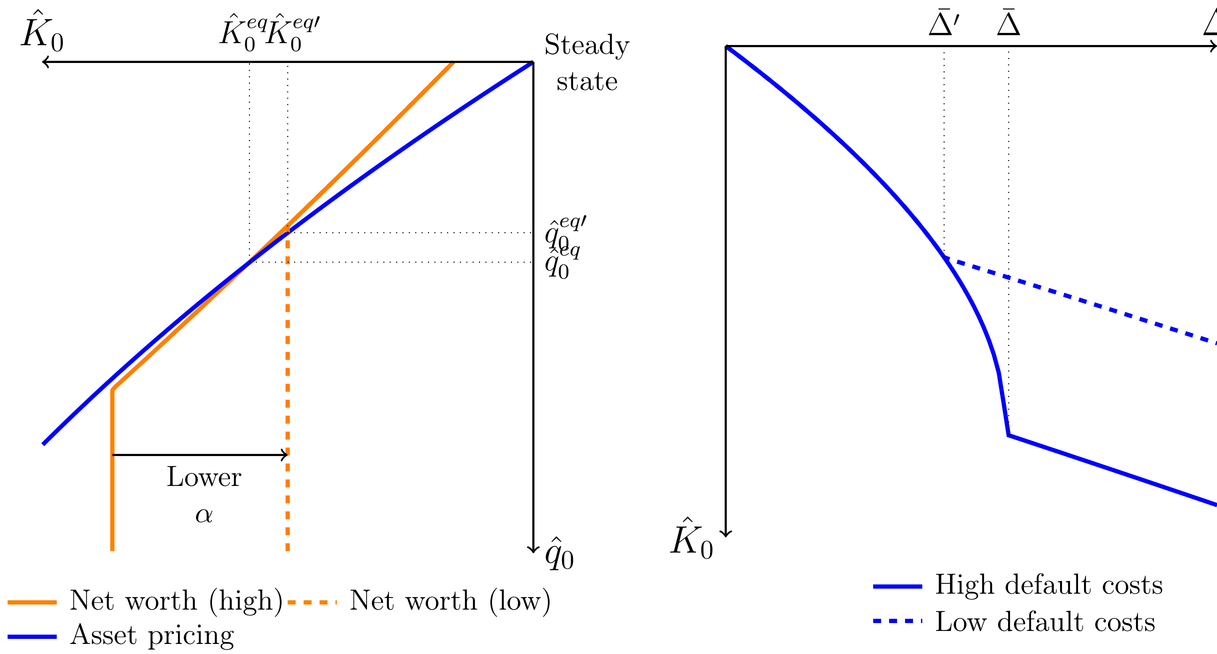
Note. This figure illustrates the date-0 equilibrium for preference shocks ϵ of two different sizes. On the left, the shock is small and entrepreneurs get no haircut ($\varphi = 0$). On the right, the shock is large and entrepreneurs get a positive haircut ($\varphi > 0$). See appendix B for details.

the fall of entrepreneur's net worth and, thus, on their capital demand, which is now equal to $\underline{\hat{K}}$. Further shocks still have a negative effect on capital prices, but the amplification via the net worth channel is now absent. As prices fall, haircuts increase, stabilizing entrepreneurs' losses at $\theta\alpha - (1-\theta)\mu$.

Preference shocks Figure 2 shows the date-0 equilibrium curves after a small (left panel) and a large (right panel) preference shock. In contrast to a technology shock, a preference shock that makes financiers more impatient moves both curves. On the one hand, a decrease in asset prices implies entrepreneurs can afford to buy more capital given their net worth (a *shift* in the net worth curve). On the other hand, since entrepreneurs are highly levered, the decrease in asset prices damages their net worth (a movement *along* the net worth curve). Which force dominates? Using equations (6) and (7), one can show that the shift in the asset-pricing curve is larger. Intuitively, this is because the change in discounting affects not only current dividends $u(K)$ but also future ones.[13] As a result, prices and entrepreneur capital decrease with preference shocks in this region.

[13] One also needs to use $\hat{K}_0 \leq 0$ to arrive at this conclusion. For further details, see appendix A.1.3.

Figure 3: Changing default costs.



Note. This figure shows, for two different levels of default costs: the date-0 equilibrium (left panel) and entrepreneurs' capital as a function of the technology shock (right panel). Smaller default costs expand the renegotiation region and lead to a milder crisis. See appendix B for details.

What about the region where renegotiation is triggered (right panel)? Here, entrepreneurs' net worth is insulated from variations in asset prices. Thus, they only profit from a decrease in asset prices and their capital increases. Since entrepreneurs' capital increases but discounting of future dividends decreases, the effect on asset prices is ambiguous. Also, given that entrepreneurs' net worth is fixed in this region, the effect on haircuts is also ambiguous.

The following proposition summarizes the comparative statics results for technology and preference shocks.

Proposition 1. *(a) For small ϵ, there exists $\underline{\Delta}$ such that for $\Delta < \underline{\Delta}$ there exists an equilibrium with no renegotiation, i.e. $\varphi = 0$. Equilibrium capital and prices are continuous in Δ and ϵ, and strictly decreasing in Δ and ϵ.*

(b) For any ϵ, there exists $\overline{\Delta}$ such that for $\Delta > \overline{\Delta}$ there exists an equilibrium with non-trivial renegotiation, i.e. $\varphi > 0$. Equilibrium capital and prices are continuous in Δ and ϵ, and strictly decreasing in Δ. The haircut φ is strictly increasing in Δ. Capital is strictly increasing in ϵ, while the effect on prices $\hat{q}$ and haircuts φ is ambiguous.

2.4 Allocating financial losses

The left panel in figure 3 shows the date-0 equilibrium for two values of default costs: “high” (solid line) and “low” (dashed line).[14] Even though default is never realized in equilibrium, a lower default cost implies a more attractive outside option and, as a result, a larger haircut. Formally, the “vertical” branch of the net worth curve shifts to the right as default costs decrease. This has two implications. First, the “amplification” region where equilibrium is characterized by the intersection of two upward-sloping curves contracts. That is, the economy features amplification for a smaller set of shocks. Second, when the shock is large enough to trigger renegotiation, the surplus extracted by entrepreneurs is larger and, as a result, the crisis is less pronounced.

The right panel in figure 3 shows the entrepreneurs’ capital stock as a function of the shock for a “high” (solid line) and “low” (dashed line) level of default costs.[15] Both economies behave similarly for small shocks. However, the low default cost economy is much more stable for large shocks: Renegotiation is triggered earlier and entrepreneurs capture a larger share of the surplus of avoiding wasteful default.

Proposition 2. *When $\theta\alpha - (1-\theta)\mu$ increases (i.e. financiers' bargaining power or entrepreneurs' default cost increases or financiers' repossession cost decreases),*

(i) $\bar{\Delta}$ strictly increases if $\bar{\Delta} \in (0, \tilde{\Delta})$.

(ii) equilibrium capital and prices strictly decrease if $\Delta > \bar{\Delta}$ (i.e. in the renegotiation region).

How does the economy recover after the financial crisis? Approximating equation (2) around the steady state yields

$$\hat{k}_t = \frac{\eta}{\eta+1}\hat{k}_{t-1},$$

where $\eta^{-1} \equiv d\ln u(K)/d\ln K|_{K=K^*}$. Thus, the rate of convergence back to the steady state is exactly the same as in KM. This implies that our model retains an attractive feature of the KM model: the ability to rationalize episodes of sluggish recovery after financial crises, such as Japan in the 1990s. Importantly, note that the speed of convergence is independent of the distribution of financial losses, which only affect the size of the initial output drop.

[14]Changes in bargaining costs and repossession costs are symmetric; see equations (6) and (7))

[15]Figure 3 depicts a case with a unique equilibrium, but the results in proposition 3 are general. Indeed, it is well-known that this kind of environment can have multiple equilibria. Since this is a well-understood topic that is orthogonal to our analysis, we center the discussion around the case with a unique equilibrium.

3 A Model with Equilibrium Default

The aftermath of a financial crisis is often characterized not only by debt restructuring negotiations but also by outright default. For example, both outcomes were observed in the hotel business during the Great Recession. This sector was particularly affected by economic conditions with revenue earned per room falling by almost 17% in 2009, and stock prices of the largest publicly traded hotel chains falling by around 80% between July 2007 and March 2009.[16] A prominent example of renegotiation was the deal that Blackstone secured for Hilton's debts in April 2010. Debt was restructured from \$20 to \$16 billion and maturity extended by two years.[17] An example of default is the case of Sunstone Hotel Investors, who defaulted on \$300 million of debt in June 2009 and had 13 hotels seized by its bank, only days later announcing its intention to buy hotels at a discount.

In this section, we extend our model to rationalize why some firms default in equilibrium and characterize its implications for the allocations of capital and asset prices.

3.1 Setup

We extend our previous model to accomodate heterogeneity in the size of the default cost α_i faced by each entrepreneur i. Crucially, default costs are private information. That is, α_i is known by the entrepreneur but unknown to the financier, who only knows the cumulative distribution function $F(\alpha) \in C^2$ with support $[0, \tilde{\alpha}]$.[18]

Since financiers ignore the type of entrepreneurs they have lent to, they face a tradeoff in the event of an unforeseen negative shock: a higher level of debt relief makes more borrowers willing to accept, but the rent extracted from each entrepreneur gets smaller. Lenders will balance the two effects, recognizing that the willingness of borrowers to accept a certain deal will be weaker for those with low default costs. For simplicity, we assume that lenders are identical and have all the ex-post bargaining power, i.e. $\theta = 1$. That is, lenders make an offer and then entrepreneurs decide whether to accept it or not.[19]

We solve the problem by backward induction. First, an entrepreneur must decide whether

[16] Starwood Hotels and Resorts Worldwide, Wyndham Worldwide Corporation, and Marriott International stock prices fell 85%, 84%, and 71% respectively in this period. Source for drop in revenue per room: https://www.economist.com/finance-and-economics/2010/03/18/you-can-check-out-any-time-you-like.

[17] Hilton's deal included the repurchase of \$1.8 billion of secured debt with a 54% discount, see Phalippou and Baum (2014). Other large hotel groups that restructured their debts were MGM Mirage in April 2009, and Harrah's in March 2010.

[18] In this section, we assume default is feasible even for the agent with the highest default cost, i.e. $\tilde{\Delta} = 1 - \frac{R^*}{R^*-1}\tilde{\alpha}K^*$. We also consider a few examples with a discrete distribution function in section 3.4.

[19] Given that lenders are risk neutral, we proceed as if each one of them faces a continuum of entrepreneurs. This makes the number of borrowers who default for a given debt reduction offer a deterministic quantity from the point of view of a single lender, and not only at an aggregate level.

to accept or decline a proposed haircut of φ, taking as given the dynamics of aggregate capital and prices. From equation (4), we know that entrepreneurs will only accept an offer if $\alpha_i \geq -(\hat{q}_0 + \varphi)$. Taking this into account lenders minimize expected losses. For a given debt offer φ, a lender incurs in a cost (in percentage terms) given by $-\hat{q}_0 + \mu$ on the fraction $F(-(\hat{q}_0 + \varphi))$ of the borrowers who default and deliver their collateral, whereas he loses φ (in percentage terms) on the complementary fraction $1 - F(-(\hat{q}_0 + \varphi))$ of credits that are renegotiated. Since individual lenders take prices, $\hat{q}_0$, as given, we can write their problem as,

$$\min_{\varphi \geq 0} \quad (\hat{q}_0 + \varphi)(1 - F(-(\hat{q}_0 + \varphi))) + \mu F(-(\hat{q}_0 + \varphi)).$$

The first order condition yields,[20]

$$1 - F(-(\varphi + \hat{q}_0)) + f(-(\varphi + \hat{q}_0))(\hat{q}_0 + \varphi - \mu) \geq 0, \qquad \text{with equality if } \varphi > 0. \tag{9}$$

Let $\bar{\alpha}$ denote the threshold default cost implied by the solution ignoring the non-negativity constraint. The full solution to financiers' problem can be written as

$$\varphi = \max\{-\hat{q}_0 - \bar{\alpha}, 0\}.$$

Note the symmetry with the derivation of the equilibrium haircut in the previous section, with $\bar{q}$ replaced by $-\bar{\alpha}$.

The net worth relation now needs to take into account that there may be default in equilibrium. Entrepreneurs with $\alpha_i < \min\{\bar{\alpha}, -\hat{q}_0\}$ default while agents with $\alpha_i \geq \min\{\bar{\alpha}, -\hat{q}_0\}$ renegotiate. Thus, the net worth relationship for an entrepreneur of type i yields

$$\frac{u(\hat{K}_0)}{a}(1 + \hat{k}_0^i) = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\min\left\{\alpha_i, \min\{\bar{\alpha}, -\hat{q}_0\}\right\}\right).$$

Integrating over individual capital holdings yields

$$\begin{aligned} \frac{u(\hat{K}_0)}{a}(1 + \hat{K}_0) =& \frac{1}{1-\epsilon}\Bigg(1 - \Delta - \frac{R^*}{R^*-1}\mathbb{E}\left(\alpha | \alpha \leq \min\{\bar{\alpha}, -\hat{q}_0\}\right)\underbrace{F(\min\{\bar{\alpha}, -\hat{q}_0\})}_{\text{defaulters}} \\ & - \frac{R^*}{R^*-1}\min\{\bar{\alpha}, -\hat{q}_0\}\underbrace{(1 - F(\min\{\bar{\alpha}, -\hat{q}_0\}))}_{\text{non-defaulters}}\Bigg) \end{aligned} \tag{10}$$

[20]We assume that (i) $(\alpha + \mu)F(\alpha)$ is strictly convex or (ii) the Mills ratio $\frac{1-F(\alpha)}{f(\alpha)}$ is weakly decreasing in α. Either of these are sufficient conditions for a unique solution to the financiers' problem. For example, a uniform distribution satisfies both requirements.

Agents that default face capital losses of α_i while agents who negotiate suffer losses of $\min\{\bar{\alpha}, -\hat{q}_0\}$. Note that all agents that do not default have the same net worth, as financiers must offer everyone the same haircut. Thus, agents with high default costs that accept the offer profit from the unobservability of their default costs.

While equation (10) seems more complicated than its counterpart in the model with no heterogeneity or private information, equation (6), it has similar properties: it still describes an upward relationship between $\hat{K}_0$ and $\hat{q}_0$ for $\hat{q}_0 \geq \bar{q}$ and a vertical line when $\hat{q}_0 < \bar{q}$ except that $\bar{q}$ is now equal to $-\bar{\alpha}$. The lower bound on entrepreneurs' capital $\underline{\hat{K}}$ is now given by,

$$\frac{u(\underline{\hat{K}})}{a}(1+\underline{\hat{K}}) = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\mathbb{E}\left(\alpha_i|\alpha_i \leq \bar{\alpha}\right)F(\bar{\alpha}) + \bar{\alpha}(1-F(\bar{\alpha}))\right)\right). \tag{11}$$

The model is closed by the same asset-pricing relationship as before.

We again make an assumption to rule out cases where the drop in asset prices triggers defaults that allow entrepreneurs to increase their capital stock (an analogue of assumption 1). In this case, we need not only a bound on the size of the preference shock, but also a (very weak) bound on the share of defaulting entrepreneurs due to redistributional considerations (i.e. entrepreneurs with low default costs benefit from fire sales by entrepreneurs with high default costs). Lemma 2 shows an equilibrium exists in this economy.

Assumption 2. *The following holds*

$$\begin{aligned} \epsilon &\leq \frac{R^*}{R^*-1}\left(\bar{\alpha}(1-F(\bar{\alpha})) + \mathbb{E}\left(\alpha_i|\alpha_i \leq \bar{\alpha}\right)F(\bar{\alpha})\right) \\ F(\bar{\alpha}) &\leq \beta'. \end{aligned}$$

Note: $\bar{\alpha}$ is a function only of μ and $F(\cdot)$ (see equation (9)). The second condition is a weak assumption insofar as β' is close to one.

Lemma 2. *An equilibrium exists. The equilibrium features $\hat{q}_0 \leq 0$ and $\hat{K}_0 \leq 0$.*

3.2 Shocks

Next, we study the response of this economy to technology and preference shocks. Proposition 3 shows that all the main results we derived in proposition 1 carry over to this environment. This follows from showing that the net worth curve has similar properties and the asset-pricing relationship is unaltered.

More interestingly, proposition 3 characterizes the behavior of the share of defaulting entrepreneurs in this new economy. Since there are some agents with very low default costs (i.e. $\alpha = 0$ is in the support), there is default even after very small shocks. As shocks become larger, asset prices decline more and an increasingly large share of entrepreneurs default. Eventually, financiers find it optimal to offer positive haircuts $\varphi > 0$. At this point, the share of defaulting entrepreneurs stabilizes and any further drops in asset prices are offset by corresponding increases in the haircut.[21]

Proposition 3. *(a) For small ϵ, there exists $\underline{\Delta}$ such that for $\Delta < \underline{\Delta}$ there exists an equilibrium with no renegotiation, i.e. $\varphi = 0$. Equilibrium capital and prices are continuous in Δ and ϵ, and strictly decreasing in Δ and ϵ. The share of defaulting entrepreneurs strictly increases with Δ and ϵ, and is strictly positive whenever either $\Delta > 0$ or $\epsilon > 0$.*

(b) For any ϵ, there exists $\overline{\Delta}$ such that for $\Delta > \overline{\Delta}$ there exists an equilibrium with non-trivial renegotiation, i.e. $\varphi > 0$. Equilibrium capital and prices are continuous in Δ and ϵ, and strictly decreasing in Δ. The haircut φ is increasing in Δ. Capital is strictly increasing in ϵ, while the effect on prices $\hat{q}$ and haircuts φ is ambiguous. The share of defaulting entrepreneurs is fixed at $F(\bar{\alpha})$.

3.3 V-shaped recoveries

To understand the differential effects of asymmetric information on the equilibrium, we compare the solution to the case of perfect information. More precisely, we consider an economy where default costs are distributed according to the same distribution F, but where the entrepreneurs' type α_i is perfectly observable by the financier. In other words, financiers can tailor the haircut to each entrepreneur, offering $\varphi_i = \max\{-\hat{q}_0 - \alpha_i, 0\}$. As a result, the individual net worth relation is given by[22]

$$\frac{u(\hat{K}_0)}{a}(1 + \hat{k}_0^i) = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^* - 1}\min\{\alpha_i, -\hat{q}_0\}\right).$$

[21] Since both curves are upward sloping, both types of equilibria may coexist for some Δ and ϵ. Our discussion in this paragraph describes a case with a unique equilibrium, but the results of proposition 3 are general.

[22] Note μ is irrelevant because financiers are assumed to have all bargaining power.

Integrating over individual capital holdings yields

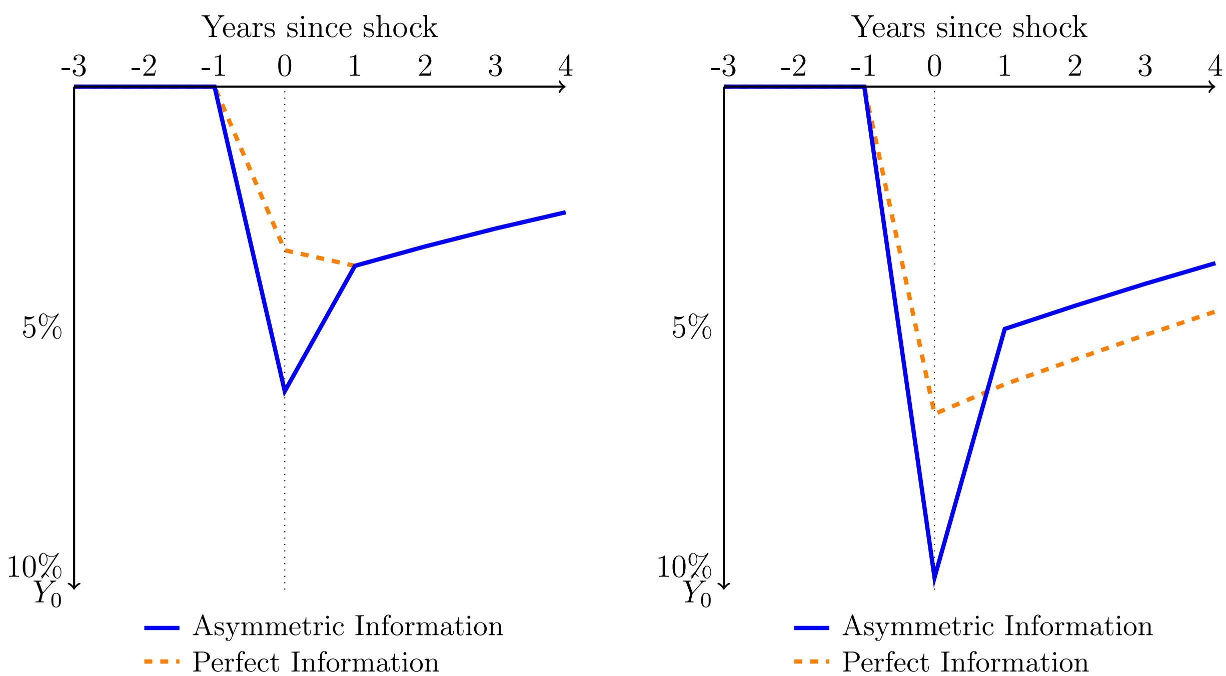
$$\frac{u(\hat{K}_0)}{a}(1+\hat{K}_0) = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\mathbb{E}\left(\alpha|\alpha \le \min\{\bar{\alpha}, -\hat{q}_0\}\right) \underbrace{F(\min\{\bar{\alpha}, -\hat{q}_0\})}_{\text{defaulters with asymmetric information}} \right.$$
$$\left. - \frac{R^*}{R^*-1}\underbrace{\mathbb{E}\left(\alpha|\alpha > \min\{\bar{\alpha}, -\hat{q}_0\}\right)}_{\text{larger than } \bar{\alpha}!} \underbrace{(1 - F(\min\{\bar{\alpha}, -\hat{q}_0\}))}_{\text{non-defaulters with asymmetric information}} \right) \tag{12}$$

Figure 4 shows the response of output to a shock in an economy with asymmetric information (blue solid line) and in one with perfect information (orange dashed line). In the left panel, the economy is hit with a small shock, i.e. the equilibrium features $\hat{q}_0 \ge -\bar{\alpha}$. In this case, agents with default costs larger than $-\hat{q}_0$ do not get any haircuts in either economy. Agents with smaller default costs get a haircut in the perfect information (PI) economy and default in the asymmetric information (AI) economy. Since lenders have all the bargaining power, entrepreneurs have the same net worth in either case, implying that they can buy the same amount of capital. Given that there is a one-period lag in production, output is the same in both economies from $t = 1$ onwards. By contrast, at $t = 0$, the PI economy has larger output since it avoids wasteful default (financiers are strictly better off).

Next, consider the case of a large shock (right panel), i.e. the equilibrium features $\hat{q}_0 < -\bar{\alpha}$. Entrepreneurs with $\alpha_i < -\bar{\alpha}$ still have the same net worth in both economies. By contrast, agents with $\alpha_i > \bar{\alpha}$ have more net worth in the AI economy. This is because financiers are forced to offer everyone the same haircut, so agents with high default costs, who would get a very small haircut under PI, now obtain a more generous haircut from financiers. In other words, asymmetric information transfers wealth from creditors to debtors. This can be seen by comparing (10) and (12): under AI all these agents lose the same amount $\bar{\alpha}$ while under PI they lose their expected default cost $\mathbb{E}(\alpha|\alpha > \bar{\alpha})$, which is larger. In addition, since entrepreneurs can afford more capital in the AI economy, in equilibrium asset prices are higher, further boosting their net worth relative to the PI economy. Since capital at $t = 0$ is higher, output is higher in the AI economy from $t = 1$ onwards. By contrast, at $t = 0$, capital used in production is predetermined (K^*) and the AI economy still features default while the PI economy does not. Thus, output at $t = 0$ in the AI economy is unambiguously lower than in the PI economy.

This result provides a rationalization of what is sometimes called a V-shaped recovery. In the context of our model, the dramatic drop in output on impact is due to default costs paid by debtors and repossession costs paid by creditors, which is then followed by a speedy recovery facilitated by the implicit transfer from creditors to debtors. By contrast,

Figure 4: V-shaped recovery.



Note. This figure simulates an event at $t = 0$ (a shock to both Δ and ϵ) and plots the response of output under asymmetric (AI) and perfect information (PI) for small shocks (left panel) and large shocks (right panel). The crisis is deeper at $t = 0$ under AI due to inefficient default but recovery is faster as entrepreneurs extract larger haircuts if the shock is large. See appendix B for details.

in economies where default costs are more easily observable, the initial recession is milder but can last significantly longer.

Proposition 4. *In an economy with asymmetric information ("AI" superscript), a shock has the following effects, relative to an equivalent economy with perfect information ("PI" superscript):*

(i) After a small shock (i.e. when $\hat{q}_0^{AI} \geq -\bar{\alpha}$*),* $\varphi^{AI} = 0$ *and* $\hat{K}_0^{AI} = \hat{K}_0^{PI}$*. Output at* $t = 0$ *is lower in the asymmetric information economy. At dates* $t \geq 1$*, output is equal in both economies.*

(ii) After a large shock (i.e. when $\hat{q}_0^{AI} < -\bar{\alpha}$*),* $\varphi^{AI} > 0$ *and* $\hat{K}_0^{AI} > \hat{K}_0^{PI}$*. Output at* $t = 0$ *is lower in the asymmetric information economy. At dates* $t \geq 1$*, the situation reverses and output is larger in the asymmetric information economy.*

3.4 Allocating financial losses

Proposition 2 also has an analogue in this economy. Here, a higher repossession cost implies it is more costly to let agents default. Hence, to prevent agents from defaulting, financiers must offer everyone a better haircut, boosting entrepreneurs' net worth across the board.

Proposition 5. *When* μ *increases,*

(i) $\bar{\Delta}$ *strictly decreases if* $\bar{\Delta} \in (0, \tilde{\Delta})$*,*

(ii) equilibrium capital and prices strictly increase if $\Delta > \bar{\Delta}$ *(i.e. in the renegotiation region),*

(iii) the share of defaulting entrepreneurs decreases if $\Delta > \bar{\Delta}$ *(i.e. in the renegotiation region).*

Note that, unlike proposition 2, proposition 5 is silent on the effect of default costs. The reason is that $\bar{\alpha}$ is an endogenous object. Indeed, many cases could arise: effective default costs could go up or down, and the recovery could be faster or slower (i.e. debtors could bear more or less losses). Next, we construct four examples to illustrate the different possible cases. For simplicity, we set $\mu = 0$. Table 1 summarizes the results.

Uniform $F\ [0, \tilde{\alpha}]$ In this case, one can show (see appendix B),

$$\bar{\alpha} = \frac{1}{2}\tilde{\alpha}, \quad , F(\bar{\alpha}) = \frac{1}{2}, \quad , \underline{\hat{K}} \propto -\tilde{\alpha}.$$

Thus, as $\tilde{\alpha}$ increases, entrepreneurs extract less haircut and the ensuing recovery slows down. While the share of defaulting entrepreneurs stays constant, each entrepreneur that defaults pays a larger cost. Thus, the crisis at $t = 0$ is also more pronounced.

Table 1: Potential default costs, effective default costs, and amplification

Distribution	Increase in (potential) default costs:	$\hat{Y}_0$	$\hat{K}_0$
Uniform $F\ [0, \tilde{\alpha}]$	$\uparrow \tilde{\alpha}$	$-$	$-$
Two types	$\uparrow p^H$, $\downarrow p^L$	$+$	$-$
Three types I	$\downarrow \alpha^M$, $\uparrow \alpha^L$, $p^L \Delta\alpha^L > -p^M \Delta\alpha^M$	$-$	$+$
Three types II	$\uparrow p^M$, $\downarrow p^L$	$+$	$+$

Two types Suppose there are two types, α^L and α^H with $\alpha^H > \alpha^L$ that arise with probability p and $1-p$, respectively. We cannot use the first order condition given by equation (9). Nevertheless, note that financiers will either offer a haircut such that $\varphi^H = -\hat{q}_0 - \alpha^H$, in which case only the high type will renegotiate, or they will offer $\varphi^L = -\hat{q}_0 - \alpha^L$, in which case both types will renegotiate. The profits of each strategy are

$$\begin{aligned} \varphi^H: &\qquad \hat{q}_0 + p\alpha^H \\ \varphi^L: &\qquad \hat{q}_0 + \alpha^L \end{aligned}$$

If $p\alpha^H > \alpha^L$, then offering a small haircut is optimal. Suppose this is the case. Note the probability of default is $1-p$, while

$$\underline{\hat{K}} = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L(1-p) + \alpha^H p\right)\right).$$

Next, consider an increase in average default costs by raising p. This does not change the net worth of any entrepreneur, but tilts the composition towards higher default cost agents. Thus, entrepreneurs as a group bear a large share of the financial losses, slowing down the recovery of output at dates $t \geq 1$. However, effective default costs decrease: There are fewer low type agents, who are the only ones that default in equilibrium. Thus, output at $t = 0$ falls less.

Three types I Suppose there are three types, α^L, α^M and α^H with $\alpha^H > \alpha^M > \alpha^L$ that arise with probability p^H, p^M, and p^L, respectively. We now need to compare three possible strategies: $\varphi^H = -\hat{q}_0 - \alpha^H$ (only high type accepts), $\varphi^M = -\hat{q}_0 - \alpha^M$ (high and medium type accept), and $\varphi^L = -\hat{q}_0 - \alpha^L$ (everyone accepts). The profits of each strategy are

$$\begin{aligned} \varphi^H\colon \quad & \hat{q}_0 + \alpha^H p^H \\ \varphi^M\colon \quad & \hat{q}_0 + \alpha^M (p^H + p^M) \\ \varphi^L\colon \quad & \hat{q}_0 + \alpha^L \end{aligned}$$

Suppose $(p^M + p^H)\alpha^M > \max\{p^H \alpha^H, \alpha^L\}$. In this case, the financier offers a haircut attractive enough to attract both the high type and the medium type, φ^M. Thus,

$$\underline{\hat{K}} = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L p^L + \alpha^M (p^M + p^H)\right)\right).$$

Next, consider an increase in average default costs that comes from increasing α^L and decreasing α^M such that $p^L \Delta\alpha^L = -p^M \Delta\alpha^M + \varepsilon$. As long as these changes are not very large, the financier will still offer a haircut that attracts both the high type and the medium type. This haircut will have to increase, since medium agents are now more prone to defaulting:

$$\underline{\hat{K}}' - \underline{\hat{K}} = -\frac{1}{1-\epsilon}\frac{R^*}{R^*-1}(p^L \Delta\alpha^L + (p^M + p^H)\Delta\alpha^M)$$

When $\varepsilon \to 0$,

$$\underline{\hat{K}}' - \underline{\hat{K}} = -\frac{1}{1-\epsilon}\frac{R^*}{R^*-1} p^H \Delta\alpha^M > 0.$$

The increase in potential default costs lowers the losses borne by entrepreneurs, speeding up the recovery from the crisis. However, defaulting entrepreneurs (i.e. low types) must now pay a higher cost. Therefore, effective default costs increase and the crisis at $t = 0$ is deeper. In sum, the path of output becomes more V-shaped.

Three types II Suppose $p^M = (\alpha^M)^{-1}(\alpha^H - \alpha^M)p^H - \frac{\varepsilon}{2}$, i.e. the financier slightly prefers offering φ^H to attract high types to offering φ^M and also attract medium types. Thus,

$$\underline{\hat{K}} = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L p^L + \alpha^M p^M + \alpha^H p^H\right)\right).$$

Next, consider an increase in average default costs that comes from increasing p^M to $p^M + \varepsilon$ and decreasing p^L to $p^L - \varepsilon$. Clearly, this increases average default costs. Furthermore, now there are enough intermediate types that it is profitable for financiers to offer a more attractive haircut to induce them to renegotiate. The lower bound on capital after renegotiation is given by

$$\underline{\hat{K}}' = \frac{1}{1-\epsilon}\left(1 - \Delta - \frac{R^*}{R^*-1}\left(\alpha^L(p^L - \varepsilon) + \alpha^M(p^M + \varepsilon + p^H)\right)\right).$$

Thus, when $\varepsilon \to 0$,

$$\underline{\hat{K}}' - \underline{\hat{K}} = \frac{1}{1-\epsilon}\frac{R^*}{R^*-1}\alpha^M p^M > 0.$$

In other words, even though the average default cost went up, the financier, in order to attract intermediate types, offers a much larger haircut. Thus, entrepreneurs have a larger net worth as a group and the recovery of output at dates $t \geq 1$ is faster. Furthermore, there is also less effective default as fewer agents default (i.e. entrepreneurs of type M stop defaulting). In other words, not only is recovery faster, but also the crisis at $t = 0$ is milder.

4 Discussion

Our model is well suited to describe events where several agents find themselves in a dire situation against which they are underinsured. For example, when a large negative shock hits and agents have sizeable noncontingent liabilities.[23] Prominent examples include: the real estate market in the U.S. after the Great Recession of 2008, where borrowers had bought property using residential mortgage loans and secured commercial property loans, which are typically nonrecourse; the South East Asian crisis of 1997/98 in which many local firms had borrowed abroad in foreign currency; and the burst of the Japanese stock market bubble in 1992, among others.

An ideal experiment for our paper would compare two economies that are identical ex-ante, but different ex-post, i.e. economies that differ only in how they allocate unforeseen capital losses. To see this, note that in our model the parameters of interest that determine the ex-post resolution of financial crises, α, μ, and θ, do not affect the steady state of the economy (q^*, K^*, B^*). That is since the threat of default and renegotiation are triggered in crisis, they do not affect the "normal" debt capacity of a borrower.

The work of Mian et al. (2015) provides a particularly apt setting to test our main results. They exploit variation in foreclosure laws across U.S. states: While some states have a judicial requirement to foreclose homes, others do not. Getting a judicial requirement consumes time and resources, making it more costly for the lender to foreclose on the property. In this sense,

[23] A separate yet interesting question is why agents do not insure in the first place. One possibility, particularly relevant for our application, is that agents underestimate the true probability of a crisis. This is consistent with the evidence in Mian et al. (2015), as we discuss below.

it is akin to an increase in μ. In addition, the authors show that there are no significant differences between judicial and nonjudicial states in terms of house prices, leverage, loan-to-value ratios, and household characteristics between 2002 and 2005, i.e. in the "pre-crisis" period. This observation is consistent with μ not affecting the steady state of the model.

In our model, a higher μ leads to fewer defaults (foreclosures), larger haircuts, and smaller amplification after a sizeable shock (proposition 5). Mian et al. (2015) show there is a strong correlation between foreclosure laws and foreclosure propensities during the Great Recession, i.e. there are fewer foreclosures in states with a judicial requirement. Then, using judicial requirements as an instrument for foreclosures, they find a strong negative effect of foreclosures on house prices.[24] Our model explains these results and clarifies the mechanism: There are fewer distressed home sales not only because the judicial requirement prevented inefficient foreclosures, but also because it allowed other agents, who would have renegotiated anyway, to extract a larger haircut.

Another relevant experiment, also in the context of the Great Recession, is provided by Agarwal et al. (2017). They exploit the exposure of different zip codes to the Home Affordable Modification Program (HAMP) to study the effect of renegotiations on economic outcomes. HAMP provided financial incentives for intermediaries to renegotiate distressed financial loans, which we interpret as a higher μ since it increases the opportunity cost of letting agents default. They use investor-owned properties, which were initially not eligible, as a control for the effect of HAMP on renegotiations. They find that the program led to a net increase in the annual rate of temporary and permanent contract modifications, and reduced the foreclosure rate. They also show that regions with higher shares of mortgage renegotiations had lower house price declines.[25] Our model is consistent with these results.

Finally, our model can also shed light on the debate regarding the speed of recovery after an economic crisis. It is often argued that recessions that follow a financial crisis are much harder to recover from than normal recessions caused by monetary tightening (see, e.g., Reinhart and Rogoff, 2009). A prominent example of such a slow recovery is Japan after its 1992 stock market crash. However, there are other episodes with financial crises that feature V-shaped recoveries, most notably some South East Asian countries after 1997. Our model suggests one reason why the latter group of countries experienced a fast recovery: That it was hard for lenders to know their borrowers' default costs. By contrast, in Japan, where it was potentially easier for lenders to infer that their counterparties had high default costs, firms decided to pay down their debts. Indeed, Ahmadjian (2006) has found, through an

[24] They also find a negative effect on other measures of economic activity, such as residential investment and auto sales.

[25] They also find lower consumer debt delinquency rates and a modest increase in auto sales.

analysis of ownership ties within business groups in Japan, that although some peripheral relationships were disrupted, the links between core firms remained robust. The image is that of a collection of physical and relational capital that incumbents find costly to take apart, and prefer to go through the painful process of debt reduction to preserve. By contrast, large-scale transfers of property and control, with many renegotiations of private debts, took place in some South East Asian countries in the late 1990s. For example, Gomez (2006) finds that in the Malaysian crisis, some prominent capitalists lost control of their corporate assets, while other business groups with better political connections thrived.[26]

5 Conclusions

Modelling financial crises requires specifying how counterparties, and the legal system itself, deal with widespread broken promises. To this end, we provided a framework to study how institutions and technological factors determine the way the economy allocates financial losses and examined their macroeconomic implications. Our model emphasized the size and observability of debtors' default costs and the size of creditors' repossession costs. These costs were meant to capture, in reduced form, the frictions surrounding bankruptcy procedures that prevent creditors from collecting debts and discourage borrowers from starting new businesses.

We found that renegotiation and default put a lower bound on the financial losses borne by the most productive sector, thereby limiting the depth of the financial crisis. This dampening mechanism is only triggered when shocks exceed a specific size, which is smaller if repossession costs are substantial. Also, the haircut increases with repossession costs, which accelerates the recovery. The converse is true of entrepreneurs' default cost, but only if these costs are perfectly observable. Indeed, we constructed examples where these costs are private information, and an increase did not lead to less favourable haircuts or more equilibrium default.

We also showed that the unobservability of default costs imprints interesting dynamics on output after a financial crisis. On the one hand, when default costs are private information, some agents default in equilibrium, depressing output on impact even more. On the other hand, asymmetric information prevents lenders from effectively extracting surplus from borrowers. In other words, haircuts are larger on average. Then, entrepreneurs' net worth does not fall as much, and the economy improves more rapidly. Compared to an econ-

[26] We do not mean to imply this was the only reason why these countries experienced different paths after the crisis, but rather suggest a new additional channel. Other reasons, such as the fiscal policy stance, are also clearly important (see, e.g. Chari and Henry (2015) for a comparison between South East Asian and European countries focusing on the fiscal aspect).

omy where costs are public information, the path of output under asymmetric information is more V-shaped.

The model is useful to interpret developments in real estate markets in the United States during the Great Recession. In particular, it suggests exceptional interventions in debt markets, such as HAMP, might have significant macroeconomic effects. These interventions may help not only by reducing inefficient liquidation but also by allowing borrowers who would have renegotiated anyway extract a more substantial haircut. Our results on V-shaped recoveries are also consistent with the experience of different countries after financial crises, such as Japan after its stock market crash in 1992 and South East Asia after the crisis of 1997.

The treatment of shocks as zero probability events can be a useful analytical device, and suitable to our purpose of studying the ex-post resolution of significant aggregate disturbances. However, our results may change if agents treat these events as non-negligible and alter their behavior ex-ante. In this case, one would expect agents to curb their borrowing for precautionary reasons, especially if ex-post they bear most of the losses. This behavior would mitigate the impact of small shocks. Then, the macroeconomy may show two types of non-linearities. First, an increase in the financial multipliers as buffer stocks are exhausted (evocative of a corridor effect; see Leijonhufvud (1973) for an earlier treatment, or Jensen et al. (2020) for a recent model of business cycle skewness) and, in the other extreme, a moderation of multipliers as debts are renegotiated in the event of a very large shocks.

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A Proofs and derivations

A.1 Section 2

A.1.1 Nash bargaining solution

Given our assumption of Nash bargaining, φ solves

$$\max_{\varphi \geq 0} (-(\hat{q}_0 + \varphi - \mu) q^* K^*)^\theta (U^R - U^D)^{1-\theta}$$

where $\theta \in [0, 1]$ is the financiers' bargaining power. Since $\hat{K}_0$ is taken as given, this program has the same solution as

$$\max_{\varphi \geq 0} \theta \ln(-\hat{q}_0 - \varphi + \mu) + (1 - \theta) \ln(\hat{q}_0 + \varphi + \alpha)$$

Note that the objective is concave. Thus, we can characterize the solution using the first order condition, which simplifies to

$$-\hat{q}_0 - \varphi - \theta\alpha + (1 - \theta)\mu \leq 0 \quad \text{with equality if } \varphi > 0.$$

Rearranging yields equation 5.

A.1.2 Lemma 1

First, guess $\hat{K}_0 = \underline{\hat{K}}$ (note $\underline{\hat{K}} \leq K^*$ since $\Delta \geq 0$ and $\epsilon \geq 0$). Plugging in $\underline{\hat{K}}$ into equation 7 implies some $\hat{q}_0^{ap}$. If $\hat{q}_0^{ap} \leq \bar{q}$, then $(\underline{\hat{K}}, \hat{q}_0^{ap})$ is an equilibrium.

If $\hat{q}_0^{ap} > \bar{q}$, we can ignore the max in the net worth relation. That is, we can define two functions $\hat{q}^{nw}(\hat{K}_0)$ and $\hat{q}^{ap}(\hat{K}_0)$ that describe asset prices that satisfy equations (6) and (7), respectively,

$$\hat{q}_0^{nw} = \frac{R-1}{R} \left(\frac{u(\hat{K}_0)}{a} (1 + \hat{K}_0^{nw})(1 - \epsilon) - (1 - \Delta) \right) \tag{13}$$

$$\hat{q}_0^{ap} = \frac{R-1}{R} (1 - \epsilon) \left(\frac{u(\hat{K}_0^{ap})}{a} + \sum_{t=1}^{\infty} \frac{1}{R^t} \frac{u(\hat{K}_s(\hat{K}_0^{ap}))}{a} \right) - 1. \tag{14}$$

Note $\hat{q}^{nw}(\underline{\hat{K}}) = \bar{q}$ by definition of $\underline{\hat{K}}$, implying $\hat{q}_0^{ap}(\underline{\hat{K}}) > \hat{q}^{nw}(\underline{\hat{K}})$. At $\hat{K} = 0$,

$$\hat{q}_0^{nw}(0) = (\Delta - \epsilon)\frac{R^* - 1}{R^*}$$
$$\hat{q}_0^{ap}(0) = -\epsilon.$$

Since $\hat{q}_0^{ap}(0) \leq \hat{q}_0^{nw}(0)$, and both $\hat{q}^{nw}$ and $\hat{q}^{ap}$ are continuous functions, the intermediate value theorem implies there exists $\hat{K}^{eq} \in (\underline{\hat{K}}, 0]$ such that $\hat{q}_0^{nw}(\hat{K}^{eq}) = \hat{q}_0^{ap}(\hat{K}^{eq})$. Thus, $(\hat{K}^{eq}, \hat{q}^{nw}(\hat{K}^{eq}))$ is an equilibrium.

A.1.3 Proposition 1

Part (a) Consider the system of equations (13) and (14). These equations define an equilibrium ignoring the possibility of renegotiation and default. When $\Delta = 0$, and $\epsilon = 0$, $\hat{q}_0 = 0$ and $\hat{K}_0 = 0$ solve this system. Note that

$$\frac{d\hat{q}^{nw}}{d\hat{K}_0}|_{\hat{K}_0=0} = (1-\epsilon)\frac{R^* - 1}{R^*}(\frac{1}{\eta} + 1) \tag{15}$$
$$\frac{d\hat{q}^{ap}}{d\hat{K}_0}|_{\hat{K}_0=0} = (1-\epsilon)\frac{R^* - 1}{R^*}(\frac{1}{\eta} + \frac{1}{(R-1)\eta + R}) \tag{16}$$

where $1/\eta \equiv \frac{d\log u(K)}{d\log K}|_{K=K^*}$ as in the original KM paper. Note $\frac{d\hat{q}^{nw}}{d\hat{K}_0}|_{\hat{K}_0=0} > \frac{d\hat{q}^{ap}}{d\hat{K}_0}|_{\hat{K}_0=0}$. Thus, we can apply the implicit function theorem at the steady state to establish the existence of a unique continuously differentiable solution $\{\hat{K}_0^{km}(\Delta,\epsilon), \hat{q}_0^{km}(\Delta,\epsilon)\}$ in an open ball $\check{B}$ around $(\Delta,\epsilon) = (0,0)$. Since $\hat{q}_0^{km}$ is continuous and $\hat{q}_0^{km}(0,0) > \bar{q}$, a subset of this ball $\tilde{B}$ is also an equilibrium of the full model.

It remains to show that entrepreneurs' capital and asset prices are decreasing in Δ and ϵ. We rely again on the implicit function theorem to compute:

$$\frac{d\hat{q}^{km}}{d\Delta} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}}|_{\hat{K}_0=0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}}|_{\hat{K}_0=0}}\frac{dq^{ap}}{d\hat{K}}|_{\hat{K}_0=0} < 0 \tag{17}$$
$$\frac{d\hat{K}^{km}}{d\Delta} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}}|_{\hat{K}_0=0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}}|_{\hat{K}_0=0}} < 0 \tag{18}$$
$$\frac{d\hat{q}^{km}}{d\epsilon} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}}|_{\hat{K}_0=0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}}|_{\hat{K}_0=0}}K^*\left(\sum_{s=0}\frac{1}{R^{*s}}\frac{dq^{nw}}{d\hat{K}^{nw}}|_{\hat{K}_0=0} - \frac{dq^{ap}}{d\hat{K}^{ap}}|_{\hat{K}_0=0}\right) < 0 \tag{19}$$
$$\frac{d\hat{K}^{km}}{d\epsilon} = -\frac{(R^*-1)/R^*}{\frac{d\hat{q}_0^{nw}}{d\hat{K}}|_{\hat{K}_0=0} - \frac{d\hat{q}_0^{ap}}{d\hat{K}}|_{\hat{K}_0=0}}\left(\sum_{s=0}\frac{1}{R^{*s}} - 1\right) < 0. \tag{20}$$

Since these derivatives are continuously differentiable, their sign is the same around $(\Delta, \epsilon) = (0, 0)$.

Part (b) In an equilibrium with renegotiation, $\hat{K}_0 = \underline{\hat{K}}$, which is defined by equation (8). Thus, an equilibrium with renegotiation exists if

$$\hat{q}^{ap}(\underline{\hat{K}}) = \frac{R-1}{R}(1-\epsilon)\left(\frac{u(\underline{\hat{K}})}{a} + \sum_{t=1}^{\infty}\frac{1}{R^t}\frac{u(\hat{K}_s(\underline{\hat{K}}))}{a}\right) - 1 \leq \bar{q} = -\theta\alpha.$$

Since $\underline{\hat{K}}$ is strictly decreasing in Δ, while $\hat{q}^{ap}$ is strictly increasing in $\underline{\hat{K}}$, it follows that if this condition is satisfied for some Δ, it is also satisfied for all $\Delta' < \Delta$. Three cases may arise. First, it could be that evaluated at $\Delta = 0$, $\hat{q}^{ap}(\underline{\hat{K}}) < \bar{q}$. In this case, there will be a renegotiation equilibrium $\forall \Delta$ so $\bar{\Delta} = 0$. Second, it could be that even evaluating the previous expression at $\Delta = \tilde{\Delta}$, $\hat{q}^{ap}(\underline{\hat{K}}) > -\theta\alpha$. In this case there will be no renegotiation equilibrium so trivially $\bar{\Delta} = \tilde{\Delta}$. If neither of these cases arise, then since $\underline{\hat{K}}$ is continuous and monotone, we can define $\bar{\Delta}$ as the solution to

$$\frac{R-1}{R}(1-\epsilon)\left(\frac{u(\underline{\hat{K}}(\bar{\Delta}))}{a} + \sum_{t=1}^{\infty}\frac{1}{R^t}\frac{u(\hat{K}_s(\underline{\hat{K}}(\bar{\Delta})))}{a}\right) - 1 = -\theta\alpha.$$

The fact that entrepreneurs' capital is strictly decreasing in Δ and increasing in ϵ is an immediate implication of $\underline{\hat{K}}$ being strictly decreasing in Δ and strictly increasing in ϵ. This also implies that asset prices fall with Δ, given that the asset-pricing curve is independent of Δ.

A.1.4 Proposition 2

When $\bar{\Delta} \in (0, \tilde{\Delta})$, i.e. when it is interior, it must solve

$$\frac{R-1}{R}(1-\epsilon)\left(\frac{u(\underline{\hat{K}}(\bar{\Delta}))}{a} + \sum_{t=1}^{\infty}\frac{1}{R^t}\frac{u(\hat{K}_s(\underline{\hat{K}}(\bar{\Delta})))}{a}\right) - 1 = -\theta\alpha.$$

Since the RHS strictly decreases with $\theta\alpha$ and $\underline{\hat{K}}$ strictly decreases with Δ and $\theta\alpha - (1-\theta)\mu$, part (i) of the result follows.

By continuity, when $\Delta > \bar{\Delta}$, if there exists a renegotiation equilibrium, then it still exists after a small perturbation. Since $\hat{K}_0 = \underline{\hat{K}}$, equation (8) immediately implies $\hat{K}_0$ strictly decreases with $\theta\alpha - (1-\theta)\mu$. Equation (7) then implies $\hat{q}_0$ strictly decreases with $\theta\alpha - (1-\theta)\mu$.

A.2 Section 3

A.2.1 Lemma 2

First, guess $\hat{K}_0 = \underline{\hat{K}}$. Note $\underline{\hat{K}} \leq 0$ since $\Delta \in [0, \tilde{\Delta}]$ and $\epsilon \in [0, \frac{R^*}{R^*-1}((\bar{\alpha}(1 - F(\bar{\alpha})) + \mathbb{E}(\alpha_i|\alpha_i \leq \bar{\alpha})F(\bar{\alpha}))]$. Plugging $\underline{\hat{K}}$ into equation (7) implies some $\hat{q}_0^{ap}(\underline{\hat{K}})$. If $\hat{q}_0^{ap}(\underline{\hat{K}}) \leq -\bar{\alpha}$, then $(\underline{\hat{K}}, \hat{q}_0^{ap}(\underline{\hat{K}}))$ is an equilibrium.

If $\hat{q}_0^{ap}(\underline{\hat{K}}) > -\bar{\alpha}$, then $\underline{\hat{K}}$ is not an equilibrium. We proceed as in the proof of Lemma 1 and define $\hat{q}_0^{nw}(\hat{K})$ as the solution to 10 "ignoring" the min, i.e. replacing $\min\{\hat{q}_0, \bar{\alpha}\}$ by $\hat{q}_0^{nw}$. Clearly, by definition, $\hat{q}_0^{nw}(\underline{\hat{K}}) = -\bar{\alpha}$. Thus, $\hat{q}_0^{nw}(\underline{\hat{K}}) < \hat{q}_0^{ap}(\underline{\hat{K}})$. Furthermore, since $\hat{q}_0^{nw}$ increases with $\hat{K}$, we have $-\hat{q}_0^{nw}(\hat{K}) \geq \bar{\alpha}\ \forall \hat{K} \in [\underline{\hat{K}}, 0]$, so the $\hat{q}_0^{nw}(\hat{K})$ actually describes the net worth relationship on this interval. In particular, at $\hat{K} = 0$, we have

$$\Delta - \epsilon = -\frac{R^*}{R^* - 1}\left(\mathbb{E}\left(\alpha|\alpha \leq -\hat{q}_0^{nw}(0)\right) F(-\hat{q}_0^{nw}(0)) + (-\hat{q}_0^{nw}(0))\left(1 - F(-\hat{q}_0^{nw}(0))\right)\right)$$
$$\hat{q}_0^{ap}(0) = -\epsilon$$

Since $\Delta \geq 0$, this implies

$$\hat{q}_0^{ap}(0) \leq \frac{R^*}{R^* - 1}\hat{q}_0^{nw}(0)\left(1 - F(-\hat{q}_0^{nw}(0))\right).$$

Using that $-\hat{q}_0^{nw}(0) \geq \bar{\alpha}$ and $R^* = 1/\beta'$,

$$\hat{q}_0^{ap}(0) \leq \hat{q}_0^{nw}(0)\frac{1 - F(\bar{\alpha})}{1 - \beta'},$$

which, using Assumption 2 implies $\hat{q}_0^{ap}(0) \leq \hat{q}_0^{nw}(0)$. Given that both $\hat{q}^{nw}(\hat{K})$ and $\hat{q}^{ap}(\hat{K})$ are continuous functions on $[\underline{\hat{K}}, 0]$, the intermediate value theorem implies there exists $\hat{K}^{eq} \in (\underline{\hat{K}}, 0]$ such that $\hat{q}_0^{nw}(\hat{K}^{eq}) = \hat{q}_0^{ap}(\hat{K}^{eq})$. Thus, $(\hat{K}^{eq}, \hat{q}^{nw}(\hat{K}^{eq}))$ is an equilibrium.

A.2.2 Proposition 3

Part (a) We start by defining the implicit relationship $\hat{q}^{nw}(\hat{K})$, which is the net worth relationship as-if agents never renegotiated, i.e

$$\frac{u(\hat{K}_0)}{a}(1 + \hat{K}_0) = \frac{1}{1 - \epsilon}\left(1 - \Delta - \frac{R^*}{R^* - 1}\int_0^{-\hat{q}_0^{nw}} \alpha dF(\alpha) - \frac{R^*}{R^* - 1}(1 - F(-\hat{q}_0^{nw}))(-\hat{q}_0^{nw})\right)$$

and consider the system formed by this equation and the asset-pricing relationship (7).

When $\Delta = 0$, and $\epsilon = 0$, $\hat{q}_0 = 0$ and $\hat{K}_0 = 0$ solve this system. Furthermore, $\frac{d\hat{q}^{nw}}{d\hat{K}_0}|_{\hat{K}_0=0}$

and $\frac{d\hat{q}^{ap}}{d\hat{K}_0}|_{\hat{K}_0=0}$ are still given by (15) and (16) (use $F(0) = 0$). We can proceed exactly like before to show the existence of a continuously differentiable solution. Entrepreneurs' capital and asset prices are also still decreasing since the derivatives with respect to Δ and ϵ are still given by (17) - (20) (using, again, $F(0) = 0$).

Finally, note that since $\hat{q}_0$ strictly decreases with Δ and ϵ, the share of defaulting entrepreneurs must strictly increase according to $F(-\hat{q}_0)$. Furthermore, since $\hat{q}_0 < 0$ whenever either $\Delta > 0$ or $\epsilon > 0$, and $f(0) > 0$, the share of defaulting entrepreneurs is always nonzero.

Part (b) The proof of the first part of the result is exactly analogous to the one of Proposition (1) since $\hat{K}_0$ is given by (8) and it has all the necessary properties for previous argument to work (with $\bar{q} = -\bar{\alpha}$).

It only remains to show that the share of defaulting entrepreneurs is constant. This is an immediate implication of the financiers' first order condition - given by equation (9) - holding with equality.

A.2.3 Proposition 4

(i) Since $\hat{q}_0^{AI} \geq -\bar{\alpha}$, agents either default or pay the full value of debt. Guess $\hat{q}_0^{PI} = \hat{q}_0^{AI}$. Then, those agents that repay fully under asymmetric information also repay fully under perfect information (since financiers have all the bargaining power). By contrast, agents that default in the AI economy now get a haircut. However, since financiers' have all bargaining power, entrepreneurs in the PI economy are not better off than their counterparts in the AI economy, i.e. they can afford the same amount of capital. Since entrepreneurs get the same amount of capital in both economies, $\hat{q}_0^{PI}$ effectively satisfies the asset-pricing relationship so $(\hat{K}_0^{PI}, \hat{q}_0^{PI})$ constitutes an equilibrium of the PI economy.

Since the path of capital is the same in both economies, and output is produced with a one-period lag, output is equal in both economies $\forall t \geq 1$. Furthermore, since both start with the same level of capital K^* but the asymmetric information wastes resources on default costs, output is lower in the AI economy at $t = 0$.

(ii) Guess $\hat{q}_0^{PI} = \hat{q}_0^{AI} < -\bar{\alpha}$. Given this asset price, agents with $\alpha_i > \bar{\alpha}$ would renegotiate their debts in the PI economy. However, since financiers' know their default costs, they extract a smaller haircut (note the haircut offered under AI makes the agent with $\bar{\alpha}$ indifferent). On the other hand, agents with $\alpha_i < \bar{\alpha}$ default under AI and renegotiate under PI. As in (i), this set of agents would end up with the same net worth. It follows that, if $\hat{q}_0^{PI} = \hat{q}_0^{AI}$, agents in the PI economy would have accumulate less capital at $t = 0$ relative to the AI economy. This implies that $\hat{q}_0^{PI} = \hat{q}_0^{AI}$ is not an equilibrium. Indeed, at $\hat{q}_0^{AI}$ the net worth curve is to the left (equivalently, above) of the asset-pricing curve, implying there is an equilibrium of the AI economy with even lower asset prices $\hat{q}_0^{PI} < \hat{q}_0^{AI}$, which in turn implies even lower capital accumulation $\hat{K}_0^{PI} < \hat{K}_0^{AI}$. Since future capital stocks are monotone in current capital stocks and economies do not differ in their continuation equilibria, it follows that the AI economy has larger output for all $t \geq 1$. By contrast, at $t = 0$ output is lower in the AI economy since, like before, the economy must pay some wasteful default costs.[27]

A.2.4 Proposition 5

Applying the implicit function theorem to equation 9, we obtain $\frac{d\bar{\alpha}}{d\mu} < 0$. Parts (i) and (ii) are then analogous to the proof of 2 with $\bar{q} = -\bar{\alpha}$ instead of $\bar{q} = -\theta\alpha + (1-\theta)\mu$. Part follows from noting the share of defaulting entrepreneurs is $F(\bar{\alpha})$ and F is a cdf.

B Parametrization and calibration for figures

B.1 Parametrization

We parametrize the financiers' production function as

$$G = -\frac{1}{2} g_2 \tilde{k}'^2 + g_1 \tilde{k}'$$

and set $\bar{K} = 1$. The steady state is given by

$$\begin{aligned}
R^* &= (\beta')^{-1} \\
K^* &= g_2^{-1} R^* a - g_2^{-1}(g_1 - g_2) \\
q^* &= \frac{R^*}{R^* - 1} a \\
B^* &= R^{*-1} q^* K^*.
\end{aligned}$$

The user cost function is, then,

$$u = a(1 + \beta' g_2 a^{-1} K^* \hat{K}_t).$$

This implies that the future path of capital solves (using equation 2),

$$\hat{K}_t = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 + 4 \frac{a}{\beta' g_2 K^*} \hat{K}_{t-1}}}{2}.$$

[27] There may be more PI (and AI) equilibria with larger capital stocks and asset prices. Our analysis compares the equilibrium with lowest capital stocks in each model to ensure it is an "apples-to-apples" comparison even if there are multiple equilibria.

For Section 3, we assume F is uniform, i.e.,

$$F(\alpha) = \frac{\alpha}{\tilde{\alpha}}.$$

This implies

$$\bar{\alpha} = \frac{1}{2}(\tilde{\alpha} - \mu).$$

To compute the path of output, note

$$\begin{aligned}
Y^* &= (a+c)K^* + G(\bar{K} - K^*) \\
Y_0 &= (a(1-\Delta)+c)K^* + G(\bar{K} - K^*) - \frac{1}{2}\frac{1}{\tilde{\alpha}}\hat{q}_0^2 q^* K^* \mathbf{1}_{\text{economy=AI}} \\
Y_t &= (a+c)K_{t-1} + G(\bar{K} - K_{t-1}) \quad \text{for } t \geq 1
\end{aligned}$$

where $\mathbf{1}_{\text{economy=AI}}$ is an indicator function that is equal to one if the economy under consideration features asymmetric information.

Finally, we define $\hat{Y}_t \equiv \frac{Y_t - Y^*}{Y_t}$.

B.2 Calibration

B.2.1 Figures from Section 2

At $t = 0$, the system of equations (6) and (7) becomes

$$\begin{aligned}
(1 + \beta' g_2 a^{-1} K^* \hat{K}_t)(1 + \hat{K}_t) &= \frac{1}{1-\epsilon}(1 - \Delta + \frac{R^*}{R^*-1}\max\{\hat{q}_0, \bar{q}\}) \\
1 + \hat{q}_0 &= (1-\beta')(1-\epsilon)\left(\sum_{t=0}^{\infty} \beta'^t (1 + \beta' g_2 a^{-1} K^* \hat{K}_t)\right).
\end{aligned}$$

The lower bound on $\underline{\hat{K}}$ is given by

$$\underline{\hat{K}} = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 - 4\frac{a}{\beta' g_2 K^*}\left(\frac{1}{1-\epsilon}(1 - \Delta + \frac{R^*}{R^*-1}\bar{q}) - 1\right)}}{2}.$$

For all the plots in this section we set

$$g_2 = 0.3; g_1 = 1; a = 0.75; c = 0.3; \beta' = 0.9; \beta = 0.8; \theta = 0.5; \mu = 0.$$

These parameters satisfy the required assumptions.

In Figure 1, we set $\Delta = 0.1$ for the small shock and $\Delta = 0.2$ for the large shock. Default costs are set to $\alpha = 0.08$ and preference shocks are zero ($\epsilon = 0$). In Figure 2, we set $\epsilon = 0.01$ for the small shock and $\epsilon = 0.02$ for the large shock. Our value for ϵ for the large shock is motivated by the evidence by Gilchrist and Zakrajšek, 2012, who find that during the Great Recession the excess bond premium for financial and non-financial firms increased by approximately 2.5 percentage points. Default costs are set to $\alpha = 0.08$ and technology shocks are zero ($\Delta = 0$). In Figure 3, low default costs correspond to $\alpha = 0.04$, while high default costs are $\alpha = 0.08$. In the left panel, shocks are set to $\Delta = 0.1$ and $\epsilon = 0$. In the right panel, the technology shock Δ varies from 0 to 0.2, while $\epsilon = 0$.

B.2.2 Figures from Section 3

With asymmetric information, equation (10) becomes

$$(1 + \beta' g_2 a^{-1} K^* \hat{K}_t^{AI})(1 + \hat{K}_t^{AI}) = \frac{R^*}{R^* - 1} \min\{\bar{\alpha}, -\hat{q}_0^{AI}\} \left(1 - \frac{1}{2} \frac{\min\{\bar{\alpha}, -\hat{q}_0^{AI}\}}{\tilde{\alpha}}\right).$$

The lower bound on $\underline{\hat{K}}^{AI}$ is given by

$$\underline{\hat{K}}^{AI} = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 - 4\frac{a}{\beta' g_2 K^*} \left(\frac{1}{1-\epsilon} \left(1 - \Delta - \frac{R^*}{R^*-1} \bar{\alpha} \left(1 - \frac{1}{2}\frac{\bar{\alpha}}{\tilde{\alpha}}\right)\right) - 1\right)}}{2}.$$

With perfect information, equation (12) becomes

$$(1 + \beta' g_2 a^{-1} K^* \hat{K}_t^{PI})(1 + \hat{K}_t^{PI}) = \frac{1}{1 - \epsilon} \left(1 - \Delta + \frac{R^*}{R^* - 1} \max\{\hat{q}_0, -\tilde{\alpha}\} + \frac{R^*}{R^* - 1} \frac{1}{\tilde{\alpha}} \frac{1}{2} \{\hat{q}_0, -\tilde{\alpha}\}^2\right).$$

The lower bound on $\underline{\hat{K}}$ is given by

$$\underline{\hat{K}}^{PI} = \frac{-(1 + \frac{a}{\beta' g_2 K^*}) + \sqrt{(1 + \frac{a}{\beta' g_2 K^*})^2 - 4\frac{a}{\beta' g_2 K^*} \left(\frac{1}{1-\epsilon} \left(1 - \Delta - \frac{1}{2}\tilde{\alpha}\frac{R^*}{R^*-1}\right) - 1\right)}}{2}.$$

We keep the same parameters for preferences and technology as in the previous section. In addition, we set $\tilde{\alpha} = 0.08$ implying $\bar{\alpha} = 0.04$. In Figure 4, we set $\Delta = 0.1$ and $\epsilon = 0.01$ for the small shock and $\Delta = 0.2$ and $\epsilon = 0.02$ for the large shock. Δ is chosen to create a drop in output in the crisis period of around 5% and 10%, respectively.