Cooperation and Retaliation in Legislative Bargaining*

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Abstract

We study a legislative-bargaining divide-the-pie game in which some legislators have the ability to affect the amount of resources to be distributed (positively or negatively). If included in the winning coalition, these legislators cooperate and increase the size of the pie. If excluded, they retaliate and decrease it. Cooperation and retaliation produce significant changes in the equilibrium allocation relative to Baron and Ferejohn (1989). We find that, i) cooperating and retaliating districts are more likely to be included in the winning coalition, ii) the equilibrium might feature larger-than-minimum winning coalitions, and iii) there exist equilibria with inefficient output losses.

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1 Introduction

Policymaking in parliaments not only allocates resources, but also resources can increase or decrease depending on the coalition of legislators that supports policy. For example, when negotiating over the budget, legislators who are satisfied with the bargaining outcome may be willing to cooperate to increase the central government's resources. Those who are less satisfied may not cooperate and may even choose to retaliate, reducing the central government's tax base. Taking these spillovers into account, in the form of cooperation or retaliation, the size of the pie to be distributed is endogenous to the legislators approving the budget. Thus, the presence of these bargaining spillovers might modify an agenda setter's proposal on how to distribute aggregate resources.

A more concrete example of bargaining spillovers that affect legislative outcomes is provided by the 1990 Clean Air Act Amendment in the U.S. This arrangement established the first environmental program to rely on tradable emission permits to control acid rain pollution by reducing sulphur dioxide (SO_2) and nitrogen oxide (NO_x) emissions from coal-powered electric generating plants. At the time, mid-west states were high emitters of SO_2 and NO_x, and delayed their support for the new regulation until they received sufficient compensation. This was achieved by giving polluting utilities free emission permits, which amounted to expected transfers of approximately two billion dollars per year in 2019 dollars.[1] Thus, polluting states had a higher threat point in bargaining and used it to secure a benefit from the new legislation.

The presence of cooperation or retaliation opens an informal platform to influence formal policy-making. We study how cooperation and retaliation affect the outcome of legislative bargaining, potentially leading to inefficient policy choices. In a first stage, some districts decide whether to become active, i.e. commit to engage in cooperation or retaliation. In a second stage, the legislature adjourns knowing districts' types and a divide-the-pie legislative bargaining game follows.[2] Importantly, active districts not included in the winning coalition cause a loss in output (retaliators would decrease the pie and cooperators would not contribute

[1]At the time of the regulation market prices were expected to be roughly \$200/ton and permits for more than five million tons were initially allocated. The value of free permits was approximately 0.5% of total federal grants to state and local governments in 1990. For Ohio and Indiana, two of the most polluting states, the expected value of free permits was equivalent to 2.8% and 4.7% of federal grants in 1990. See Cooper et al. (2010) for a detailed analysis of the political negotiations that resulted in the free allocation of pollution permits.

[2]Throughout, we denote the resources to be divided in legislative bargaining as pie, rents, or output.

to increase it). Hence, the resources to be divided are endogenous, and depend on the number of active districts that are excluded from the winning coalition.

The inclusion of cooperating and retaliating legislators introduces asymmetries across legislators' bargaining power. Active districts have a higher threat point, giving the agenda setter incentives to make them part of the winning coalition more often than the remaining districts. This rationale may hold even when the agenda setter already has a sufficient number of votes to pass her policies, resulting in larger-than-minimal winning coalitions.[3] There is a countervailing force, districts with a higher threat point are also more expensive to buy. This is why some active districts may be excluded from the coalition. Thus, in some equilibria there are inefficient output losses, as either the gains of including cooperating legislators are not realized, or retaliation takes place.

The type of equilibria and expected output losses depend on the voting rule, the number of active districts, and patience. When the agenda setter is choosing the composition of her winning coalition, she trades off the higher cost of active districts against the increase in output they produce. If there is a relatively large discounting, all active districts are included in the coalition. The continuation value of active districts increases with patience. Therefore, as patience increases, active districts eventually stop being called into the winning coalition with certainty, larger-than-minimal winning coalitions are less likely, and there are output losses. We also show that all legislators that have the option to become active in the first stage decide to do so.[4]

An increase in the required supermajority makes minimum winning coalitions more likely, and reduces output losses. Larger supermajorities do not necessarily benefit active districts. Although marginal increases of the needed majority may increase active districts' continuation values, the effect is non-monotonic. For instance, in the extreme case of unanimity rule, all active districts must be included in the coalition. This rule makes all legislators, from active and passive districts, equally needed and ex-ante payoffs must be identical for all legislators. That an increase in the supermajority, or a reduction in patience, increases efficiency suggests that procedural rules should be made contingent on the number of cooperating or retaliating legislators.

In legislatures, the districts' representatives are agents of their constituencies. While we model the decision to become active as made by the legislator, in some

[3] These results are in line with the empirical literature, in which larger-than-minimal winning coalitions are the norm. See Knight (2008) and references therein.

[4] The presence of output losses for some equilibria render this decision non trivial.

circumstances this is the result of grassroots movements. For example, the Great Recession, and the slow recovery from it, produced an outburst of protests in established democracies around the world. Occupy Wall Street in the United States, "indignados" in Spain, the anti-austerity movement in Greece are examples of demonstrations that can have an impact on economic activity, and may have affected legislators' actions.[5]

Our results do not only apply to bargaining in formal legislatures. Environmental negotiations that take place in the international arena are subject to different rules of engagement. A polluting country which does not support the outcome of an international environmental agreement may threaten to sustain pollution (imposing a negative externality on all other countries) unless it obtains a better deal. Conversely, countries may allocate more effort in reducing contamination if they perceive a benefit from cooperation. An example of how cooperation and retaliation forces might shape international agreements is the clean development mechanism set up in the aftermath of the 1997 Kyoto Protocol.[6]

We contribute to the literature on multilateral bargaining in legislatures.[7] In particular our work relates to Baron and Ferejohn's (1989) application of Rubinstein's (1982) bargaining model to legislative bargaining. The former is a non-cooperative zero-sum game that shows how the final distribution of resources is affected by the majority rule, the recognition probabilities of the agenda setter, and the exact details of the bargaining rules (for instance, the presence or absence of amendments).

Recent papers on multilateral bargaining in legislatures study the effect of institutional changes, e.g. different procedural rules, on policy outcomes. Snyder et al. (2005) look at voting power and recognition probabilities when legislators voting weights depend on the party shares in the election. Banks and Duggan (2000, 2006) generalize the work-horse models by looking at multidimensional policies and general status quo.[8] Other papers have considered the effect of endogenous status quo: in Riboni (2010) yesterday's policy is today's policy if

[5]Other recent cases were the protests seen in Iran, Lebanon, Hong Kong, Colombia and Chile.

[6]The clean development mechanism allows countries to implement part of their committed emission abatement targets through projects in countries that have ratified the Kyoto protocol but are not subject to such targets. This gives incentives to ratify the protocol both to countries that have to reduce emissions, as they can do so at a lower cost, and to countries that do not have to reduce emissions, as they will be recipients of foreign investment. See Beccherle and Tirole (2011).

[7]Examples of early works include Austen-Smith and Banks (1988), Baron (1991), Romer and Rosenthal (1978), and Romer and Rosenthal (1979).

[8]Most of these models account for inefficiencies when a proposal is passed with delay. Eraslan (2002) provides a general model with heterogeneous recognition probabilities and discounting.

an agreement is not reached, and Diermeier and Fong (2011) add persistence in agenda-setting power. Dynamic macroeconomic models have also incorporated a streamlined model of legislative bargaining for fiscal policy choice (Battaglini and Coate, 2007, 2008; Leblanc et al., 2000; Piguillem and Riboni, 2015, 2018).

Our model has an exogenous status quo. A notable feature of this type of models is that a proposal is passed with the minimum amount of votes required, i.e., with minimum winning coalitions.[9] Banks (2000) and Groseclose and Snyder (2000) study larger-than-minimal winning coalitions in a setup with sequential voting, which allows for buying cheaper coalitions than minimum winning ones. This issue is also studied in a model of "pivotal bribing" in committees in Dal Bó (2007), and in a model of lobbying in Hummel (2009). Our work is also related to the theory of political failure by which an inefficient allocation of resources can be caused by politically determined policy choices (Acemoglu, 2003).

To our knowledge, ours is the first paper to study legislative bargaining in which the size of rents depends on the composition of the winning coalition.[10] There are other applications of non-cooperative games that consider externalities in bargaining or an endogenous pie, outside the scope of this paper (see Ray and Vohra (2015) or Dasgupta and Maskin (2007)). For instance, in Stole and Zwiebel (1996) bargaining is centralized by the firm, in which its "owner" splits the surplus with potentially productive workers, one by one, with an exogenously given bargaining power; Manzini (1999) models a firm which bargains with an union that can harm the firm's profitability. Similarly, Calvert and Dietz (2006) and Cardona and Rubí-Barceló (2014) consider consumption externalities in the bargaining stage. The latter shows that these externalities affect ex-ante investment, leading to inefficient outcomes.[11]

Baranski (2016) is the paper closest to ours. Players make costly contributions to a common surplus after having bargained over its division, thus the size of rents

[9]In his classical work, Riker (1962) poses that bargaining games with zero sum games must only feature minimum winning coalitions in equilibrium. Although this has been disputed since Shepsle (1974) and it does not hold empirically, there are few papers that can account for larger-than-minimum winning coalitions. That is, in single shot games, these larger winning coalitions can be explained with "open bargaining rules" that allow for amendments (for instance, Fréchette et al., 2003), while in dynamic games, unanimity can sometimes be achieved in steady state (for instance, in Baron and Bowen, 2018 larger than minimal coalitions could be an equilibrium outcome). Similarly with endogenous status quo, like Anesi and Seidmann (2015).

[10]Eraslan and Merlo (2017) consider a model in which players are heterogeneous with respect to the potential surplus they bring to the bargaining table, and thus the size of the pie depends on the (random) identity of the agenda setter.

[11]On the same lines, in Harstad (2005), ex-ante investment and, therefore, the size of the pie, diminish with the majority rule (furthermore, Harstad (2005) follows Riker (1962) in modeling legislative bargaining, and restricts attention to minimum winning coalitions).

is also endogenous to the agenda setters' equilibrium proposals. Differently, in Baranski (2016) players receive an endowment and are identical when negotiations begin. Output is the result of joint production, and bargaining is over equity shares instead of quantities. While some results are qualitatively similar to ours (e.g. winning coalitions are typically formed by two types of players), we delve deeper in two directions: we allow for non-minimal winning coalitions and for agents that can decrease rents. Additionally, we characterize efficiency and show that output losses are increasing in patience.

In most papers, policy making takes place exclusively within formal institutions, disregarding informal channels of influence. An exception is Scartascini and Tommasi (2012), where political actors can choose to play in the legislative arena, or outside of it.[12] If they stay outside parliament, they become active in the informal arena and they channel their demands through mobilizations, riots, strikes, etc. Protests are placated with transfers from the formal institution. The authors focus on the long run determinants of institutionalization of policy making, understood as the fraction of actors choosing the formal arena. Contrary to Scartascini and Tommasi (2012), in our paper all demands are channeled inside the parliament, the size of the pie depends on the winning coalition, and the legislative game is repeated until there is an agreement. Also, we allow for positive and negative actions, which can take place simultaneously.

Other studies on political actions outside the parliament focus on the causes of protests, broadly defined. Ray and Esteban (2017) discuss how excluded factions (e.g. ethnic groups) can cause conflict and retaliation. Moreover, they link conflict with inequality, lower economic activity and development. In terms of our setup, the exclusion of an ethnic group from the winning coalition can backlash into conflict. Edmond (2013) is a recent example of theoretical work on the coordination aspects of protesting, emphasizing (weak) institutional quality as a catalyst for protesting. Battaglini (2017) focuses on whether protests (or petitions) have the power of the wisdom of the commons in influencing policy makers through aggregation of preferences.

The rest of the paper is organized as follows. Section 2 describes the environment, and defines the equilibrium concept. Section 3 characterizes equilibria, and section 4 presents the main results and provides some comparative statics. Section 5.1 solves for the decision to commit to become active. Section 6 concludes, and an appendix collects all proofs.

[12] The legislative bargaining game has one round, equivalent to imposing $\delta = 0$ in our setup.

2 Model

We consider an economy with n districts represented by n legislators who have to decide how to divide aggregate resources, $\tilde{Y}$. Following Baron and Ferejohn (1989), legislators bargain over the distribution of resources using closed rules (i.e., no amendments) with equal probabilities of recognition and discounting. From the set of legislators N with $|N| = n$, one is randomly chosen to make a proposal $x \in X \subset R^n$, where X is the set of all proposals that satisfy the budget constraint. That is, a proposal assigns $x_j \geq 0$ to each district $j \in N$, represented by a legislator who is also indexed with j, such that $\sum_j x_j \leq \tilde{Y}$. Let the voting rule q be such that if a (super) majority of $n/2 < q \leq n$ votes to approve the proposal, resources are distributed and the game is over. If the proposal is not approved, the game begins again, and there can be an infinite number of sessions. We assume $u_j(x) = x_j$ for all j, and all players discount the future with $0 < \delta < 1$.

Before the legislature convenes, districts' types are drawn, and remain fixed throughout the game. Districts are either "active", if they can affect the pie $\tilde{Y}$, or "passive" otherwise. There are r active districts, out of which r^+ are "productive", if they may increase rents, and r^- are "destructive", if they may decrease them. In terms of the model primitives, if and only if a legislator from a productive district votes in favor of the proposal, then aggregate resources increase by η. If and only if a legislator from a destructive district votes against the proposal, then there is a reduction of aggregate resources by η.[13] We interpret these changes in aggregate output as cooperation and retaliation.

All legislators who support the proposal are considered to be in the winning coalitions. Thus, productive districts in the winning coalition cooperate, while destructive ones outside the winning coalition retaliate.[14] In Section 5 we consider model extensions that address this assumption. First, we endogeneize districts' types by giving them a choice to commit to their types before the legislature convenes, and show that those districts that have this option will excercise it. Second, we show that when legislators are given a choice to cooperate or retaliate our results follow under minor additional assumptions.

[13] Restricting output changes to be of the same magnitude for productive and destructive districts simplifies the characterization of equilibrium, see lemma 2. As we will argue in section 4.1 after characterizing equilibria, assuming instead asymmetries, such that e.g. $\eta^- > \eta^+$, would result in retaliating districts' continuation value being greater, and them being more likely to be called in the winning coalition.

[14] The relationship between transfers an actions that affect output is documented in different strands of the political economy literature (as bribing, ear-marked spending, etc.) and it is specially salient in the papers on conflict, even in developed countries. For instance, see Gillezeau (2015) on the effect of anti-poverty spending on the abatement of the 1960s riots in the U.S.

Total rents to be distributed, $\tilde{Y}$, are given by an exogenous pie Y plus or minus the production or destruction of the active legislators (feasibility requires that $n\eta < Y$). The presence of cooperation and retaliation introduces two innovations with respect to Baron and Ferejohn (1989): first, the resources to be distributed, $\tilde{Y}$, are endogenous and depend on the profile of districts' votes. Second, ex ante payoffs are not necessarily the same across districts' types, even if they have the same probability of being agenda setters.

2.1 Strategies and Equilibrium concept

At every node t, each player is deciding a proposal strategy, in case she is the agenda setter, or a voting strategy otherwise. Because we are interested in history independent equilibria, at every node t the available actions and strategies must be the same. Hence, we drop t from all notation that follows.

A pure proposal strategy for an agenda setter, $s_j \in X$, determines how much to offer to each of the N districts, indexed by j. A pure voting strategy for all legislators j is defined by $a_j : X \to \{\texttt{yes}, \texttt{no}\}$. Following Baron and Ferejohn (1989), we assume that legislators will vote to accept the proposal when indifferent, i.e. we prevent them from mixing between `yes` and `no` strategies.

A proposal in mixed strategies is characterized by a probability distribution over feasible pure strategy proposals, $\pi_j(s_j)$, such that $\pi_j \geq 0$, and $\int_{s_j \in X} \pi_j(s_j) = 1$ for all j. A subgame-perfect Nash equilibrium specifies the equilibrium strategies for any player, at any node. Stationarity implies that the equilibrium strategies are the same in every node, up to the agenda setter's type.[15] Since a pure strategy is a degenerate mixed strategy, we define our equilibrium only in terms of the latter, such that the pair $\sigma_j = (\pi_j, a_j)$ is a mixed (stationary) strategy.

Definition 1 (Stationary Subgame-Perfect Nash Equilibria)**.** The n-tuple $(\sigma_j^*)^n$ is a stationary subgame perfect Nash Equilibria, for all $j \in N$, if

$$u_j(\sigma_j^*, \sigma_{-j}^*) \geq u_j(\sigma_j, \sigma_{-j}^*),$$

for all σ_j and for all σ_{-j}^*.

We focus on symmetric stationary equilibria. A direct implication of symmetry is that the continuation values of all the legislators of the same type are identical,

[15] Note that this equilibrium concept is analogous to a Markov Equilibrium in which the only state variable is the agenda setter's type (which is independent of the previous round or period's state).

and have the same set of possible strategies. In terms of notation, this allows us to replace the j indexes with an index for the legislators' types. Let i index the agenda setter's type and the supra index k the floor-legislators' types. i and k refer to whether a legislator (agenda setter or floor legislator) comes from a passive, retaliating or cooperating district, i.e. $i, k \in \{0, -, +\}$ respectively. In particular, under symmetry, all districts of type k that are offered a positive payoff should receive the same amount, $x^k(i)$. Thus we can characterize pure strategy proposals, s_i, by how much to offer ($x^k(i)$ for all i and k), to how many legislators ($m^k(i)$ for all i and k), such that $\sum_k m^k(i)x^k(i) \leq \tilde{Y}$. More generally, with mixed strategy proposals, $m^k(i)$ denotes the *expected* number of legislators of type k to whom an agenda setter of type i offers a positive payoff.[16]

A direct implication of stationarity is that any agenda setter, following any history of the game, would solve the same optimization problem. That is, an agenda setter of type i chooses a proposal π_i that maximizes her objective function $\tilde{Y}(\pi_i, a_i) - \sum_k m^k(i)x^k(i)$ subject to participation constraints, feasibility constraints and the resource constraint.

The participation constraints require that the proposal made by the agenda setter $i \in \{0, +, -\}$ induces at least $q - 1$ floor legislators to vote `yes`. In other words, these floor legislators obtain at least the value of waiting, which is the continuation value δv^k, i.e., $x^k(i) \geq \delta v^k$. The feasibility constraints require that (i) no more than the available active districts are offered to be in the coalition and (ii) at least the minimum amount of required active districts are offered to be in the coalition. Suppose the agenda setter comes from a productive district, then the former constraint for floor legislators from productive districts is $m^+(+) \leq r^+ - 1$ and the latter is $m^+(+) = \max\{0, q + r^+ - n - 1\}$. Finally, the resource constraint captures how the profile of expected votes determines the pie $\tilde{Y}$. Any solution to the maximization problem, and the continuation values, is then a symmetric stationary equilibrium.

3 Analysis

We begin the analysis with the characterization of the set of winning coalitions that must be considered in equilibrium. A minimum winning coalition is one in which $m^+(i) + m^-(i) + m^0(i) = q - 1$, i.e. exactly $q - 1$ legislators plus the agenda setter vote `yes`. Larger-than-minimal winning coalitions might arise in equilibrium

[16] For simplicity we omit the argument $\pi_{i,j}$ of the underlying proposal in mixed strategy.

if the benefit of adding a district to the coalition outweighs its cost. Note that no agenda setter will consider winning coalitions in which "additional legislators" come from passive districts when they have positive continuation values. Doing so would suppose a cost for the agenda setter with no gain. Thus, we are led to the following lemma:

Lemma 1. If $\sum_{k\in\{0,+,-\}} m^k(i) > q-1$, and $v^0 > 0$, then $m^0(i) = 0$ for all i. That is, when an agenda setter considers larger-than-minimal winning coalitions, all members, except perhaps the agenda setter, come from active districts.

Proof. All proofs are in the appendix. □

Note that if $v^0 = 0$, there is a large number of "trivial" larger-than-minimal winning coalitions in which passive districts are offered 0 and vote `yes`. We conjecture, and later verify, that in equilibrium $v^0 > 0$. Under the conjecture, lemma 1 reduces the set of larger-than-minimum winning coalitions to be considered in equilibrium. This simplifies the maximization problem. In particular, the expected number of passive districts called into a minimal coalition is $q-1-m^+(i)-m^-(i)$ for all i. More generally, irrespective of the coalition, the expected number of passive legislators is $\max\{0, q-1-m^+(i)-m^-(i)\}$.

Given that the agenda setter's utility is decreasing in $x^k(i)$, constraints for $x^k(i)$ are always binding, and $x^k(i) = \delta v^k$, for all i and k. Since the agenda setter takes as given continuation values, her strategy is then reduced to choosing $m^+(i)$, and $m^-(i)$, i.e. the composition of her coalition. Moreover, any systematic difference between the expected number of retaliating and cooperating districts, must be due to systematic differences in their continuation values. The following lemma shows that both types of active districts have the same continuation values.

Lemma 2. For all $1 \leq r^+ \leq n$, $1 \leq r^- \leq n - r^+$, and for all $0 < \delta < 1$, it is always the case that $v^+ = v^-$.

Lemma 2 implies that, in equilibrium, the agenda setter is indifferent about the composition of her coalition as long as $m^+(i) + m^-(i)$ is constant.[17] Since productive and destructive districts have the same values, we distinguish districts only whether they are active or passive. We will denote active districts as $i, k = 1$, and passive districts we keep denoting as $i, k = 0$. Along the equilibrium path all legislators that receive at least their continuation value vote in favor of a

[17]Note that if r^+, or r^-, is zero then the problem can also trivially be cast in terms of $m^+(i) + m^-(i)$.

proposal. Thus, expected output is a function of the expected number of active legislators who would vote `yes` following a proposal from an agenda setter of type i, i.e. $\tilde{Y}$ is fully determined by the agenda setter's type and her choice of $m(i) \equiv m^+(i) + m^-(i)$.

Therefore, the problem of an agenda setter of type $i = 0, 1$ is given by

$$\begin{aligned} \max_{m(i)} \quad & \tilde{Y}(m(i)+i) - m(i)\delta v^1 - \max\{0, q-1-m(i)\}\delta v^0 \\ \text{s.t.} \quad & \tilde{Y}(m(i)+i) = Y - r^-\eta + (m(i)+i)\eta, \\ & m(i) \leq r - i, \\ & m(i) \geq \max\{0, q+r-n-i\}. \end{aligned} \tag{1}$$

The corresponding Lagrangian is given by

$$\begin{aligned} \mathcal{L}^i(m(i)) \;=\; & Y - r^-\eta + (m(i)+i)\eta - m(i)\delta v^1 - \max\{0, q-1-m(i)\}\delta v^0 \\ & +\overline{\lambda}(i)[r-i-m(i)] - \underline{\lambda}(i)[\max\{0, q+r-n-i\} - m(i)], \end{aligned}$$

where $\overline{\lambda}(i)$ and $\underline{\lambda}(i)$ are, respectively, the multipliers on the upper and lower bounds of $m(i)$. The first order condition of the agenda setter's problem, which reflects the effects of a marginal increase in the expected number of active districts willing to vote in favor of the proposal, is given by

$$\eta - \delta v^1 + \mathbb{1}_{m(i)<q-1}\delta v^0 - \overline{\lambda}(i) + \underline{\lambda}(i) = 0. \tag{2}$$

The indicator function shows that for minimum winning coalitions the agenda setter contemplates reducing the expected number of passive legislators increasing one-for-one the expected number of active legislators. A marginal increase in $m(i)$ affects payoffs for the agenda setter threefold: they increase by η as the increase in the expected number of active districts increases rents, are reduced by δv^1 reflecting the cost of added active districts, and they increase by δv^0 from cost savings from less passive districts. Thus, the first order condition for an interior equilibrium with a minimum winning coalition is characterized by $\eta - \delta v^1 + \delta v^0 = 0$.

For larger-than-minimal coalitions, the indicator function shows that the decision is on enlarging the coalition with new members from active districts. There are thus two changes in the agenda setter's payoff from a marginal increase in $m(i)$: they increase by η as the increase in the expected number of active districts increases rents, and are reduced by δv^1 reflecting the cost of these added active districts. Thus, the first order condition for an interior equilibrium with a larger-than-minimum winning coalition is characterized by $\eta - \delta v^1 = 0$.

We denote "corner-equilibria" those equilibria in which either $\underline{\lambda}(i) > 0$, $\bar{\lambda}(i) > 0$, or $m(i) = q - 1$, and "interior equilibria" those equilibria in which at least one type of agenda setter' choice is unconstrained, i.e., one for which $\underline{\lambda}(i) = \bar{\lambda}(i) = 0$, and $m(i) \neq q - 1$. Since in interior equilibria $m(i)$ generically will not be an integer, we are led to the following characterization of equilibria.[18]

Remark 1. Generically, interior equilibria are mixed-strategy equilibria, and corner equilibria are pure strategy equilibria.

Note that the second order conditions trivially hold due to the linearity of the objective function and the constraints. For the same reason, uniqueness is generally not warranted. In the case of corner equilibria, the constraints are solved for a unique value of $m(i)$, which guarantees uniqueness. In the case of interior equilibria, where multiplicity arises, the equilibria are payoff equivalent: v^0, v^1, and the expected number of active districts in a winning coalition, $E(m) \equiv \frac{n-r}{n} m(0) + \frac{r}{n}(m(1) + 1)$, do not depend on the particular mix that leads to $m(i)$ for $i = 0, 1$.[19]

For any v^0 and v^1, let the expected cost of forming a coalition of $m(i)$ active districts be: $e(m^i) = m(i)\delta v^1 + \max\{0, q - 1 - m(i)\}\delta v^0$. Let ρ^i be the probability of a type i legislator being called into a coalition by the agenda setter. Then, taking into account that the probability of recognition as an agenda setter is the same for all types, stationarity implies that, for $i = 0, 1$, we can write valuations as follows:

$$v^i = \frac{1}{n}(\tilde{Y} - e(m^i)) + \frac{n-1}{n}\rho^i \delta v^i. \tag{3}$$

From the equations above we can solve for v^i as a function of ρ^i, which depends on the coalitions proposed by legislators of type i, summarized in $m(i)$. That is, we need to calculate $\rho^i(m(0), m(1))$.

Given a pair of strategies $m(0)$ and $m(1)$, the construction of ρ^i for $i = 0, 1$ is mechanical. For instance, in the case of a passive legislator, we construct the probability that he is called into a coalition, ρ^0, as follows. With probability $r/(n-1)$, the agenda setter comes from an active district, hence, the probability that a passive legislator is called in the coalition depends on how many passive districts the active agenda setter needs to call, $\max\{0, q - m(1) - 1\}$, divided by

[18] Our distinction between pure and mixed-strategy equilibria relates to whether strategies call for an integer number of legislator of each type, or if there is randomization between different integers. In legislative bargaining, due to anonymity, strategies are usually mixing in the sense that there is randomization between legislators of a given type.

[19] Payoff equivalence for mixing strategies is shown in the proof of propositions 4 and 5.

the total number of available passive districts $(n-r)$. With probability $\frac{n-r-1}{n-1}$ the agenda setter is from a passive district, and the probability that a passive legislator is called in the coalition depends on how many passive districts the passive agenda setter needs to call, $\max\{0, q-m(0)-1\}$, divided the total number of available passive districts $(n-r-1)$. Similarly for the case of a legislator from an active district that is not the agenda setter. Hence,

$$\rho^0(m(0), m(1)) = \frac{r}{n-1}\frac{q-m(1)-1}{n-r} + \frac{n-r-1}{n-1}\frac{q-m(0)-1}{n-r-1}, \tag{4}$$

$$\rho^1(m(0), m(1)) = \frac{r-1}{n-1}\frac{m(1)}{r-1} + \frac{n-r}{n-1}\frac{m(0)}{r}. \tag{5}$$

Any stationary subgame-perfect Nash equilibrium must solve the system of equations (2), and (3) for all $i \in \{0,1\}$, with ρ^i given by (4) and (5). We now show that that to form a winning coalition active districts are more expensive than passive ones, i.e. $v^1 > v^0$, and gain intuition by considering a simple example that shows on the one hand the optimality of mixing strategies, and on the other the possibility of output losses.

Lemma 3. For all $1 \leq r < n$, and for all $0 < \delta < 1$, it is always the case that $v^1 > v^0$.

Example 1. Consider the case in which there is only one destructive active district, i.e., $r = r^- = 1$. For simplicity, let also assume that all legislators are patient, i.e., $\delta \to 1$. The only non-trivial choice is that of a passive agenda setter who must choose $m(0) = \rho^1$. Suppose that in equilibrium the active district is always included, i.e. $m(0) = 1$. Plugging $m(0) = 1$ in equations (3), we obtain $v^1 \to Y$ and $v^0 \to 0$.[20] The first order condition (2) is met if

$$\delta(v^1 - v^0) \leq \eta,$$

but with the strategy above $\delta(v^1 - v^0) \to Y$, which is greater than η (due to feasibility requiring $n\eta < Y$). Then, it is not optimal to have $m(0) = 1$, and $m(0) < 1$. Since there is a positive probability that the active district will be left out of the winning coalition, i.e. $1 - \rho^1 > 0$, there are expected output losses in equilibrium.

Similarly, suppose the passive agenda setter never includes the active district in the winning coalition, i.e. $m(0) = 0$. Following the same procedure as above,

[20] This result is not surprising, as $m(0) = 1$ gives the active district veto power, and thus can extract all the surplus as players become perfectly patient.

after some algebra we find

$$\left(n - \frac{q-1}{n-1}\right)(v^1 - v^0) = -\frac{q-1}{n-1}\left(Y - \eta\frac{q(n-1)}{n(q-1)}\right) < 0.$$

which contradicts lemma 3. Thus, $m(0) = 0$ is not optimal and the equilibrium is in mixed strategies, i.e. $0 < m(0) < 1$.

The example shows that, when patience is high, active districts must be left out of the winning coalition with positive probability, such that their continuation values decrease enough for the agenda setter to find it profitable to include them in a winning coalition. This creates inefficient output losses in equilibrium.

4 Results

In what follows we restrict the analysis to q-supermajorities that exclude the unanimity rule. Since all legislators must receive their continuation value to approve a proposal, with $q = n$ they all have to be included in a winning coalition. Hence, they all have the same continuation value and the distinction between types disappears. In that case, equilibrium is the same as in Baron and Ferejohn (1989) with unanimity rule.[21]

Remark 2. With $q = n$ the equilibrium in this game is identical to Baron and Ferejohn (1989), and the expected payoff of all legislators is $\frac{1}{n}(Y + r^+\eta)$.

In light of lemma 1, all minimum winning coalitions that include more than q members are composed of active districts, except perhaps for the agenda setter. As a consequence, if there are less active districts than $q - 1 + i$, it must be the case that coalitions are minimal. Therefore, larger-than-minimal winning coalitions are a potential phenomena only when there is a relatively large number of active districts. Taking these issues into account, in proposition 1 we first determine the threshold for which larger-than-minimal winning coalitions are an equilibrium. Then we analyze the case of a relative low number of active districts in proposition 2, and finally we study the case of a large number of active districts in proposition 3.

Proposition 1 provides our first result. It establishes that, when allowing for retaliation and cooperation that may change the size of rents, larger-than-minimum winning coalitions are possible in equilibrium.

[21] Equal ex ante values are also the outcome in the limit as $\delta \to 0$.

Proposition 1. Winning coalitions are minimal if and only if $\delta \geq \delta_q$. If $r \leq q - 1 + i$, $\delta_q = 0$, for $i = 0, 1$.[22]

Relative to the voting rule, q, the number of active districts, r, and the potential change in output, η, the discount factor determines how costly it is to get a legislator's support. When the discount factor is large enough, only minimum winning coalitions can be sustained in equilibrium. Indeed, for high δ, $\delta \geq \delta_q$, since legislators give a relatively large weight to the future, their continuation values are large. Thus, the cost of adding a non-necessary legislator into the winning coalition is large as well. In this case, the agenda setter does not want to form a larger-than-minimal winning coalition. Conversely, for low δ, $\delta < \delta_q$, the legislators' continuation values are small, and the cost of including an extra active district in the coalition might be smaller than the output loss if excluded. Since the agenda setter acts as a residual claimant, she is willing to add a non-necessary active district, even though the cost of the coalition increases by δv^1, because rents increase by η, and so her utility increases by $\eta - \delta v^1$.[23]

Since Riker (1962), the literature on bargaining has tried to reconcile the theoretical prediction of minimum winning coalitions with evidence that larger-than-minimum coalitions are frequently observed.[24] Proposition 1 provides a rationale for larger than-minimum winning coalitions: they are an equilibrium if and only if for the agenda setter the cost of additional legislators is lower than the increase in rents from including them in the coalition.

The following proposition characterizes equilibria when there are so few active districts that there are no incentives to have larger-than-minimum winning coalitions.

Proposition 2. For $r \leq q - 1 + i$, there exists a unique $\bar{\delta}$ such that if and only if $\delta \leq \bar{\delta}$ then $m(i) = r - i$, for $i = 0, 1$.

From Proposition 1, $\delta_q = 0$ and so all coalitions are minimal. For $\delta \leq \bar{\delta}$, there exists a unique corner solution in which all active districts are offered their

[22]Note that δ_q only depends on i when $r = q$. In this case when $i = 0$, $\delta_q > 0$, but when $i = 1$, $\delta_q = 0$. To avoid cumbersome notation we have decided to drop i as a determinant of δ_q.

[23]Notice that the description of the equilibrium assures that the budget constraint is always satisfied in equilibrium. We can study feasibility under individual deviations in which an active legislator is offered his continuation value, and the proposal is approved without his vote (because initially there was a larger-than-minimal coalition). It can be shown that the budget constraint would still be always satisfied, with the agenda setter absorbing the reduction in resources. Moreover, a sensible alternative assumption rules out these deviations: that the agenda setter distributes x_j to district j if an only if (i) the proposal is approved, and (ii) legislator j voted `yes`.

[24]See Knight (2008) and references therein.

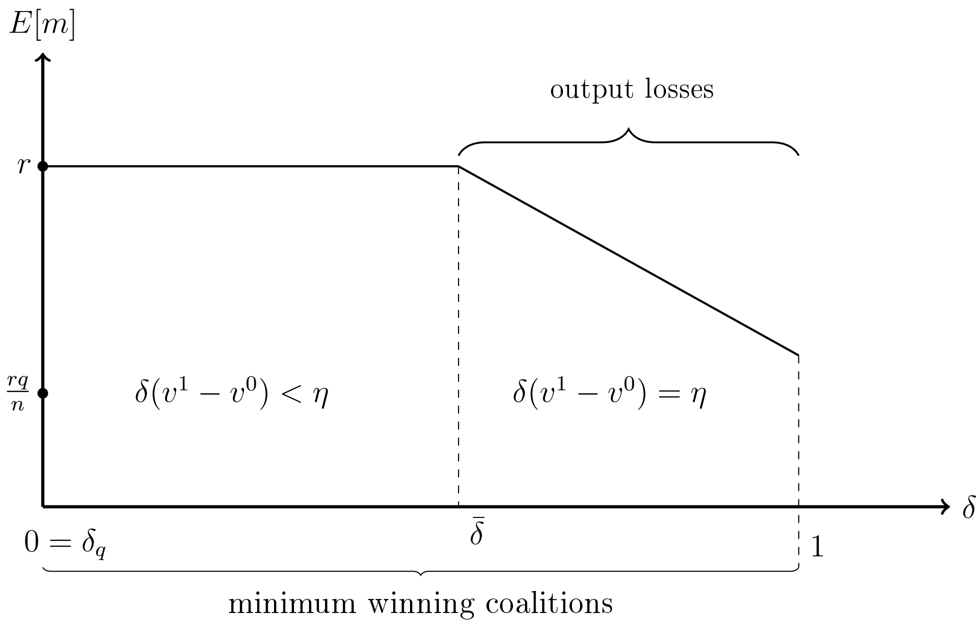

Figure 1: Equilibria when $r \leq q - 1 + i$

continuation value, they all vote yes and there is no output loss. When legislators are more patient, including them in the coalition is more costly. In this case, even though the agenda setter has the technology to avoid inefficiencies, she does not use it. She reduces the probability of calling active districts to lower their continuation value, up to the point in which she is indifferent between active and passive districts, i.e. $\delta(v^1 - v^0) = \eta$.

Thus, in equilibrium, some active districts might be left out of the coalition, as shown in figure 1. Therefore, proposition 2 presents our second result, that it is possible for output to be inefficient in equilibrium (which happens whenever $m(i) + i < r$). This result reflects the fact that in models of legislative bargaining with linear utility, the agenda setter's actions can be interpreted as if she only cared about the welfare of the winning coalition. Thus, if the cost of replacing a passive by an active one, is higher than the output gain, not all active districts will be called into the coalition. In contrast, a social planner that cared for aggregate social welfare would never exclude active districts, as this implies an inefficient loss of output.

The following proposition characterizes equilibria when the presence of a large number of active districts raises the possibility of having larger-than-minimum winning coalitions.

Proposition 3. For $i = 0, 1$, let $r > q - 1 + i$, then
(i) there exists a unique $\bar{\delta}$ such that if and only if $\delta \in [\delta_q, \bar{\delta}]$ then $m(i) = q - 1$.

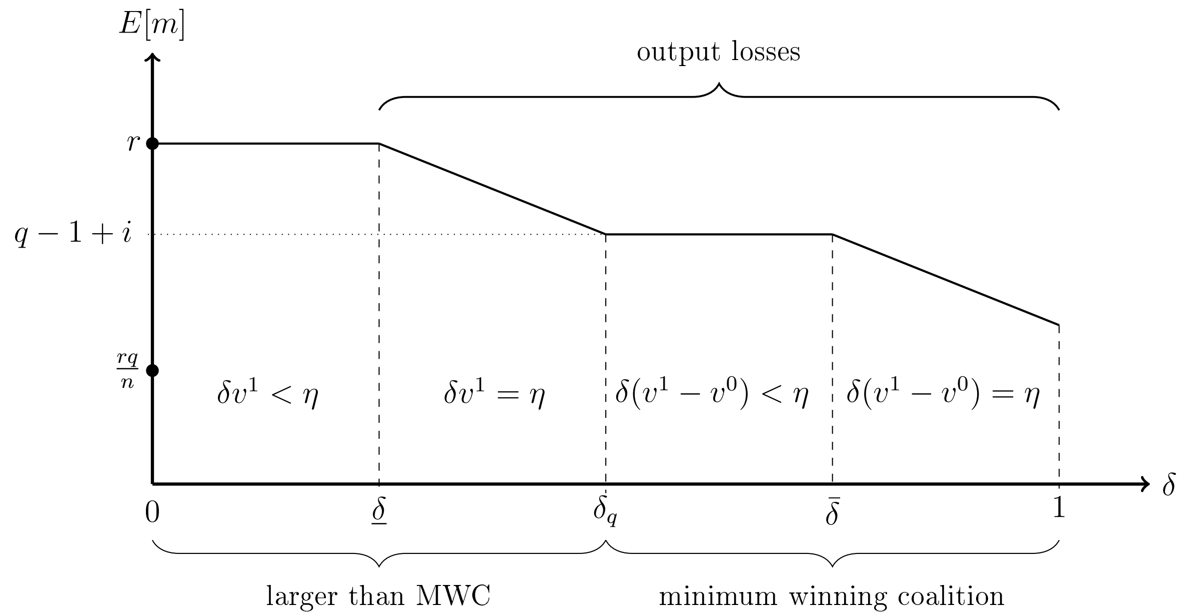

Figure 2: Equilibria when $r > q - 1 + i$

Otherwise, for $\delta > \bar{\delta}$ there exists an interior equilibria with $m(i) < q - i$.
(ii) there exists a unique $\underline{\delta}$ such that if and only if $\delta \in [0, \underline{\delta}]$, then $m(i) = r - i$. Otherwise, for $\delta \in (\underline{\delta}, \delta_q)$, there only exists interior equilibria with $q - i < m(i) < r - i$.

In (i) equilibria are similar as those in proposition 2, as the agenda setter only proposes minimum winning coalitions, and for high δ these imply output losses. In (ii), when $m(i) < r - 1$, some active districts are left out of the winning coalition and the equilibrium is also inefficient. When $m(i) > q-1$, the first order condition for an interior equilibrium is

$$\delta v^1 = \eta.$$

Similarly to proposition 1, for δ large enough, the benefits to include active districts beyond the minimum-winning coalition must be in balance with the costs. Therefore, for $\underline{\delta} < \delta < \delta_q$, there are mixed strategy equilibria with larger-than-minimum winning coalitions. If δ becomes so small that the benefits of including active districts beyond $q - 1$ is always larger than its cost, then there is a unique pure strategy equilibrium in which all active districts are called into the coalition. Figure 2 describes equilibria for (i) and (ii).

Corollary 1. (i) Legislators from active districts have a higher probability of being in the winning coalition. (ii) For all $1 \leq r < n$, and for all $0 < \delta < 1$, it is always the case that $v^0 > 0$.

Corollary 1 presents our final results. It shows that active districts are more likely to be called into a winning coalition. In fact, as shown in the proof, it is precisely their higher probability of being in the winning coalition that leads them to have higher ex ante payoffs. We also verify our conjecture that $v^0 > 0$. Thus, we can rule out trivial larger-than-minimum coalitions formed by calling into them legislators with zero continuation values.

4.1 Comparative Statics

The next propositions summarize some comparative static results. In particular, we are interested in the effect of parameter changes on the likelihood of having minimum winning coalitions and on output losses, defined as $(r - E(m))\eta$, where recall the definition of the expected number of active districts in a winning coalition, $E(m) = \frac{n-r}{n}m(0) + \frac{r}{n}(m(1) + 1)$. As expected, these results depend on the effect of parameter changes on the continuation values of active and passive districts.

Proposition 4. (i) An increase in the required supermajority q, increases the range of parameters for which minimum winning coalitions are an equilibrium outcome, and reduces the expected output losses. Additionally, active districts' payoffs increase, except when $r \leq q - 1 + i$ and $\delta < \bar{\delta}$, for $i = 0, 1$. (ii) Output losses (weakly) increase with δ.

An increase in the needed supermajority (weakly) raises the number of both types of legislators in the winning coalition. The increase in the expected number of active districts reduces output losses in equilibrium. The mechanism by which active legislators are (weakly) more likely to be part of the winning coalition depends on whether the number of active legislators is higher or lower than q.

First, consider the case of a large number of active legislators ($r > q - 1 + i$), depicted in figure 3. An increase in q has a direct effect on the region of δ for which there are minimum winning coalitions because more legislators are needed in these coalitions. This mechanical effect implies that both corresponding thresholds, δ_q and $\bar{\delta}$, decrease. On the contrary, q has no effect on $\underline{\delta}$, nor on $m(i)$ for $\delta \in (\underline{\delta}, \delta_q)$. Thus, the supermajority does not affect the equilibrium, in particular output losses, for $\delta \in [0, \delta_q)$. According to lemma 1, in the region of minimum winning coalitions, $\delta \in [\delta_q, \bar{\delta}]$, additional legislators needed to achieve the new supermajority come from active districts. Thus, in this region, an increase in q reduces output losses. Finally, for $\delta > \bar{\delta}$, $E(m)$ increases with q (since otherwise v^0 would increase more than v^1), thus also reducing output losses.

If there is a small number of legislators ($r \leq q-1+i$), the agenda setter's initial response to an increase in q is to call more passive districts into the minimum winning coalition (if $\delta \in [0, \bar{\delta})$, there is no other course of action as all active districts are already in the coalition). This increases passive districts' continuation values, giving the agenda setter incentives to increase the probability of calling active districts when using a mixing strategy. As a result, $\bar{\delta}$ increases with q, as does $E(m)$ for $\delta > \bar{\delta}$. Thus, an increase in q reduces output losses.

The different mechanisms that explain the decrease in output losses with greater supermajorities are then consistent with a non-monotonic effect of q on the continuation values of active districts. For $r > q-1+i$ some active districts are left out from the minimum winning coalition, thus increasing the needed supermajority increases the probability that they are called into it, rising their continuation value. For $r \leq q-1+i$, when $\delta < \bar{\delta}$, the effect of an increase in the supermajority reverses, as this now increases the probability that passive districts are called into the coalition. The increase in passive districts continuation values must be met, due to feasibility, by a decrease in active players' continuation values. Since when $r \leq q-1+i$, $\bar{\delta}$ is increasing in q, there is always a supermajority above which the continuation values of active districts is decreasing in q.[25]

Finally, note that contrary to results in other dynamic games (Piguillem and Riboni, 2015), more patience leads to more inefficient outcomes. Higher values of δ increase the continuation value of active districts inducing the agenda setter to call them less often into the winning coalition. This increases output losses.

Proposition 5. An increase in the potential damage η or in the number of active districts (either constructive or destructive, r^+ and r^- respectively) decreases the range of parameters for which minimum winning coalitions are an equilibrium outcome. Additionally, the effect on output losses is ambiguous.

An increase in η increases the agenda setter's incentives to include active districts in the winning coalition. In particular, the thresholds for larger-than-minimum winning coalitions including all active districts, $\underline{\delta}$, and for minimum winning coalitions, δ_q, increase. On the one hand, a larger η increases the expected number of active districts in the coalition ($E(m)$), on the other hand, it increases the damage of active districts left out from it. Hence, the effect on output losses, $(r - E(m))\eta$, is generally ambiguous. For example, with a large number of active districts ($r > q-1+i$), when mixed strategies are an equi-

[25]Formally, this threshold supermajority corresponds to q such that $\bar{\delta} = 1$. See (11) in the appendix.

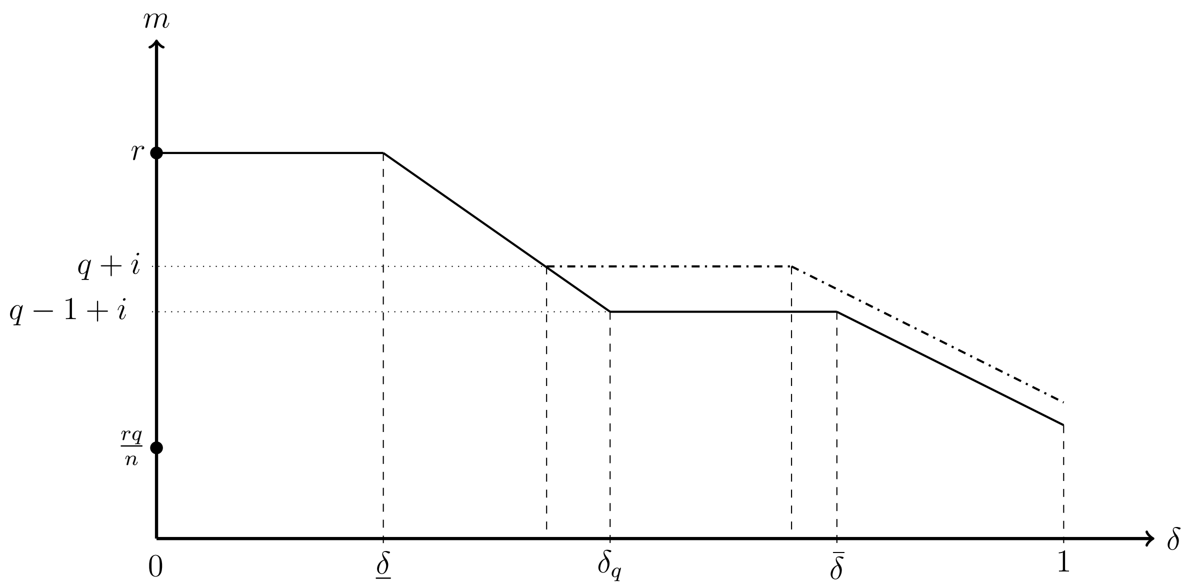

Figure 3: Comparative statics in q: $r > q + i$

librium with larger-than-minimum winning coalitions, an increase in η reduces output losses. With minimum winning coalitions that do not include all active districts, an increase in η increases output losses.[26]

Alternatively, this exercise could have been performed over the ratio $\frac{\eta}{Y}$, with similar results. That is, an increase in η can also be interpreted as a reduction of Y.[27] Thus, changes in η can be interpreted as comparing different economies in a cross-section, or the same economy over the business cycle (for the latter an increase in η reflects a fall in Y). Then, since in a recession (boom) more (less) active districts are included in the winning coalition, endogenous rents dampen output shocks, i.e. $\frac{d\tilde{Y}}{dY} < 1$.

Consider now an increase in the number of active districts. There are two effects. First, it gives the agenda setter incentives to increase the number of districts called into the winning coalition. Second, it reduces the probability of a given active district to be called into the winning coalition. These two effects have opposite effects on the continuation value of active districts. When $r > q - 1 + i$, $\underline{\delta}$, $E(m)$ for $\delta \in (\underline{\delta}, \delta_q)$, and δ_q increase with r^- or r^+. From the latter, minimum winning coalitions are part of the equilibrium for a smaller set of δ. While an increase in r^+ reduces output losses for $\delta \in (\underline{\delta}, \delta_q)$, the effect of r^- is ambiguous (the higher is r^- the more likely output losses increase). For $\delta \in [\delta_q, \bar{\delta}]$, an increase

[26]When $\delta > \bar{\delta}$, the effect is ambiguous. It can be shown that when $\delta \approx 1$, output losses increase with η.

[27]If both Y and η were to increase proportionally, it would result in proportional increases of v^k. Thus, there would not be an effect on thresholds or optimal strategies.

in r^- or r^+ increases output losses, and for $\delta > \bar{\delta}$ the effect is ambiguous (it can be shown that output losses increase with r^- or r^+ when $\delta \approx 1$).

We can conjecture how our equilibria would be affected if we lift some of our model's assumptions. If one of the types of active districts has a larger effect on rents (a higher threat point), e.g. $\eta^- > \eta^+$, then retaliators will have a higher probability than cooperators to be called into the winning coalition and thus higher ex ante values (corollary 1). Finally, we conjecture what would happen if one of the types of districts, e.g. the passive, are more likely to be selected as agenda setter. In this case, passive districts' continuation value would (weakly) increase (as in Eraslan (2002)). This would reduce the probability that they are called into a winning coalition, and thus unambiguously would increase expected output. Note that the converse would happen if active districts are more likely to be agenda setters.

5 Extensions

5.1 Choice of Becoming Active

In the setup of the game, productive districts cooperate at no cost if included in the winning coalition and destructive ones retaliate if they are not included. In this section, having characterized the equilibria for a given number of active districts, we endogeneize districts' commitment to behave in that way.

We assume that by default prohibitively large collective action costs prevent districts from committing to cooperate or retaliate as above. Each district can solve its collective action problem with an exogenous probability, and we assume this stochastic process to be i.i.d. across districts. An interpretation is that shocks can spur mobilization or agreement in societies (for instance, in Acemoglu and Robinson (2001) a shock may cause revolutions). We say that the collective action problem is solved with probability $\beta^+ + \beta^-$ and it is not solved with the remaining probability $1 - \beta^+ - \beta^-$. In particular, with probability β^+, district j has the option of committing to the productive strategy. Similarly, with probability β^- district j has the option of committing to the destructive strategy. Finally, with the remaining probability district j commits to take no action, regardless of the outcome of bargaining.[28] We will now show that all districts for which the

[28]Thus, parameters β^- and β^+ can be seen as measures of institutional quality, or as measures of the degree of discretion that districts have to shield regional output from national taxation, or to promote growth opportunities with spillovers.

collective action problem is solved, will choose to commit to their types.

With a bit of an abuse in notation, let's assume that r districts have the option to either become productive or destructive, and denote by $v^i(\cdot)$ ex ante payoffs as a function of the number of active districts. Without loss of generality, we consider the decision problem in one of these districts, that takes as given that the other $r-1$ districts will become active. Thus, this district is in effect comparing payoffs $v^1(r)$ and $v^0(r-1)$. Given that becoming active is assumed to be costless, it will be in the districts interest to do so whenever $v^1(r) > v^0(r-1)$. Note that the presence of output losses for some equilibria renders this condition non trivial.

Proposition 6. For all $1 \leq r \leq n$, and for all $0 < \delta < 1$, it is always the case that $v^1(r) > v^0(r-1)$.

We thus verify that all districts that have an option to become active will do so. The assumption that becoming active is costless allows to characterize this decision without having to find explicit expressions for $v^1(r)$ and $v^0(r)$. If instead we assume that the action is costly, then each district, upon observing how many districts managed to solve their collective action problems, would have to compare the expected gain from becoming active with the cost. Furthermore, if information is imperfect, such that each district only observes if they can solve their collective action problem, the expected gain, $E[v^1(r) - v^0(r-1)]$, depends on the distribution of r (which depends on parameters β^+ and β^-). Thus, the decision on becoming active requires knowing $v^i(r)$ for all i and r.[29]

Denote by z the cost of becoming active.[30] For small z, e.g. $z < \min_r[v^1(r) - v^0(r-1)]$, proposition 6 continues to hold, and all districts that have the option to become active will do so. Propositions 1, 2, 3, 4, and 5, and corollary 1 would hold as well.

5.2 Cooperation and Retaliation as Choices

The assumption that cooperators and retaliators are committed to cooperate when they are in the winning coalition and committed to riot when they are not was

[29] A microfoundation for actions with imperfect information is to have citizens (or a subgroup of them, such as public servants or scientists) in district i observe a noisy signal of the realization of a variable θ_i that summarizes institutional quality or growth opportunities in their district and decide non cooperatively whether to engage in destructive/productive action or not. If the mass of citizens choosing to act is larger than θ_i then the action is successful and we say that the district is active. See Edmond (2013) for a detailed analysis in an application to street protests.

[30] This might include legislators' ex ante expected reputational costs from later destroying resources or not increasing output when able to do so.

made for expositional clarity and serves to highlight the model's results. If productive/destructive actions are costless, we can allow legislators to choose whether to cooperate or retaliate by having a tie-breaking rule that does not favor the economy: Retaliators retaliate, and cooperators do not cooperate, if left out of the winning coalition.[31]

The model setup in section 2 would now include an additional stage in which legislators of type $i \in \{+, -\}$ decide whether to cooperate/retaliate, depending on this type and the state of the game. Then, an agenda setter is drawn, she makes an offer which is then voted by the legislature. If the proposal is passed, legislators decide whether to cooperate/retaliate and then transfers are made. If cooperators cooperate, they are paid the promised resources. Similarly if retaliators do not retaliate.

Corollary 2. In equilibrium, (i) retaliators who are in the winning coalition prefer to not retaliate, while those left out of the winning coalition choose to retaliate. Similarly, (ii) cooperators in the winning coalition choose to cooperate, while those left out of the winning coalition do not cooperate.

6 Conclusions

We introduce a simple, and arguably natural, assumption in Baron and Ferejohn's (1989) canonical model of legislative bargaining: some legislators have the ability to either "grease" or "sand" the wheels of policy-making. These legislators, if satisfied with the outcome of the bargaining, cooperate to increase output, rents, or resources available for taxation. Conversely, if unsatisfied, they may retaliate reducing output, rents, or the tax base. With this assumption, the pie to be distributed in the legislative bargaining game becomes endogenous, and determined by the composition of the winning coalition.

Given their ability to affect the level of aggregate resources, active districts are more likely to be called into a winning coalition than passive districts, thus the cost to buy them is larger. When the agenda setter is choosing the composition of her winning coalition, she trades off the higher cost of active districts against the increase in output they produce. Therefore, as patience increases, active districts eventually stop being called into the winning coalition with certainty. This produces output losses, as either the gains of including cooperating legislators are not realized, or retaliation takes place.

[31] We would need additional assumptions for our model results to hold if productive/destructive actions are costly.

When there is a relatively large number of active districts, larger-than-minimum winning coalitions are possible in equilibrium. This feature of our model resonates with the large empirical evidence on larger-than-minimum winning coalitions, and fills a gap in theoretical models of legislative bargaining, where only minimum winning coalitions are possible. In our model, larger-than-minimum winning coalitions lessens the trade-off between expropriation of minorities and decision-making costs (Buchanan and Tullock (1962); Harstad (2005)). With impatient agents and a large number of active districts, larger-than-minimum coalitions only include active districts and exclude the passive minority while increasing the size of rents.

The intuition from our model allows us to conjecture what would happen in a different institutional setting in which not all active districts have the same effect on rents. For example, it is claimed that transitions to democratization in Western Europe and Latin America were fostered by negative economic shocks that allowed for organizing revolutions (as in Acemoglu and Robinson (2001)). If we interpret that negative shocks have a larger effect on resources than positive shocks, then those who have the larger potential to affect resources, retaliators, would be more likely to be called into government.

The mechanisms highlighted in our model may be informative of policymaking more generally. For example, one may interpret retaliation as produced by ethnic conflict, and patience as inversely related to mandate duration. Hence, a dictator can reduce conflict by co-opting opposing ethnic groups that are not necessarily needed to govern, if these perceive that ascension to power is unlikely and thus have few demands. In transition to democracy, as expected mandates shorten, more conflict arises (see Ray and Esteban (2017)).

Our finding that an increase in the supermajority, or a reduction in the discount factor, increases efficiency has normative implications. Our model suggests that procedural rules should be made contingent such that voting on legislation is delayed (by e.g. requiring that more committees evaluate a proposal), or the effective supermajority be increased (e.g. by increasing the number of navette rounds in a bicameral legislature before a conference committee is convened), when there are a large number of active districts. This would reduce expected output losses in the presence of retaliators or cooperators, while having no effect on legislative rules when rents are exogenous.

Districts that have the opportunity to commit to the retaliating/cooperating strategies will do so in our setup. Such opportunistic behavior links our results to the literature on institutional strength (Scartascini and Tommasi (2012); Levitsky and Murillo (2009)). Districts only cooperate if they get transfers, incentivizing only conditional cooperation. A weak institutional setting, with large potential damage, many active districts, and/or very impatient agents sustains an equilibrium with systematic transfers to active districts. In turn, this leads to greater incentives to become an active member, weakening the institutional framework even further.

Our work provides the foundations for a dynamic game, in which a share of available resources can be used to invest in strengthening institutions, e.g. by reducing the probability that a district can engage in retaliating activities in the following period. Legislative bargaining can thus introduce persistence to output shocks. Similarly, if damages have permanent effects, a dynamic extension can be employed to study climate negotiations. We leave the analysis of such extensions for future work.

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7 Appendix

7.1 Proof of Lemma 1

Suppose that $m^+(i) + m^-(i) + m^0(i) > q - 1$, such that there is a larger-than-minimum coalition. Consider the alternative strategy in which $m^0(i)$ is reduced by Δm. As long as $m^+(i) + m^-(i) + m^0(i) - \Delta m \geq q - 1$, and $m^0(i) - \Delta m \geq 0$, the proposal is feasible, will be approved and the agenda setter's payoff increases in $\Delta m \delta v^0 \geq 0$. Thus, it must be the case that, when $v^0 > 0$, all equilibria with larger-than-minimum coalitions feature $m^0(i) = 0$.

7.2 Proof of Lemma 2

The proof proceeds by contradiction. Suppose $v^+ > v^-$. Then the agenda setter can increase her payoff by reducing m^+ by Δm, and increasing m^- by Δm, keeping $m^+ + m^-$ unaffected. This has no impact on resources to be distributed (excluded productive districts will not increase output by $\eta \Delta m$, but included destructive district will refrain from destroying resources by $\eta \Delta m$). And the change in the composition of the winning coalition increases the agenda setter's payoff by $\Delta m \delta(v^+ - v^-) > 0$. Thus, the agenda setter will try to replace productive by destructive districts as much as possible. If no agenda setter includes productive districts unless they are needed to reach a minimum winning coalition, that is, $m^+(i) = \max\{0, q + r^+ - n - \mathbb{1}_{i=+}\}$.

If $m^+(i) = 0$, then a destructive district as agenda setter would have the same surplus output as a productive one (at least when $m^-(i) \leq r^- - 1$, otherwise proof mirrors the case $m^+(i) = q + r^+ - n - \mathbb{1}_{i=+}$, see below), the same recognition probability, and would be called into a winning coalition with higher, positive, probability, $\frac{m^-(i)}{r^-} > 0 = \frac{m^+(i)}{r^+}$. Thus, it must be the case that $v^- \geq v^+$.

If $m^+(i) = q + r^+ - n - \mathbb{1}_{i=+}$, then a productive district as agenda setter would have a higher surplus output so we need to evaluate the value functions. These would be given by,

$$
\begin{aligned}
v^+ &= \frac{1}{n}\left[Y - (r^+ - (q + r^+ - n))\eta - r^-\delta v^- - m^0(+)\delta v^0 - (q + r^+ - n - 1)\delta v^+\right] \\
&\quad + \frac{q + r^+ - n - r^+/n}{r^+}\delta v^+, \\
v^- &= \frac{1}{n}\left[Y - (r^+ - (q + r^+ - n))\eta - (r^- - 1)\delta v^- - m^0(-)\delta v^0 - (q + r^+ - n)\delta v^+\right] \\
&\quad + \frac{n-1}{n}\delta v^-.
\end{aligned}
$$

Given that $m^0(+) = m^0(-)$ substracting we get

$$(v^- - v^+)\left[1 - \frac{\delta}{n} - \delta \frac{q + r^+ - n - r^+/n}{r^+}\right] = \delta \left[\frac{n-1}{n} - \frac{q + r^+ - n - r^+/n}{r^+}\right] v^-.$$

Since the terms in square brakes in the LHS and RHS are both positive, and $v^- \geq 0$, it must again be the case that $v^- \geq v^+$. Thus, we cannot have $v^+ > v^-$. Similar reasoning rules out $v^+ < v^-$, and we conclude that it must be the case that $v^+ = v^-$.

7.3 Proof of Lemma 3

From first order condition (2) it is immediate that, if $v^0 \geq v^1$, an agenda setter would never choose to have a passive district in her coalition when an active one is available. If $r \geq q$, no passive is called into the winning coalition, so the value of a passive legislator is just the recognition probability, $\frac{1}{n}$, times the proposer's payoff of a passive agenda setter. But an active agenda setter would have a larger surplus output (since she comes from an active district the proposer's payoff, conditional on the same voting majority, is higher), the same recognition probability, and would be called into a winning coalition with higher, positive, probability. Thus, it must be the case that $v^1 > v^0$. Consider now the case that $r < q$. A passive district then has positive probability of being called into the winning coalition. But, this probability is 1 for active districts and thus higher than for passive districts (proposer's payoff, conditional on the same voting majority, is higher for an active agenda setter). Therefore, it is also the case that $v^1 > v^0$.

7.4 Proof of Proposition 1

The threshold $\delta_q(r)$ is zero when even including all active districts the winning coalition is minimal. This is always the case when $r \leq q - 1$, and is also the case when $r = q$ and the agenda setter is from a active district.

When $r > q$ or $r = q$ and the agenda setter is from a passive district, the threshold $\delta_q(r)$ will be determined by solving the equilibrium under the assumption that $m(i) = q - 1$, and verifying that the agenda setter does not prefer to increase the expected number of active districts into the coalition by Δm. For this case, from equations (3), values must satisfy

$$v^0 = \frac{1}{n}\left[Y - r^-\eta + (q-1)\eta - (q-1)\delta v^1\right], \tag{6}$$

$$v^1 = \frac{1}{n}\left[Y - r^-\eta + q\eta - (q-1)\delta v^1\right] + \frac{n-1}{n}\left[\frac{n-r}{n-1}\frac{q-1}{r} + \frac{r-1}{n-1}\frac{q-1}{r-1}\right]\delta v^1 \tag{7}$$

From the second equation we can solve for v^1

$$v^1 = \frac{r\left(Y - r^-\eta + q\eta\right)}{nr - \delta(n-r)(q-1)}. \tag{8}$$

Whenever $\delta v^1 > \eta$, the agenda setter will be unwilling to increase by $\Delta m > 0$ the expected number of active districts into the coalition, as this would reduce her payoff by $\Delta m(\delta v^1 - \eta) > 0$. Thus the agenda setter form a minimum winning coalition calling $q-1$ active districts into it. Thus, δ_q is implicitly determined by $\delta_q v^1 = \eta$,

$$\eta = \delta_q \frac{r\left(Y - r^-\eta + q\eta\right)}{nr - \delta_q(n-r)(q-1)}. \tag{9}$$

Since then RHS of the last equation is increasing in $\delta_q(r)$, the coalition will be minimal when $\delta \geq \delta_q$. Note that it might be the case that $v^1|_{\delta=1} < \eta$, and thus $\delta_q > 1$ If $\delta_q \geq 1$, which might happen for high r^-, coalitions are always larger-than-minimum.

7.5 Proof of Proposition 2

To determine the threshold $\bar{\delta}$ we solve for a corner equilibrium with $m(i) = r - i$, and verify that the agenda setter does not prefer to reduce the expected number of active districts included in the coalition. To solve for the value functions, from equations (3),

$$v^0 = \frac{1}{n}\left[Y + r^+\eta - r\delta v^1 - (q-r-1)\delta v^0\right] + \frac{n-1}{n}\left[\frac{n-r-1}{n-1}\frac{q-r-1}{n-r-1} + \frac{r}{n-1}\frac{q-r}{n-r}\right]\delta v^0,$$

$$v^1 = \frac{1}{n}\left[Y + r^+\eta - (r-1)\delta v^1 - (q-r)\delta v^0\right] + \frac{n-1}{n}\delta v^1.$$

Where the last equation shows that in this case all active districts are included in the winning coalition with probability one. Solving we find

$$\begin{aligned} v^0 &= \frac{(Y+r^+\eta)(1-\delta)(n-r)}{n(n-r)(1-\delta)+r\delta(n-q)}, \\ v^1 &= \frac{Y+r^+\eta-(q-r)\delta v^0}{n-(n-r)\delta}. \end{aligned} \tag{10}$$

Note that $\frac{dv^0}{d\delta} < 0$. Since expected output is independent of δ, and feasibility implies $rv^1(\delta)+(n-r)v^0(\delta) = Y+r^+\eta$, it must be the case that

$$r\frac{dv^1}{d\delta}+(n-r)\frac{dv^0}{d\delta} = 0.$$

Thus, $\frac{dv^1}{d\delta} > 0$ and $\frac{d\delta(v^1-v^0)}{d\delta} > 0$. To show that $0 < \bar{\delta} < 1$ we note that $v^0|_{\delta=0} = v^1|_{\delta=0} = \frac{Y+r^+\eta}{n}$, while $v^0|_{\delta=1} = 0$, and $v^1|_{\delta=1} = \frac{Y+r^+\eta}{r}$, implying $v^1|_{\delta=1}-v^0|_{\delta=1} > \eta$. Thus, $\bar{\delta}$ is determined by

$$\begin{aligned} \bar{\delta}\left(v^1|_{\bar{\delta}}-v^0|_{\bar{\delta}}\right) &= \eta, \\ \frac{\bar{\delta}(Y+r^+\eta)}{n(1-\bar{\delta})+r\bar{\delta}}\left[1-\frac{1-\bar{\delta}+\bar{\delta}q/n}{1+\frac{r\bar{\delta}(n-q)}{(1-\bar{\delta})(n-r)}}\right] &= \eta. \end{aligned} \tag{11}$$

and for $\delta > \bar{\delta}$, the expected number of active districts that the agenda setter would choose satisfies $m(i) < r-i$, as the cost or including all active districts in the coalition is higher than the resource cost of excluding some of them.

7.6 Proof of Proposition 3

(i) Since we assume $\delta \geq \delta_q$, from proposition 1 we are only considering minimum winning coalitions. To determine the threshold $\bar{\delta}$ we solve for a corner equilibrium with $m(i) = q-1$ and verify that the agenda setter does not prefer to reduce the expected number of active districts included in the coalition. Equations (6) and (7) characterize v^0 and v^1, from which we get

$$v^1-v^0 = \frac{\eta}{n}+\frac{q-1}{n}\left[\frac{(n-r)}{r}+1\right]\delta v^1$$

From (8) we have that $\frac{dv^1}{d\delta} > 0$ which implies $\frac{d\delta(v^1 - v^0)}{d\delta} > 0$. The threshold $\bar{\delta}$ is characterized by

$$\eta = \bar{\delta}\left[\frac{\eta}{n} + \bar{\delta}\frac{(q-1)[Y - r^{-}\eta + q\eta]}{nr - (n-r)(q-1)\bar{\delta}}\right]. \tag{12}$$

Taking into account that $v^1|_{\delta=1} - v^0|_{\delta=1}$ might be lower than η, it might be the case that $\bar{\delta} > 1$. If $\bar{\delta} \geq 1$, then the equilibrium is always at a corner with $m(i) = q-1$. This will be the case when η is large enough, and for high values of r^{-}. When $\bar{\delta} < 1$, and $\bar{\delta} < \delta$, the agenda setter prefers to exclude some active districts from the minimum-winning coalition, and the equilibrium is interior.

(ii) To determine the threshold $\underline{\delta}$ we solve for a corner equilibrium with $m(i) = r - i$ and verify that the agenda setter does not prefer to reduce the expected number of active districts included in the coalition.

$$\begin{aligned} v^0 &= \frac{1}{n}\left[Y + r^{+}\eta - r\delta v^1\right], \\ v^1 &= \frac{1}{n}\left[Y + r^{+}\eta - (r-1)\delta v^1\right] + \frac{n-1}{n}\delta v^1. \end{aligned}$$

From the second equation we derive

$$v^1 = \frac{Y + r^{+}\eta}{n - \delta(n-r)}.$$

It is immediate that $\frac{dv^1}{d\delta} > 0$. An agenda setter will be willing to include all active districts in the coalition as long as the cost of doing so is lower than the damage they could produce on output. Thus, the threshold $\underline{\delta}$ is determined by $\underline{\delta}v^1 = \eta$,

$$\underline{\delta}\frac{Y + r^{+}\eta}{n - (n-r)\underline{\delta}} = \eta \implies \underline{\delta} = \frac{n\eta}{Y + (n - r^{-})\eta}. \tag{13}$$

When $\underline{\delta} < \delta < \delta_q$ the agenda setter will form a coalition with $q - 1 < m(i) < r - i$ active districts in expectation.

7.7 Proof of Corollary 1

It is straightforward that active districts have a higher probability of being in the winning coalition when $r > q-1$, and $\delta \in [0, \delta_q)$, as in this case passive districts are never called into a winning coalition. When $\delta \in [\delta_q, \bar{\delta}]$ such that we have corner equilibria including $m(i) = \max\{q-1, r-i\}$, if $m(i) = q-1$, active districts have a positive probability of being in the winning coalition while passive districts are never called into it, while if $m(i) = r - i$ an active district's probability of being in the winning coalition is 1, thus higher than for a passive district.

We are thus left with the case $\delta \geq \bar{\delta}$. To prove that active districts must have a higher probability of being in the winning coalition we proceed by contradiction and assume that this probability is the same for every district (also see proof of lemma 3). If this were the case, the probability of being in the winning coalition must be $\frac{q-1}{n}$. This implies

$$m(0) = \frac{rq}{n}, \qquad m(1) = \frac{rq}{n} - 1.$$

We now use equations (3) to estimate v^0 and v^1:

$$\begin{aligned} v^0\left(1-\delta\frac{q-1}{n}\right) &= \frac{1}{n}\left[Y - r^-\eta + \frac{rq}{n}\eta - (\frac{rq}{n})\delta v^1 - (q - \frac{rq}{n} - 1)\delta v^0\right], \\ v^1\left(1-\delta\frac{q-1}{n}\right) &= \frac{1}{n}\left[Y - r^-\eta + \frac{rq}{n}\eta - (\frac{rq}{n} - 1)\delta v^1 - (q - \frac{rq}{n})\delta v^0\right]. \end{aligned}$$

These equations imply

$$(v^1 - v^0)\left(1 - \delta\frac{q-1}{n} - \frac{\delta}{n}\right) = 0. \tag{14}$$

But for an interior solution, as must be the case when $\delta \geq \bar{\delta}$, first order condition (2) implies

$$\delta(v^1 - v^0) = \eta. \tag{15}$$

Equation (14) is generically inconsistent with (15), and would imply that if districts have the same probability of being in the winning coalition they should have the same continuation values, i.e. $v^1 = v^0$. Thus, this tells us that the source of higher ex ante payoffs for active districts is precisely their higher probability of being in the winning coalition.

To prove that $v^0 > 0$ we start by assuming $v^0 = 0$, such that equations (3) give

$$\begin{aligned} v^0 &= \frac{1}{n}[Y - r^-\eta + m(0)\eta - m(0)\delta v^1] = 0, \\ v^1 &= \frac{1}{n}[Y - r^-\eta + (m(1)+1)\eta] + \frac{n-r}{nr}m(0)\delta v^1, \end{aligned}$$

From (2) we have that in equilibrium either $\delta v^1 < \eta$ and $m(0) = m(1) + 1 = r$, or $\delta v^1 = \eta$. The latter case can immediately be discarded as it would imply, from value function for v^0, that $Y - r^-\eta = 0$ and by assumption $Y - n\eta > 0$.

Let's consider then that $\delta v^1 < \eta$. The value function for v^1 gives $v^1 = \frac{Y+r^+\eta}{n-\delta(n-r)}$. Replacing this in the value function for v^0 gives

$$v^0 = \frac{1}{n}[Y + r^+\eta]\left(\frac{n(1-\delta)}{n-\delta(n-r)}\right).$$

But this is positive if $\delta < 1$. Thus, we prove that $v^0 > 0$.

7.8 Proof of Propositions 4 and 5

We start by characterizing equilibria for the two types of interior equilibria: a) for minimum winning coalitions, $\delta > \bar{\delta}$, and b) for larger-than-minimum winning coalitions, $\delta \in [\underline{\delta}, \delta_q)$.

a) We expect to find multiple interior equilibria since we have a system of three equations, (3), and the indifference condition $\delta(v^1 - v^0) = \eta$, in four unknowns, v^0, v^1, $m(0)$, and $m(1)$. Using these three equations leads to a continuum of equilibria characterized by a relation between strategies, say $m(1) = f(m(0))$. This allows us to write $v^0(m(0))$ and $v^1(m(0))$, which from (3) are given by:

$$\begin{aligned} v^0(m(0)) &= \frac{\frac{1}{n}\left[\tilde{Y}\left(m(0)\right) - e\left(C_m^0\right)\right]}{1 - \frac{n-1}{n}\rho^0(m(0))\delta} \\ v^1(m(0)) &= \frac{\frac{1}{n}\left[\tilde{Y}\left(f(m(0)\right) - e\left(C_m^1\right)\right]}{1 - \frac{n-1}{n}\rho^1(m(0))\delta} \end{aligned} \tag{16}$$

Because strategies $m(0)$ and $f(m(0))$ satisfy $\delta(v^1 - v^0) = \eta$, for all feasible $m(0)$ we must have that

$$\frac{d\left[\tilde{Y}\left(m(0)\right) - e\left(C_m^0\right)\right]}{dm(0)} = \frac{d\left[\tilde{Y}\left(f(m(0)\right) - e\left(C_m^1\right)\right]}{dm(0)} = 0,$$

since agenda setters are indifferent with respect to the composition of their coalitions. We must also have that the total derivatives $\frac{dv^1(m(0))}{dm(0)} = \frac{dv^0(m(0))}{dm(0)}$ (to satisfy $\delta(v^1(m(0)) - v^0(m(0))) = \eta$). Using expressions (16), after some algebra, this implies

$$\frac{dv^0(m(0))}{dm(0)} = \frac{n-1}{n}\delta\frac{v^0}{1-\frac{n-1}{n}\rho^0\delta}\frac{d\rho^0}{dm(0)} = \frac{n-1}{n}\delta\frac{v^1}{1-\frac{n-1}{n}\rho^1\delta}\frac{d\rho^1}{dm(0)} = \frac{dv^1(m(0))}{dm(0)}.$$

Taking total derivatives for the probabilities of being called into the winning coalition, (4), and (5), and replacing above we get

$$\frac{v^0}{1-\frac{n-1}{n}\rho^0\delta}\left(-\frac{r}{n-r}\frac{df(m(0))}{dm(0)}-1\right)=\frac{v^1}{1-\frac{n-1}{n}\rho^1\delta}\left(\frac{df(m(0))}{dm(0)}+\frac{n-r}{r}\right)$$

If $\frac{df(m(0))}{dm(0)}=-\frac{n-r}{r}$ then $\frac{dv^1(m(0))}{dm(0)}=\frac{dv^0(m(0))}{dm(0)}=0$. Otherwise we can eliminate from both sides the term $\left(\frac{df(m(0))}{dm(0)}+\frac{n-r}{r}\right)$ and

$$-\frac{n-r}{r}\frac{v^0}{1-\frac{n-1}{n}\rho^0\delta}=\frac{v^1}{1-\frac{n-1}{n}\rho^1\delta}.$$

But this is absurd since the LHS is negative and the RHS is positive. Thus the only possible solution is that $\frac{df(m(0))}{dm(0)}=-\frac{n-r}{r}$, and v^0 and v^1 are independent of $m(0)$. The intuition for this result comes from the fact that these strategies give legislators the same ex ante probability of being called into the winning coalition, and thus the same ex ante value since the probability of being agenda setter is always $\frac{1}{n}$.

Given that all solutions feature the same ex ante values we can apply a refinement to have a system of four equations in four unknowns. We choose that expected output be independent of the identity of the agenda setter:

$$Y-(r^{-}-m(0))\eta=Y-(r^{-}-m(1)-1)\eta.$$

Using the indifference condition $\delta(v^1-v^0)=\eta$ to replace v^1 as a function of v^0 in equation (3) for $i=0$, and the feasibility constraint (which can be used instead of (3) for $i=1$) we get the following system of two equations in two unknowns

$$v^0\left(1-\frac{\delta}{n}\left(\frac{rq-nm(0)}{n-r}\right)\right)) \quad = \quad \frac{1}{n}\left[Y-r^{-}\eta\right] \tag{17}$$

$$nv^0 \quad = \quad Y-\left(\frac{r+\delta r^{-}}{\delta}-m(0)\right)\eta \tag{18}$$

b) The proof mirrors a), with the indifference condition now given by $\delta v^1=\eta$. From (3) for $i=1$ we can get the expression for v^1 as a function of $m(0)$ and $m(1)$. Imposing the condition $\delta v^1=\eta$ for a mixed strategy equilibrium gives a continuum of equilibria characterized by a relation between strategies, $m(1)=f(m(0))$. This allows us to write $v^1(m(0))$, which from (5) is given by:

$$v^1(m(0)) = \frac{\frac{1}{n}\left[\tilde{Y}\left(f(m(0)) - e\left(C_m^1\right)\right]\right.}{1 - \frac{n-1}{n}\rho^1(m(0))\delta}$$

A parallel reasoning as before tells us that both v^1, and the numerator of the expression above are invariant to changes in $m(0)$ as long as $\delta v^1(m(0)) = \eta$. Thus ρ^1 is independent of $m(0)$, which implies that, as before, $\frac{df(m(0))}{dm(0)} = -\frac{n-r}{r}$. As a corollary we have that v^0 is also independent of $m(0)$ ($\rho^0 = 0$ since passive districts are never called into a winning coalition when this is larger-than-minimum). We apply the same refinement that expected output be independent of the identity of the agenda setter.

Replacing the indifference condition, $v^1 = \frac{\eta}{\delta}$, into (3) for $i = 0$, and into the feasibility constraint, the latter results in

$$\begin{aligned}(n-r)v^0 + r\frac{\eta}{\delta} &= Y - r^-\eta + m(0)\eta \\ (n-r)\frac{1}{n}\left[Y - r^-\eta\right] + r\frac{\eta}{\delta} &= Y - r^-\eta + m(0)\eta \\ -\left[Y - r^-\eta\right]\frac{r}{n} + r\frac{\eta}{\delta} &= m(0)\eta. \qquad (19)\end{aligned}$$

We now continue the proof or our comparative static results with the following lemma, for which $E(m)$ is the expected number of active districts present in interior equilibria. Note that under our refinement, $E(m) \equiv m_0$.

Lemma 4. For the thresholds characterizing equilibrium types in proposition 3,

$$\begin{aligned}
&\text{i) } r > q-1: && \frac{d\delta_q}{dq} < 0, \quad \frac{d\delta_q}{d\eta} > 0, \quad \frac{d\delta_q}{dr^-} > 0, \quad \frac{d\delta_q}{dr^+} > 0,\\
& && \frac{d\underline{\delta}}{dq} = 0, \quad \frac{d\underline{\delta}}{d\eta} > 0, \quad \frac{d\underline{\delta}}{dr^-} > 0, \quad \frac{d\underline{\delta}}{dr^+} = 0,\\
& && \frac{d\bar{\delta}}{dq} < 0, \quad \frac{d\bar{\delta}}{d\eta} > 0, \quad \frac{d\bar{\delta}}{dr^-} > 0, \quad \frac{d\bar{\delta}}{dr^+} > 0, \quad (m(i) = q-1)\\
&\text{ii) } r \leq q-1: && \frac{d\bar{\delta}}{dq} > 0, \quad \frac{d\bar{\delta}}{d\eta} > 0, \quad \frac{d\bar{\delta}}{dr^-} \lesseqgtr 0, \quad \frac{d\bar{\delta}}{dr^+} \lesseqgtr 0. \quad (m(i) = r-i)
\end{aligned}$$

For interior equilibria,

$$\begin{aligned}
&\text{iii) } \delta > \bar{\delta}: && \frac{dE(m)}{d\delta} < 0, \quad \frac{dE(m)}{dq} > 0, \quad \frac{dE(m)}{d\eta} > 0, \quad \frac{dE(m)}{dr^-} > 0, \quad \frac{dE(m)}{dr^+} > 0,\\
&\text{iv) } \delta \in [\underline{\delta}, \delta_q): && \frac{dE(m)}{d\delta} < 0, \quad \frac{dE(m)}{dq} = 0, \quad \frac{dE(m)}{d\eta} > 0, \quad \frac{dE(m)}{dr^-} > 0, \quad \frac{dE(m)}{dr^+} > 0.
\end{aligned}$$

Note that i) is straightforward from (9), (12), and (13), and iv) is straightforward from (19). Note that (19) also allows to sign, when possible, the effect on output losses. The proof of iii) is a bit more complicated as there are two equations in the two unknowns, m_0 and v_0. Nevertheless, after some algebra to replace the derivatives of v_0 with respect to the different parameters we find the above results, which hold since $nm_0 > rq$ for all interior equilibria when $\delta > \bar{\delta}$ (otherwise it would not be the case that $v_1 > v_0$). For ii) the effect of q is straightforward from (11). For η this follows since we established that $\frac{d\delta(v^1 - v^0)}{d\delta} > 0$ in the proof of proposition 3. For r^- and r^+ the effects are ambiguous, as can be see from (11). The intuition is that an increase in the number of active districts has a negative effect on the continuation value of both active and passive districts. For the former due to the dilution of agenda setter rents, while for the latter due to lower probability of being in the winning coalition. Higher values of δ increase the continuation value of active districts inducing the agenda setter to call them less often into the winning coalition.

7.9 Proof of Proposition 6

We consider first the case with $r > q-1$ and $\delta \in [0, \underline{\delta})$, i.e. when all active districts are included in the winning coalition and this is larger-than-minimum. Since there are no output losses, the feasibility constraint implies that for all r

$$rv^1(r) + (n-r)v^0(r) = Y + r^+\eta. \tag{20}$$

Since, from lemma 3, $v^1(r) > v^0(r)$, equation (20) implies that $v^1(r) > \frac{Y + r^+\eta}{n} > v^0(r)$ for all r. Thus, it must be the case that $v^1(r) > v^0(r-1)$ for all r.

Next we consider the case $r > q-1$, and $\delta \in [\underline{\delta}, \delta_q)$, i.e. when there is a larger-than-minimum winning coalition but not all active districts are included in it. Since the agenda setter in these equilibria satisfies the first order condition (2) for an interior equilibrium, and $m(i) > q-1$, it must be the case that

$$\eta - \delta v^1(r) = 0.$$

Considering that the RHS of equation (20) now reflects an output loss, $Y - r^-\eta + m(r)\eta$, we infer that

$$v^1(r) = \frac{\eta}{\delta} > \frac{Y}{n} - \frac{r^- - m(r)}{n}\eta > v^0(r).$$

Thus, we find that $v^1(r) > v^0(r-1)$ for all r.

We now consider the case $\delta \in [\delta_q, \bar{\delta}]$ such that we have corner equilibria including $m(i) = \max\{q-1, r-i\}$ active districts. If $m(i) = r-i$ then output is efficient and we can apply the logic of the case with $r > q-1$ and $\delta \in [0, \underline{\delta})$. Thus, we consider that $m(i) = q-1$ and there are output losses. The value functions $v^1(r)$ and $v^0(r)$ for this case must satisfy equations (6) and (7). Thus,[32]

$$\begin{aligned} v^1(r) &= \frac{r\left(Y - r^-\eta + q\eta\right)}{nr - \delta(n-r)(q-1)}, \\ v^0(r-1) &= \frac{1}{n}\left[Y - (r^- - 1)\eta + (q-1)\eta - \frac{(q-1)\delta(r-1)\left(Y - (r^- - 1)\eta + q\eta\right)}{n(r-1) - \delta(n-r+1)(q-1)}\right], \\ &= \frac{\left(Y - r^-\eta + q\eta\right)\left[r - 1 - \delta(q-1)\right] - \delta(q-1)(r-1)\eta}{n(r-1) - \delta(n-r+1)(q-1)}. \end{aligned}$$

Thus,

$$\begin{aligned} v^0(r-1) &= v^1(r)\frac{r - (1+\delta(q-1))}{r}\frac{nr - \delta(n-r)(q-1)}{n(r-1) - \delta(n-r+1)(q-1)} \\ &\quad - \frac{\delta(q-1)(r-1)\eta}{n(r-1) - \delta(n-r+1)(q-1)} \\ &< v^1(r)\frac{r - (1+\delta(q-1))}{r}\frac{nr - \delta(n-r)(q-1)}{n(r-1) - \delta(n-r+1)(q-1)}. \end{aligned}$$

Where the inequality in the last step follows since $r > 1$. Finally, the term multiplying $v^1(r)$ in the last expression is smaller than one (this follows since $r > q-1$). Thus, it is always the case that $v^1(r) > v^0(r-1)$ for all r.

We are left now with the last case, $\delta \geq \bar{\delta}$, i.e. interior equilibria that imply minimum winning coalitions. Since the agenda setter in these equilibria satisfies the first order condition (2) for an interior equilibrium, and $m(i) < q-1$, it must be the case that

$$\eta - \delta(v^1(r) - v^0(r)) = 0. \tag{21}$$

The RHS of the feasibility constraint, (20), now is given by $Y - r^-\eta + m(r)\eta$. Using equation (21) to write the LHS of the feasibility constraint in terms of $v^0(r)$ we have

$$nv^0(r) + r\frac{\eta}{\delta} = Y - r^-\eta + m(r)\eta.$$

[32]In what follows we assume that the district evaluating the action would be a destructive type. The analysis is similar for a district with the option to be productive.

Using this last equation for r and $r-1$ and equation (21) we get[33]

$$n[v^1(r) - v^0(r-1)] = \eta\left(\frac{n-1}{\delta} - 1\right) + \eta(m(r) - m(r-1)).$$

Since the first term in the RHS is positive, if $m(r) \geq m(r-1)$, then $v^1(r) > v^0(r-1)$. We prove this by contradiction. If $m(r) < m(r-1)$, we can show that there are strategies that result in higher values v^0 and v^1, implying that a choice of $m(r) < m(r-1)$ is suboptimal. For this we consider strategies that imply $m'(r) = m(r-1)$, which is a feasible option. We write feasibility constraints for $m(r)$ and $m'(r)$, using (21) to substitute $v^1(r)$ in terms of $v^0(r)$,

$$\begin{aligned} nv^0(r) + r\frac{\eta}{\delta} &= Y - r^-\eta + m(r)\eta, \\ nv^{0'}(r) + r\frac{\eta}{\delta} &= Y - r^-\eta + m(r-1)\eta \end{aligned}$$

Subtracting these two equations we get

$$n[v^{0'}(r) - v^0(r)] = [m(r-1) - m(r)]\eta > 0.$$

Thus proving that $m(r) < m(r-1)$ is not optimal. This completes the proof that $v^1(r) > v^0(r-1)$ for all $r \geq 1$ and all δ.

7.10 Proof of Corollary 2

Retaliators who are in the winning coalition, i.e. they are offered a positive amount of resources if they suppport the proposal, prefer to not retaliate because otherwise they would not receive resources. Retaliators left out of the winning coalition are indifferent between rioting or not, hence under our tie-breaking-rule, in equilibrium they choose to retaliate. Cooperators in the winning coalition choose to cooperate, otherwise they would not receive any transfer. Finally, cooperators left out of the winning coalition are indifferent between cooperating or not hence, under the tie-breaking rule, in equilibrium they choose not to cooperate.

[33] Again, in what follows we assume that the district evaluating the action would be a destructive type. The analysis is similar for a district with the option to be productive.